

22W-EC ENGR-113-LEC-1 Final Exam

MATTHEW FIORELLA

TOTAL POINTS

89 / 100

QUESTION 1

1 Q1 15 / 15

- ✓ - **0 pts** Correct
- **2 pts** Correct modes & sequences, partial correct ROC.
- **2 pts** Correct modes & ROC(right/left side), partial correct sequences.
- **5 pts** Correct modes & ROC(right/left side), wrong sequences.
- **8 pts** Correct modes, partial correct ROC/sequences.
- **10 pts** Correct ROC(right/left side), wrong modes/sequences
- **15 pts** No Attempt

QUESTION 2

Q2 15 pts

2.1 Q2.a 5 / 5

- ✓ - **0 pts** Correct
- **1 pts** Wrong phase+magnitude. Or no plot. Or didn't simply exponential representation.
- **2 pts** Correct DFT formulation with additional coefficient multiplier or calculation error.
- **3 pts** Correct DFT formulation.
- **4 pts** Wrong DFT formulation.

2.2 Q2.b 4 / 5

- **0 pts** Correct
- ✓ - **1 pts** Coefficient 1/4 or incomplete answer
- **1 pts** Correct $X(k-1)$ based on wrong $X(k)$
- **1 pts** Wrong $X(k-1)$ based on correct $X(k)$
- **2 pts** Wrong $X(k-1)$ based on wrong $X(k)$
- **2 pts** wrong inverse DFT formulation/calculation.
- **5 pts** No Attempt

2.3 Q2.c 5 / 5

- ✓ - **0 pts** Correct
- **1 pts** Wrong calculation
- **2 pts** Wrong circular convolution formulation. Or linear convolution with correct answer.
- **3 pts** Linear convolution with wrong answer.
- **5 pts** No Attempt/Completely Wrong

QUESTION 3

Q3 10 pts

3.1 Q3.a 5 / 5

- ✓ - **0 pts** Correct
- **2 pts** Partially Correct
- **5 pts** No Attempt/Completely Wrong

3.2 Q3.b 5 / 5

- ✓ - **0 pts** Correct
- **1 pts** Calculation error.
- **2 pts** Partially Correct
- **3 pts** Wrong inverse DTFT.
- **5 pts** No Attempt / Completely Wrong

QUESTION 4

Q4 15 pts

4.1 Q4.a 5 / 5

- ✓ - **0 pts** Correct
- **2 pts** Partially Correct

4.2 Q4.b 5 / 5

- ✓ - **0 pts** Correct
- **2 pts** Correct $h(n)$ based on wrong partial fractional expansion.
- **2 pts** Wrong $h(n)$ based on correct partial fractional expansion.

- **4 pts** Wrong $h(n)$ based on wrong partial fractional expansion.
- **5 pts** No Attempts.

4.3 Q4.c 4 / 5

- **0 pts** Correct
- ✓ - **1 pts** Incomplete answer / Calculation Error
- **2 pts** Identify eigenfunction.
- **3 pts** Wrong results using conv or CCDE with some correct procedures.
- **4 pts** Wrong direction with some work.
- **5 pts** No attempt

QUESTION 5

Q5 20 pts

5.1 Q5.a 2.5 / 4

- **0 pts** Correct
- **0.5 pts** $u(n)$ representation but not specifying $n \geq 1$ / calculation error.
- ✓ - **0.5 pts** Correct $h(n)$ based on wrong partial expansion.
- **1 pts** Wrong $h(n)$ based on wrong partial expansion.
- ✓ - **1 pts** Wrong $h_1(n)$ z transform.
- **1 pts** Wrong $h_2(n)$ z transform.
- **2 pts** No impulse sequence.

5.2 Q5.b 2 / 4

- **0 pts** Correct
- **1 pts** Calculation Error / Incomplete
- ✓ - **2 pts** Correct CCDE based on wrong $H(z)$
- **3 pts** Partially Correct with some work.
- **4 pts** No Attempt

5.3 Q5.c 4 / 4

- ✓ - **0 pts** Correct
- **1 pts** Inaccurate explanation.
- **3 pts** Correct justification based on wrong modes.
- **4 pts** No Attempt

5.4 Q5.d 3 / 4

- **0 pts** Correct
- **1 pts** Incorrect choice of $x(n)$ based on correct modes.
- **2 pts** Incorrect $x(n)$ based on wrong modes. / No $x(n)$ with correct modes.
- ✓ - **1 pts** One Incorrect modes.
- **2 pts** Two incorrect modes.
- **4 pts** No Attempt / Complete wrong

5.5 Q5.e 1 / 4

- **0 pts** Correct
- **2 pts** Incorrect initial conditions based on correct unilateral z-transform.
- **2 pts** Correct initial conditions based on wrong unilateral z-transform
- ✓ - **3 pts** Incorrect initial conditions based on wrong unilateral z-transform.
- **3 pts** Incorrect initial conditions with temporal approach
- **4 pts** No Attempt/Completely Wrong

QUESTION 6

Q6 15 pts

6.1 Q6.a 4.5 / 5

- **0 pts** Correct
- ✓ - **0.5 pts** Correct shape, wrong magnitude.
- **1 pts** Wrong $x_1(n)$
- **1 pts** Wrong $x_2(n)$
- **1 pts** Wrong $x_3(n)$
- **1 pts** Wrong $x_4(n)$
- **5 pts** No attempt

6.2 Q6.b 4 / 5

- **0 pts** Correct
- **1 pts** Series of convolution with correct filter sequence in sinc
- ✓ - **1 pts** Correct with wrong magnitude.
- **2 pts** Series of convolution with minor filter sequence error or didn't specify HPF/LPF Inverse DTFT
- **3 pts** Wrong with some work.

- **5 pts** No Attempt / Completely wrong.

6.3 Q6.c 5 / 5

✓ - **0 pts** Correct, as long as correct justification is made based on your right or wrong plot in your (a).

- **2 pts** Correct justification based wrong discrete frequency.

- **2 pts** Wrong justification based on correct discrete frequency.

- **4 pts** Wrong/no justification based on wrong discrete frequency.

- **5 pts** No Attempt / Complete wrong.

QUESTION 7

Q7 10 pts

7.1 Q7.a 5 / 5

✓ - **0 pts** Correct

- **2 pts** Partially Correct.

- **4 pts** Wrong with some work.

- **5 pts** No Attempt

7.2 Q7.b 5 / 5

✓ - **0 pts** Correct

- **2 pts** Partially Correct

- **4 pts** Wrong with some work.

- **5 pts** No Attempt

ECE113 Digital Signal Processing, Winter 2022

Final Exam

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Note to Proctors:

- This examination is closed book but two 8.5 x 11 cheat sheets (double sided) are allowed.
- Electronics (including calculators or smart phones) are not allowed. 3 hrs in duration.

Note to Students:

- Please write in dark pen or pencil so there is adequate contrast for readability, especially for remote students whose exams may be sent in by fax.
- Please explain or justify your work. Answers without justification will not receive full credit.

Points:

1. _____

6. _____

2. _____

7. _____

3. _____

4. _____

5. _____

Total: _____



FINAL EXAMINATION

1. (15PTS) Determine all possible sequences $x(n]$ with z -transform

$$X(z) = \frac{36z - 11}{12z^2 - 7z + 1}$$

$$X(z) = \frac{36z - 11}{12z^2 - 7z + 1} = \frac{36z - 11}{(3z - 1)(4z - 1)} = \frac{A}{3z - 1} + \frac{B}{4z - 1}$$

$$A = \left. \frac{36z - 11}{4z - 1} \right|_{z = \frac{1}{3}} = \frac{1}{\frac{1}{3}} = 3 \quad B = \left. \frac{36z - 11}{3z - 1} \right|_{z = \frac{1}{4}} = \frac{-2}{-\frac{1}{4}} = 8$$

$$X(z) = \frac{36z - 11}{(3z - 1)(4z - 1)} = \frac{3}{3z - 1} + \frac{8}{4z - 1}$$

~~$$X(z) = \frac{1}{z - \frac{1}{3}} + \frac{2}{z - \frac{1}{4}}$$~~

Possible ROC's: $|z| < \frac{1}{4}$, $\frac{1}{4} < |z| < \frac{1}{3}$, $|z| > \frac{1}{3}$

if ROC is $|z| < \frac{1}{4}$:

$$x(n) = -\frac{1}{3} u(-n) - 2 \left(\frac{1}{4}\right)^{n-1} u(-n)$$

$$\boxed{x(n) = -\left(\frac{1}{3}\right)^{n-1} u(-n) - 2\left(\frac{1}{4}\right)^{n-1} u(-n)}$$

if ROC is $\frac{1}{4} < |z| < \frac{1}{3}$:

$$\boxed{x(n) = 2\left(\frac{1}{4}\right)^{n-1} u(n-1) - \left(\frac{1}{3}\right)^{n-1} u(-n)}$$

if ROC is $|z| > \frac{1}{3}$:

$$\boxed{x(n) = 2\left(\frac{1}{4}\right)^{n-1} u(n-1) + \left(\frac{1}{3}\right)^{n-1} u(n-1)}$$



2. (15 PTS) Part (c) of this question is independent of the other two parts.

(a) Consider the sequence $x(n) = \delta(n) - 2\delta(n-2)$. Find the 4-point DFT $X(k)$ and plot it.

(b) Plot $X((k-1) \bmod 4)$ and find its 4-point inverse DFT.

(c) Evaluate the circular convolution between $x(n) = \{1, 2, 0, 1\}$ and $h(n) = \{-1, 1, 2, 1\}$.

(a) $x(n) = \delta(n) - 2\delta(n-2)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^3 (\delta(n) - 2\delta(n-2)) e^{-j\frac{2\pi}{4}kn}$$

~~$X(k) = \delta(k) - 2\delta(k-2)$~~ $X(k) = \delta(k) - 2\delta(k-2) e^{-j\pi k}$

$$X(k) = 1 - 2(\cos(\pi k) - j\sin(\pi k))$$

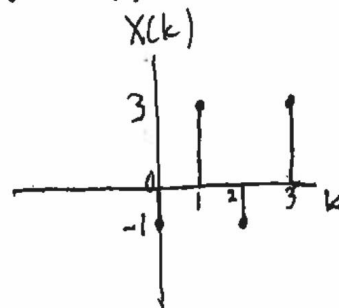
$$X(k) = 1 - 2\cos(\pi k)$$

$$X(0) = 1 - 2 = -1$$

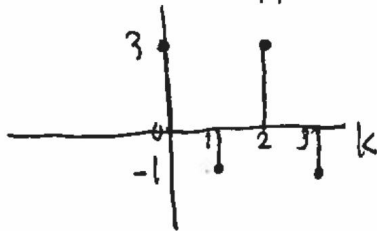
$$X(1) = 1 + 2 = 3$$

$$X(2) = 1 - 2 = -1$$

$$X(3) = 1 + 2 = 3$$



(b) $X((k-1) \bmod 4)$



$$x(n) = \frac{1}{4} \sum_{k=0}^3 X((k-1) \bmod 4) e^{j\frac{2\pi}{4}kn}$$

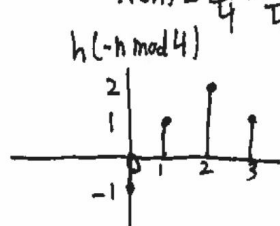
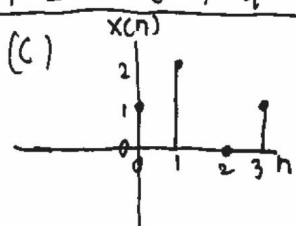
$$x(n) = \frac{1}{4}(3) + \frac{1}{4}(-e^{j\frac{\pi}{2}n}) + \frac{1}{4}(3e^{j\pi n}) + \frac{1}{4}(-e^{j\frac{3\pi}{2}n})$$

$$x(n) = \frac{3}{4} - \frac{1}{4}e^{j\frac{\pi}{2}n} + \frac{3}{4}e^{j\pi n} - \frac{1}{4}e^{j\frac{3\pi}{2}n}$$

$$x(n) = \frac{3}{4} - \frac{1}{4}e^{j\pi n} (e^{-j\frac{\pi}{2}n} + e^{j\frac{\pi}{2}n} - 3)$$

$$x(n) = \frac{3}{4} - \frac{1}{4}e^{j\pi n} (2\cos(\frac{\pi}{2}n) - 3)$$

$x(n) = \frac{3}{4} - \frac{1}{2}e^{j\pi n} \cos(\frac{\pi}{2}n) + \frac{3}{4}e^{j\pi n}$



~~$y(n) = (1)(-1) + (2)(1) + (0)(2) + (1)(1) = 2$~~
 ~~$y(n) = (1)(3) + (2)(-1) + (0)(1) + (1)(2) = 1$~~
 $y(0) = (1)(-1) + (2)(1) + (0)(2) + (1)(1) = 2$
 $y(1) = (1)(1) + (2)(-1) + (0)(1) + (1)(2) = 1$
 $y(2) = (1)(2) + (2)(1) + (0)(1) + (1)(-1) = 3$
 $y(3) = (1)(1) + (2)(2) + (0)(1) + (1)(-1) = 4$

$y(n) = \{2, 1, 3, 4\}$



3. (10 PTS) The two parts of this question are independent of one another.

(a) Find the DTFT of $x(n) = \cos(\frac{\pi}{6}n) + \sin(\frac{\pi}{4}n)$

(b) Calculate $\sum_{n=-\infty}^{+\infty} \frac{\sin^2(\frac{\pi}{6}n)}{n^2}$. (Hint: Using DTFT and Parseval's Relation may be helpful.)

$$(a) X(n) = \cos(\frac{\pi}{6}n) + \sin(\frac{\pi}{4}n)$$

$$X(n) = \frac{1}{2}(e^{j\frac{\pi}{6}n} + e^{-j\frac{\pi}{6}n}) + \frac{1}{2j}(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n})$$

$$X(n) = \frac{1}{2}e^{j\frac{\pi}{6}n} + \frac{1}{2}e^{-j\frac{\pi}{6}n} + \frac{1}{2j}e^{j\frac{\pi}{4}n} - \frac{1}{2j}e^{-j\frac{\pi}{4}n}$$

$$X(e^{j\omega}) = \pi \delta(\omega - \frac{\pi}{6}) + \pi \delta(\omega + \frac{\pi}{6}) + \frac{\pi}{j} \delta(\omega - \frac{\pi}{4}) - \frac{\pi}{j} \delta(\omega + \frac{\pi}{4})$$

$$X(e^{j\omega}) = \pi (\delta(\omega - \frac{\pi}{6}) + \delta(\omega + \frac{\pi}{6}) - j\delta(\omega - \frac{\pi}{4}) + j\delta(\omega + \frac{\pi}{4}))$$

$$(b) \sum_{n=-\infty}^{\infty} \frac{\sin^2(\frac{\pi}{6}n)}{n^2} = \sum_{n=-\infty}^{\infty} \left(\frac{\sin(\frac{\pi}{6}n)}{n} \right)^2 = \sum_{n=-\infty}^{\infty} \left(\frac{\pi}{6} \text{sinc}\left(\frac{\pi}{6}n\right) \right)^2$$

Parseval's Relation: $\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

$$X(n) = \frac{\pi}{6} \text{sinc}\left(\frac{\pi}{6}n\right) \Leftrightarrow X(e^{j\omega}) = \begin{cases} \pi, & |\omega| < \frac{\pi}{6} \\ 0, & \frac{\pi}{6} \leq |\omega| \leq \pi \end{cases}$$

$$\sum_{n=-\infty}^{\infty} \frac{\sin^2(\frac{\pi}{6}n)}{n^2} = \sum_{n=-\infty}^{\infty} \left(\frac{\pi}{6} \text{sinc}\left(\frac{\pi}{6}n\right) \right)^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \frac{1}{2\pi} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \pi^2 d\omega = \frac{1}{2\pi} (\pi^2(\frac{\pi}{6}) - \pi^2(-\frac{\pi}{6}))$$

$$= \frac{1}{2\pi} \left(\frac{\pi^3}{3} \right) = \frac{\pi^2}{6}$$

$$\sum_{n=-\infty}^{\infty} \frac{\sin^2(\frac{\pi}{6}n)}{n^2} = \frac{\pi^2}{6}$$



4. (15 PTS) Consider the relaxed and causal system

$$6y(n] - 5y[n-1] + y[n-2] = 2x[n-1].$$

- (a) Find the frequency response of the above system.
 (b) Find the impulse response $h[n]$ by inverting the frequency response in part (a)
 (c) If the signal $x[n] = \cos(\frac{\pi}{2}n + \frac{\pi}{4})$ is the input to the system, how much would its magnitude be attenuated at the output?

$$(a) 6y[n] - 5y[n-1] + y[n-2] = 2x[n-1]$$

$$6Y(e^{j\omega}) - 5e^{-j\omega}Y(e^{j\omega}) + e^{-j2\omega}Y(e^{j\omega}) = 2e^{-j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega})(2 - e^{-j\omega})(3 - e^{-j\omega}) = X(e^{j\omega})(2e^{-j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2e^{-j\omega}}{(2 - e^{-j\omega})(3 - e^{-j\omega})}$$

$$(b) H(e^{j\omega}) = \frac{2e^{-j\omega}}{(2 - e^{-j\omega})(3 - e^{-j\omega})} = \frac{A}{2 - e^{-j\omega}} + \frac{B}{3 - e^{-j\omega}}$$

$$A = \frac{2e^{-j\omega}}{3 - e^{-j\omega}} \Big|_{e^{-j\omega}=2} = 4 \quad B = \frac{2e^{-j\omega}}{2 - e^{-j\omega}} \Big|_{e^{-j\omega}=3} = -6 \quad H(e^{j\omega}) = \frac{2e^{-j\omega}}{(2 - e^{-j\omega})(3 - e^{-j\omega})} = \frac{4}{2 - e^{-j\omega}} - \frac{6}{3 - e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{2}{1 - \frac{1}{2}e^{-j\omega}} - \frac{2}{1 - \frac{1}{3}e^{-j\omega}}$$

$$h[n] = 2\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n]$$

$$(c) x[n] = \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right) = \frac{1}{2}\left(e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)} + e^{-j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}\right) = \frac{1}{2}e^{j\frac{\pi}{4}}e^{j\frac{\pi}{2}n} + \frac{1}{2}e^{-j\frac{\pi}{4}}e^{-j\frac{\pi}{2}n}$$

$$X(e^{j\omega}) = \frac{1}{2}e^{j\frac{\pi}{4}}(2\pi\delta(\omega - \frac{\pi}{2})) + \frac{1}{2}e^{-j\frac{\pi}{4}}(2\pi\delta(\omega + \frac{\pi}{2})) = \pi e^{j\frac{\pi}{4}}\delta(\omega - \frac{\pi}{2}) + \pi e^{-j\frac{\pi}{4}}\delta(\omega + \frac{\pi}{2})$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{2e^{-j\omega}}{(2 - e^{-j\omega})(3 - e^{-j\omega})} \left(\pi e^{j\frac{\pi}{4}}\delta(\omega - \frac{\pi}{2}) + \pi e^{-j\frac{\pi}{4}}\delta(\omega + \frac{\pi}{2}) \right) = \frac{2\pi e^{j\frac{\pi}{4}}e^{-j\frac{\pi}{2}}}{(2 - e^{-j\frac{\pi}{2}})(3 - e^{-j\frac{\pi}{2}})} + \frac{2\pi e^{-j\frac{\pi}{4}}e^{j\frac{\pi}{2}}}{(2 - e^{j\frac{\pi}{2}})(3 - e^{j\frac{\pi}{2}})}$$

$$Y(e^{j\omega}) = \frac{2\pi e^{-j\frac{\pi}{4}}}{(2 - e^{-j\frac{\pi}{2}})(3 - e^{-j\frac{\pi}{2}})} + \frac{2\pi e^{j\frac{\pi}{4}}}{(2 - e^{j\frac{\pi}{2}})(3 - e^{j\frac{\pi}{2}})} = \frac{4\pi \cos(\frac{\pi}{4})}{(2 - e^{-j\frac{\pi}{2}})(3 - e^{-j\frac{\pi}{2}})} = \frac{4\pi}{12} = \frac{\pi}{3} \left(\frac{1}{\sqrt{2}}\right) = \frac{\pi\sqrt{2}}{6}$$



The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that every sale, purchase, and payment must be properly documented to ensure the integrity of the financial statements. This includes recording the date, amount, and purpose of each transaction.

Furthermore, it is crucial to reconcile the accounts regularly to identify any discrepancies or errors. This process involves comparing the internal records with the bank statements and other external sources. Any differences should be investigated and corrected promptly to avoid any potential issues.

In addition, the document highlights the need for transparency and accountability in financial reporting. All stakeholders, including management and investors, should have access to accurate and timely information about the company's financial performance. This helps in making informed decisions and maintaining the trust of the stakeholders.

Finally, it is recommended to consult with a professional accountant or auditor to ensure compliance with all applicable laws and regulations. They can provide valuable insights and guidance on the best practices for financial management and reporting.

The second part of the document focuses on the implementation of internal controls to prevent fraud and mismanagement. It outlines the key components of a robust internal control system, including segregation of duties, authorization procedures, and regular monitoring.

Segregation of duties is a fundamental principle that ensures no single individual has control over all aspects of a transaction. By dividing responsibilities among different employees, the risk of errors and fraud is significantly reduced.

Authorization procedures are also essential to ensure that all transactions are approved by the appropriate management personnel. This helps in maintaining the accuracy and reliability of the financial data.

Regular monitoring and reporting are crucial to identify any weaknesses or areas for improvement in the internal control system. Management should conduct periodic reviews and audits to assess the effectiveness of the controls and take corrective actions as needed.

Overall, a strong internal control system is vital for the success and sustainability of any organization. It provides a framework for managing risks and ensuring the integrity of the financial reporting process.

In conclusion, the document provides a comprehensive overview of the key aspects of financial management and internal controls. It emphasizes the importance of accuracy, transparency, and accountability in all financial transactions and reporting. By following the guidelines and best practices outlined in this document, organizations can ensure the reliability and integrity of their financial statements and maintain the trust of their stakeholders.

Parts (b) (c) (d) (e) on back!

5. (20 PTS) A casual system is composed of the series cascade of two LTI systems with impulse response sequences given by $h_1(n) = (\frac{1}{2})^n u(2n)$ and $h_2(n) = (\frac{1}{4})^n u(n-1)$.
- Use the z-transform to determine the impulse response sequence and transfer function of the overall system.
 - Determine a description for the overall system in terms of a constant-coefficient difference equation. Denote its input and output sequences by $x(n]$ and $y[n)$, respectively.
 - Decide the stability of the system from the transfer function and its ROC.
 - What are the system's modes? Determine an input sequence such that only the largest mode appears at the corresponding output sequence.
 - Consider the difference sequence in part (a). Use the unilateral z-transform to determine initial conditions $y(-1)$ and $y(-2)$ such that only the smallest mode appears at the output of the system when the input is $x[n) = \delta[n)$.

$$(a) h_1(n) = (\frac{1}{2})^n u(2n) = (\frac{1}{4})^n u(2n) \Leftrightarrow \frac{1}{2} \sum_{k=0}^{\infty} X(e^{-j2\pi k/2} z^{1/2}) = \frac{1}{2} (X(z^{1/2}) + X(\frac{1}{z^{1/2}}))$$

$$\text{we can use } H_1(z) = \frac{1}{2} \left(\frac{1}{1 - \frac{1}{4}z^{-2}} + \frac{1}{1 + \frac{1}{4}z^{-2}} \right) = \frac{1}{2} \left(\frac{1 + \frac{1}{4}z^{-2} + 1 - \frac{1}{4}z^{-2}}{1 - \frac{1}{16}z^{-4}} \right) = \frac{1}{1 - \frac{1}{16}z^{-4}}$$

$$h_2(n) = (\frac{1}{4})^n u(n-1) = \frac{1}{4} (\frac{1}{4})^{n-1} u(n-1) \Leftrightarrow H_2(z) = \frac{1}{4} \frac{z^{-1}}{1 - \frac{1}{4}z^{-1}}$$

$$H(z) = H_1(z) H_2(z) = \frac{1}{4} \frac{z^{-1}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{16}z^{-4})}$$

~~Handwritten work showing partial fraction decomposition of $H(z)$ with various scribbles and corrections.~~

$$H(z) = H_1(z) H_2(z) = \frac{1}{4} \frac{z^{-1}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{16}z^{-4})} \text{ ROC } |z| > \frac{1}{4}$$

$$H(z) = \frac{A}{z - \frac{1}{4}} + \frac{B}{z - \frac{1}{16}}$$

$$H(z) = \frac{A}{z - \frac{1}{4}} + \frac{B}{z - \frac{1}{16}} \Rightarrow A = \frac{\frac{1}{4}z}{z - \frac{1}{16}} \Big|_{z=\frac{1}{4}} = \frac{\frac{1}{16}}{\frac{3}{16}} = \frac{1}{3}, \quad B = \frac{\frac{1}{4}z}{z - \frac{1}{4}} \Big|_{z=\frac{1}{16}} = \frac{\frac{1}{64}}{-\frac{3}{16}} = -\frac{1}{12}$$

$$H(z) = \frac{1}{3} \frac{1}{z - \frac{1}{4}} - \frac{1}{12} \frac{1}{z - \frac{1}{16}}, \text{ ROC } |z| > \frac{1}{4} \Leftrightarrow h[n) = \frac{1}{3} (\frac{1}{4})^{n-1} u(n-1) - \frac{1}{12} (\frac{1}{16})^{n-1} u(n-1)$$

Remaining parts on back

$$(b) H(z) = \frac{\frac{1}{4}z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{6}z^{-1})}, \text{ ROC: } |z| > \frac{1}{4}$$

$$\frac{Y(z)}{X(z)} = \frac{\frac{1}{4}z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{6}z^{-1})}$$

$$Y(z) - \frac{5}{6}z^{-1}Y(z) + \frac{1}{64}z^{-2}Y(z) = \frac{1}{4}z^{-1}X(z)$$

$$\boxed{y(n) - \frac{5}{6}y(n-1) + \frac{1}{64}y(n-2) = \frac{1}{4}x(n-1)}$$

$$Y(z)(1-\frac{1}{4}z^{-1})(1-\frac{1}{6}z^{-1}) = \frac{1}{4}z^{-1}X(z)$$

(c) The ^{system} function is stable because $|z|=1$ is in the ROC. In other words the unit circle is in the ROC, so we know the system is stable.

(d) The modes are $\frac{1}{4}$ and $\frac{1}{6}$.

To make only the largest mode appear we need to make the smallest disappear.

Need input with z transform $1-\frac{1}{6}z^{-1}$ so it will be cancelled out when multiplied by $H(z)$

$$1 - \frac{1}{6}z^{-1} \Leftrightarrow \boxed{\delta(n) - \frac{1}{6}\delta(n-1)}$$

$$(e) y(n) - \frac{5}{6}y(n-1) + \frac{1}{64}y(n-2) = \frac{1}{4}x(n-1)$$

$$Y^+(z) - \frac{5}{6}z^{-1}Y^+(z) + \frac{1}{64}z^{-2}Y^+(z) = \frac{1}{4}z^{-1}X^+(z) + \frac{1}{4}x(-1)$$

$$\cancel{X^+(z) = \delta(n)} \Rightarrow \cancel{X^+(z) = 1} \Rightarrow \cancel{x(n) = \delta(n)} \Rightarrow \cancel{x(n-1) = 0} \Rightarrow \cancel{X^+(z) = 1}$$

$$Y^+(z) - \frac{5}{6}z^{-1}Y^+(z) - \frac{5}{6}y(-1) + \frac{1}{64}z^{-2}Y^+(z) + \frac{1}{64}y(-1)z^{-1} + \frac{1}{64}y(-2) = \frac{1}{4}z^{-1}$$

$$\cancel{\frac{1}{64}z^{-2}Y^+(z)} \quad Y^+(z)(1-\frac{1}{4}z^{-1})(1-\frac{1}{6}z^{-1}) - \frac{5}{6}y(-1) + \frac{1}{64}y(-1)z^{-1} + \frac{1}{64}y(-2) = \frac{1}{4}z^{-1}$$

$$\text{we want } Y^+(z) = 1 - \frac{1}{6}z^{-1} \Rightarrow (1-\frac{1}{4}z^{-1}) - \frac{5}{6}y(-1) + \frac{1}{64}y(-1)z^{-1} + \frac{1}{64}y(-2) = \frac{1}{4}z^{-1}$$

$$\frac{5}{6}y(-1) + 1 + \frac{1}{64}y(-2) = 0$$

$$-\frac{1}{4}z^{-1} + \frac{1}{64}y(-1)z^{-1} = \frac{1}{4}z^{-1}$$

$$\boxed{y(-1) = 32, y(-2) = -576}$$

6. (15 PTS) The figure shows the DTFT of a real-valued sequence $x(n)$, which is fed into a cascade of two filters with modulators. The first filter is an ideal lowpass with cutoff frequency $\omega_c = \frac{\pi}{2}$ radians/sample. The second filter is an ideal highpass with cutoff frequency $\omega_h = \frac{3\pi}{4}$ radians/sample.

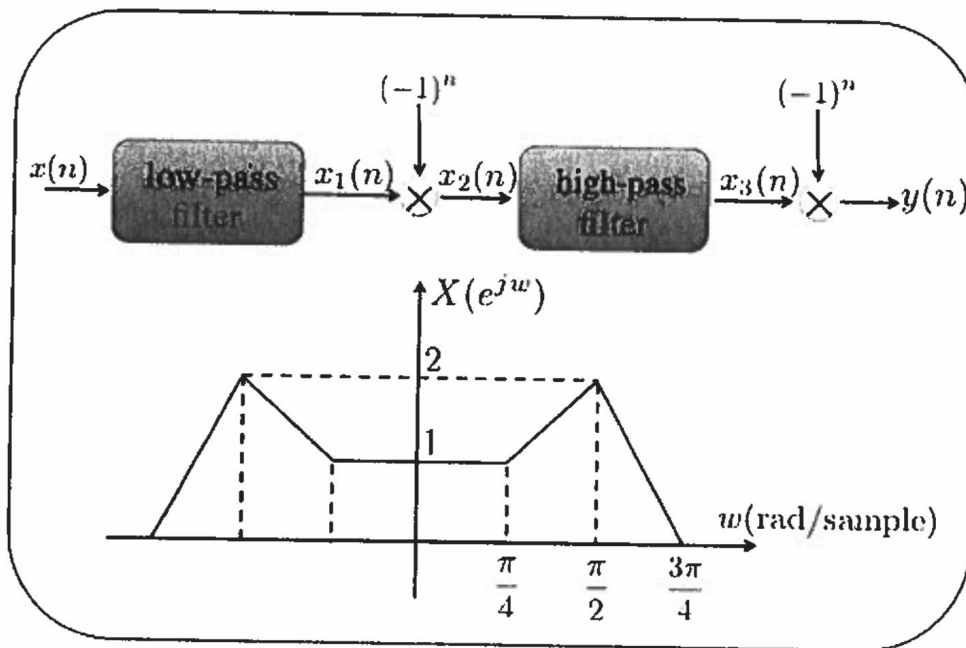
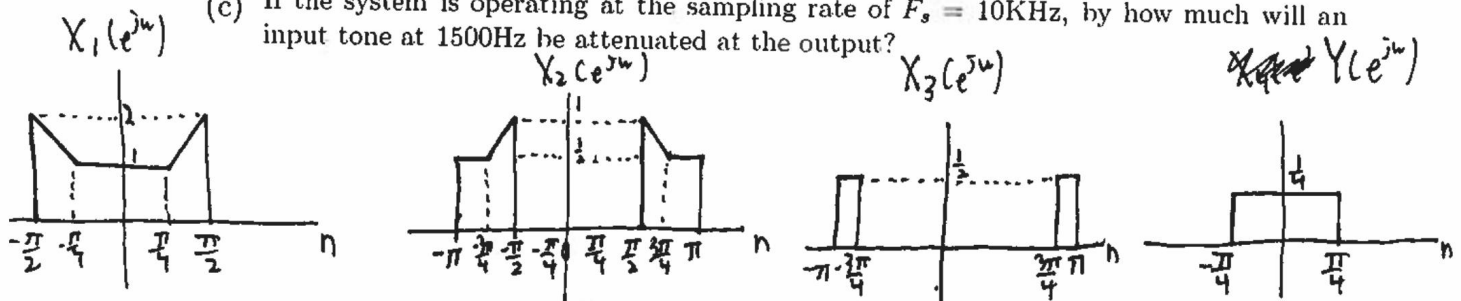


Figure 1: Block diagram for problem 6.

- (a) Draw the DTFT of the sequences $x_1(n)$, $x_2(n)$, $x_3(n)$ and $y(n)$.

- (b) Write the expression for the sequence $y(n)$?

- (c) If the system is operating at the sampling rate of $F_s = 10\text{KHz}$, by how much will an input tone at 1500Hz be attenuated at the output?



$$(b) Y(e^{j\omega}) = \frac{1}{4} \begin{cases} 1, & |\omega| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} \leq |\omega| \leq \pi \end{cases} \Leftrightarrow y(n) = \frac{1}{4} \begin{cases} \frac{1}{4}, & n=0 \\ \frac{1}{4} \text{sinc}(\frac{\pi}{4}n), & n \neq 0 \end{cases}$$

$$|y(n)| = \begin{cases} \frac{1}{16}, & n=0 \\ \frac{1}{16} \text{sinc}(\frac{\pi}{4}n), & n \neq 0 \end{cases}$$

- (c) $\omega = \frac{1500}{10000} (2\pi) = \frac{3\pi}{10} > \frac{\pi}{4}$, so the input tone will be fully attenuated by the low pass filter. This input yields no output.

7. (10 PTS) Three finite-length discrete-time signals $x(n)$, $x_1(n)$ and $x_2(n)$ are shown in Figure 2. In signal $x(n)$, the value of $x(3)$ is an unknown constant c . The sample with amplitude c is not necessarily drawn to scale.

- (a) Let $X(k)$ and $X_1(k)$ be the 5-point DFTs of $x(n)$ and $x_1(n)$. Suppose $X(k)$ and $X_1(k)$ satisfy the following relation

$$X_1(k) = X(k)e^{j2\pi 3k/5}$$

With the plots of $x(n)$ and $x_1(n)$ and the above relation, derive the value of c .

- (b) Let $X_1(k)$ and $X_2(k)$ be the N -point DFTs of $x_1(n)$ and $x_2(n)$. Suppose $X_1(k)$ and $X_2(k)$ satisfy the following relation

$$X_1(k) = X_2(k)e^{-j2\pi 3k/N}$$

With the plots of $x_1(n)$ and $x_2(n)$ and the above relation, derive the value of N .

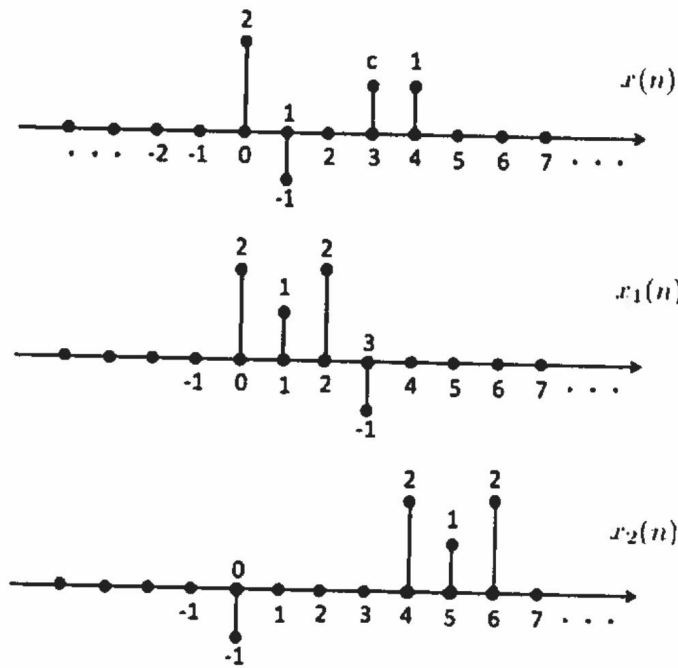


Figure 2: Plots of $x(n)$, $x_1(n)$ and $x_2(n)$.

(a) $X_1(k) = e^{j2\pi(3k/5)} X(k) \Leftrightarrow x_1(n) = x((n+3) \bmod 5)$ ~~implies~~ $x_1(0) = x(3 \bmod 5)$
 ~~$x_1(0) = x(3)$~~ $x_1(0) = x(3)$
 $c = 2$

(b) ~~$X_1(k) = e^{-j2\pi(3k/N)} X_2(k)$~~ $X_1(k) = e^{-j2\pi(3k/N)} X_2(k) \Leftrightarrow x_1(n) = x_2((n-3) \bmod N)$
 $x_1(0) = x_2(-3 \bmod N)$
 $2 = x_2(-3 \bmod N)$
 need $-3 \bmod N = 4 \Rightarrow N = 7$



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