MIDTERM EXAMINATION

1. (30 PTS) **True** or **False**? Explain or give counter-examples:
   (a) If $x(2n)$ is an energy sequence, then $x(n)$ is also an energy sequence.
   (b) If $x(n)$ is a periodic sequence then $x(2n+5)$ is also periodic.
   (c) Every causal system is relaxed.
   (d) Every time-invariant system is causal.
   (e) The series cascade of two time-variant linear systems can be LTI.
   (f) The system $y(n) = y(n-1) + x^2(2n)$, $y(-1) = 0$, $n \geq 0$, is time-invariant.
   (g) $\{z : \frac{1}{2} < |z| < 2\}$ is the ROC of an anti-causal stable LTI system.
   (h) The zero-state response of a system can be described using a convolution operation.
   (i) Doubling the sampling period of a signal doubles the number of samples.
   (j) The same constant-coefficient difference equation with boundary conditions can describe at most two systems.

2. (30 PTS) The following information is known about the behavior of a causal LTI system: (a) it has two modes at $\lambda_1 = 1/2$ and $\lambda_2 = 1/4$; (b) its response to $x(n) = (1/3)^n u(n)$ is an exponential sequence of the form $y(n) = \alpha^n u(n)$ with the largest possible energy value.
   (a) Determine the value of α.
   (b) Determine a constant coefficient difference equation for the system.
   (c) If $y(n)$ is applied to the input of the system, what would its response be?

3. (20 PTS) Consider a causal system described by the difference equation

$$y(n) = y(n-1) - \frac{1}{4}y(n-2) + \left(\frac{1}{4}\right)^n u(2n-1), \quad y(0) = a, \quad y(1) = b$$

   where a and b are real numbers.
   (a) Find a particular solution to the given system.
   (b) Find the complete solution in terms of a and b.
   (c) Given that $2b - a = -\frac{1}{2}$, compute the energy of $y(n)$. For what values of a and b, $y(n)$ has the smallest energy possible?

4. (20 PTS) Consider the following system

$$y(n) = \sum_{k=1}^{n} \lambda^{n-k} x(k)x(k+1), \quad n \geq 1$$
$$y(0) = 0$$

   where $\lambda \in (0, 1)$.

(a) Find an expression relating $y(n)$ to $y(n-1)$.

(b) Is the system causal? linear? stable?

(c) Is the system relaxed? time-invariant?