22W-EC ENGR-113-LEC-1 Final Exam

MATTHEW FIORELLA

TOTAL POINTS

89 / 100

QUESTION 1

1 Q1 **15 / 15**

✓ **- 0 pts** Correct

- 2 pts Correct modes & sequences, partial correct ROC.

- 2 pts Correct modes & ROC(right/left side), partial correct sequences.

- 5 pts Correct modes & ROC(right/left side), wrong sequences.

- 8 pts Correct modes, partial correct ROC/sequences.

- 10 pts Correct ROC(right/left side), wrong modes/sequences

- 15 pts No Attempt

QUESTION 2

Q2 15 pts

2.1 Q2.a **5 / 5**

✓ **- 0 pts** Correct

- 1 pts Wrong phase+magnitude. Or no plot. Or didn't simply exponential representation.

- 2 pts Correct DFT formulation with additional coefficient multipliler or calculation error.

- 3 pts Correct DFT formulation.

- 4 pts Wrong DFT formulation.

2.2 Q2.b **4 / 5**

- 0 pts Correct

✓ **- 1 pts** **Coefficient 1/4 or incomplete answer**

- 1 pts Correct X(k-1) based on wrong X(k)

- 1 pts Wrong X(k-1) based on correct X(k)

- 2 pts Wrong X(k-1) based on wrong X(k)

- 2 pts wrong inverse DFT formulation/calculation.

- 5 pts No Attempt

2.3 Q2.c **5 / 5**

✓ **- 0 pts** Correct

- 1 pts Wrong calculation

- 2 pts Wrong circular convolution formulation. Or linear convolution with correct answer.

- 3 pts Linear convolution with wrong answer.

- 5 pts No Attempt/Completely Wrong

QUESTION 3

Q3 10 pts

3.1 Q3.a **5 / 5**

✓ **- 0 pts** Correct

- 2 pts Partially Correct

- 5 pts No Attempt/Completely Wrong

3.2 Q3.b **5 / 5**

✓ **- 0 pts** Correct

- 1 pts Calculation error.

- 2 pts Partially Correct

- 3 pts Wrong inverse DTFT.

- 5 pts No Attempt / Completely Wrong

QUESTION 4

Q4 15 pts

4.1 Q4.a **5 / 5**

✓ **- 0 pts** Correct

- 2 pts Partially Correct

4.2 Q4.b **5 / 5**

✓ **- 0 pts** Correct

- 2 pts Correct h(n) based on wrong partial fractional expansion.

- 2 pts Wrong h(n) based on correct partial fractional expansion.

- **4 pts** Wrong h(n) based on wrong partial fractional expansion.
- **5 pts** No Attempts.

4.3 Q4.c **4 / 5**

- **0 pts** Correct

✓ - **1 pts Incomplete answer / Calculation Error**

- **2 pts** Identify eigenfunction.
- **3 pts** Wrong results using conv or CCDE with some correct procedures.
- **4 pts** Wrong direction with some work.
- **5 pts** No attempt

QUESTION 5

Q5 20 pts

5.1 Q5.a **2.5 / 4**

- **0 pts** Correct
- **0.5 pts** u(n) representation but not specifying n>=1 / calculation error.

✓ - **0.5 pts Correct h(n) based on wrong partial expansion.**

- **1 pts** Wrong h(n) based on wrong partial expansion.

✓ - **1 pts Wrong h1(n) z transform.**

- **1 pts** Wrong h2(n) z transform.
- **2 pts** No impulse sequence.

5.2 Q5.b **2 / 4**

- **0 pts** Correct
- **1 pts** Calculation Error / Incomplete

✓ - **2 pts Correct CCDE based on wrong H(z)**

- **3 pts** Partially Correct with some work.
- **4 pts** No Attempt

5.3 Q5.c **4 / 4**

✓ - **0 pts Correct**

- **1 pts** Inaccurate explanation.
- **3 pts** Correct justification based on wrong modes.
- **4 pts** No Attempt

5.4 Q5.d **3 / 4**

- **0 pts** Correct
- **1 pts** Incorrect choice of x(n) based on correct modes.
- **2 pts** Incorrect x(n) based on wrong modes. / No x(n) with correct modes.

✓ - **1 pts One Incorrect modes.**

- **2 pts** Two incorrect modes.
- **4 pts** No Attempt / Complete wrong

5.5 Q5.e **1 / 4**

- **0 pts** Correct
- **2 pts** Incorrect initial conditions based on correct unilateral z-transform.
- **2 pts** Correct initial conditions based on wrong unilateral z-transform

✓ - **3 pts Incorrect initial conditions based on wrong unilateral z-transform.**

- **3 pts** Incorrect initial conditions with temporal approach
- **4 pts** No Attempt/Completely Wrong

QUESTION 6

Q6 15 pts

6.1 Q6.a **4.5 / 5**

- **0 pts** Correct

✓ - **0.5 pts Correct shape, wrong magnitude.**

- **1 pts** Wrong x1(n)
- **1 pts** Wrong x2(n)
- **1 pts** Wrong x3(n)
- **1 pts** Wrong x4(n)
- **5 pts** No attempt

6.2 Q6.b **4 / 5**

- **0 pts** Correct
- **1 pts** Series of convolution with correct filter sequence in sinc

✓ - **1 pts Correct with wrong magnitude.**

- **2 pts** Series of convolution with minor filter sequence error or didn't specify HPF/LPF Inverse DTFT
- **3 pts** Wrong with some work.

- **5 pts** No Attempt / Completely wrong.

6.3 Q6.c **5 / 5**

✓ **- 0 pts Correct, as long as correct justification is made based on your right or wrong plot in your (a).**

- **2 pts** Correct justification based wrong discrete frequency.
- **2 pts** Wrong justification based on correct discrete frequency.
- **4 pts** Wrong/no justification based on wrong discrete frequency.
- **5 pts** No Attempt / Complete wrong.

QUESTION 7

Q7 10 pts

7.1 Q7.a **5 / 5**

✓ **- 0 pts Correct**

- **2 pts** Partially Correct.
- **4 pts** Wrong with some work.
- **5 pts** No Attempt

7.2 Q7.b **5 / 5**

✓ **- 0 pts Correct**

- **2 pts** Partially Correct
- **4 pts** Wrong with some work.
- **5 pts** No Attempt

ECE113 Digital Signal Processing, Winter 2022

Final Exam

Instructor Prof. Alwan, A.

Name: Matthew Fiorella UID: 605593729 Email: mfiorell@ucla.edu

Note to Proctors:

- **This examination is closed book but two 8.5 x 11 cheat sheets (double sided) are allowed.**
- **Electronics (including calculators or smart phones) are not allowed. 3 hrs in duration.**

Note to Students:

- **Please write in dark pen or pencil so there is adequate contrast for readability, especially for remote students whose exams may be sent in by fax.**
- **Please explain or justify your work. Answers without justification will not receive full credit.**

Points:

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________

Total: __________

FINAL EXAMINATION

1. (15PTS) Determine all possible sequences $x(n)$ with z-transform

$$X(z) = \frac{36z - 11}{12z^2 - 7z + 1}$$

$$X(z) = \frac{36z-11}{12z^2-7z+1} = \frac{36z-11}{(3z-1)(4z-1)} = \frac{A}{3z-1} + \frac{B}{4z-1}$$

$$A = \frac{36z-11}{4z-1}\Big|_{z=\frac{1}{3}} = \frac{1}{\frac{1}{3}} = 3 \qquad B = \frac{36z-11}{3z-1}\Big|_{z=\frac{1}{4}} = \frac{-2}{-\frac{1}{4}} = 8$$

$$X(z) = \frac{36z-11}{(3z-1)(4z-1)} = \frac{3}{3z-1} + \frac{8}{4z-1}$$

$$X(z) = \frac{1}{z-\frac{1}{3}} + \frac{2}{z-\frac{1}{4}}$$

Possible ROC's: $|z| < \frac{1}{4}$, $\frac{1}{4} < |z| < \frac{1}{3}$, $|z| > \frac{1}{3}$

if ROC is $|z| < \frac{1}{4}$:

$$x(n) = -\frac{1}{3}^{n-1} u(-n) - 2\left(\frac{1}{4}\right)^{n-1} u(-n)$$

$$\boxed{x(n) = -\left(\frac{1}{3}\right)^{n-1} u(-n) - 2\left(\frac{1}{4}\right)^{n-1} u(-n)}$$

if ROC is $\frac{1}{4} < |z| < \frac{1}{3}$:

$$\boxed{x(n) = 2\left(\frac{1}{4}\right)^{n-1} u(n-1) - \left(\frac{1}{3}\right)^{n-1} u(-n)}$$

if ROC is $|z| > \frac{1}{3}$:

$$\boxed{x(n) = 2\left(\frac{1}{4}\right)^{n-1} u(n-1) + \left(\frac{1}{3}\right)^{n-1} u(n-1)}$$

2. (15 PTS) Part (c) of this question is independent of the other two parts.

(a) Consider the sequence $x(n) = \delta(n) - 2\delta(n-2)$. Find the 4-point DFT $X(k)$ and plot it.

(b) Plot $X((k-1) \bmod 4)$ and find its 4-point inverse DFT.

(c) Evaluate the circular convolution between $x(n) = \{\boxed{1}, 2, 0, 1\}$ and $h(n) = \{\boxed{-1}, 1, 2, 1\}$.

(a) $x(n) = \delta(n) - 2\delta(n-2)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k}{N}n} = \sum_{n=0}^{3} \left(\delta(n) - 2\delta(n-2)\right) e^{-j\frac{2\pi k}{4}n}$$

$$X(k) = \delta(0) - 2\delta(0) \cdot e^{-j\pi k}$$

$$X(k) = 1 - 2(\cos(\pi k) - j\sin(\pi k))$$

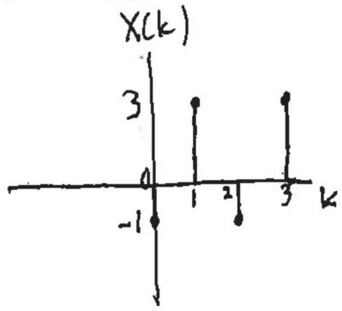
$$X(k) = 1 - 2\cos(\pi k)$$

$X(0) = 1 - 2 = -1$

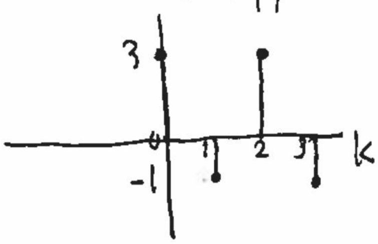
$X(1) = 1 + 2 = 3$

$X(2) = 1 - 2 = -1$

$X(3) = 1 + 2 = 3$



(b) $X((k-1) \bmod 4)$



$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X((k-1) \bmod 4) e^{j\frac{2\pi k}{4}n}$$

$$x(n) = \frac{1}{4}(3) + \frac{1}{4}\left(-e^{j\frac{\pi}{2}n}\right) + \frac{1}{4}\left(3e^{j\pi n}\right) + \frac{1}{4}\left(-e^{j\frac{3\pi}{2}n}\right)$$

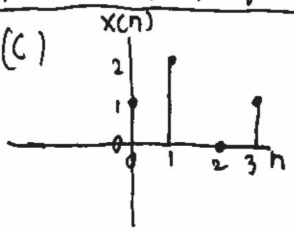
$$x(n) = \frac{3}{4} - \frac{1}{4}e^{j\frac{\pi}{2}n} + \frac{3}{4}e^{j\pi n} - \frac{1}{4}e^{j\frac{3\pi}{2}n}$$

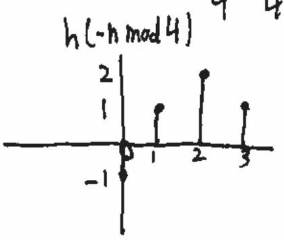
$$x(n) = \frac{3}{4} - \frac{1}{4}e^{j\pi n}\left(e^{-j\frac{\pi}{2}n} + e^{j\frac{\pi}{2}n} - 3\right)$$

$$x(n) = \frac{3}{4} - \frac{1}{4}e^{j\pi n}\left(2\cos\left(\frac{\pi}{2}n\right) - 3\right)$$

$$\boxed{x(n) = \frac{3}{4} - \frac{1}{2}e^{j\pi n}\cos\left(\frac{\pi}{2}n\right) + \frac{3}{4}e^{j\pi n}}$$

(c)





$y(0) = (1)(-1) + (2)(1) + (0)(2) + (1)(1) = 2$

$y(1) = (1)(1) + (2)(-1) + (0)(1) + (1)(2) = 1$

$y(2) = (1)(2) + (2)(1) + (0)(-1) + (1)(1) = 5$

$y(3) = (1)(1) + (2)(2) + (0)(1) + (1)(-1) = 4$

$$\boxed{y(n) = \{\boxed{2}, 1, 5, 4\}}$$

3. (10 PTS) The two parts of this question are independent of one another.

(a) Find the DTFT of $x(n) = \cos(\frac{\pi}{6}n) + \sin(\frac{\pi}{4}n)$

(b) Calculate $\sum_{n=-\infty}^{+\infty} \frac{\sin^2(\frac{\pi}{6}n)}{n^2}$. (Hint: Using DTFT and Parseval's Relation may be helpful.)

(a) $x(n) = \cos(\frac{\pi}{6}n) + \sin(\frac{\pi}{4}n)$

$x(n) = \frac{1}{2}\left(e^{j\frac{\pi}{6}n} + e^{-j\frac{\pi}{6}n}\right) + \frac{1}{2j}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right)$

$x(n) = \frac{1}{2}e^{j\frac{\pi}{6}n} + \frac{1}{2}e^{-j\frac{\pi}{6}n} + \frac{1}{2j}e^{j\frac{\pi}{4}n} - \frac{1}{2j}e^{-j\frac{\pi}{4}n}$

$X(e^{jw}) = \pi\delta(w-\frac{\pi}{6}) + \pi\delta(w+\frac{\pi}{6}) + \frac{\pi}{j}\delta(w-\frac{\pi}{4}) - \frac{\pi}{j}\delta(w+\frac{\pi}{4})$

$$\boxed{X(e^{jw}) = \pi\left(\delta(w-\frac{\pi}{6}) + \delta(w+\frac{\pi}{6}) - j\delta(w-\frac{\pi}{4}) + j\delta(w+\frac{\pi}{4})\right)}$$

(b) $\sum_{n=-\infty}^{\infty} \frac{\sin^2(\frac{\pi}{6}n)}{n^2} = \sum_{n=-\infty}^{\infty} \left(\frac{\sin(\frac{\pi}{6}n)}{n}\right)^2 = \sum_{n=-\infty}^{\infty}\left(\frac{\pi}{6}\text{sinc}(\frac{\pi}{6}n)\right)^2$

Parseval's Relation: $\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{jw})|^2 dw$

$x(n) = \frac{\pi}{6}\text{sinc}(\frac{\pi}{6}n) \Longleftrightarrow X(e^{jw}) = \begin{cases} \pi, & |w| < \frac{\pi}{6} \\ 0, & \frac{\pi}{6} \le |w| \le \pi \end{cases}$

$\sum_{n=-\infty}^{\infty} \frac{\sin^2(\frac{\pi}{6}n)}{n^2} = \sum_{n=-\infty}^{\infty}\left(\frac{\pi}{6}\text{sinc}(\frac{\pi}{6}n)\right)^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw = \frac{1}{2\pi}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \pi^2 dw = \frac{1}{2\pi}\left(\pi^2(\frac{\pi}{6}) - \pi^2(-\frac{\pi}{6})\right)$

$= \frac{1}{2\pi}\left(\frac{\pi^3}{3}\right) = \frac{\pi^2}{6}$

$$\boxed{\sum_{n=-\infty}^{\infty} \frac{\sin^2(\frac{\pi}{6}n)}{n^2} = \frac{\pi^2}{6}}$$

4. (15 PTS) Consider the relaxed and causal system

$$6y(n) - 5y(n-1) + y(n-2) = 2x(n-1).$$

(a) Find the frequency response of the above system.

(b) Find the impulse response $h(n)$ by inverting the frequency response in part (a)

(c) If the signal $x(n) = \cos(\frac{\pi}{2}n + \frac{\pi}{4})$ is the input to the system, how much would its magnitude be attenuated at the output?

(a) $6y(n) - 5y(n-1) + y(n-2) = 2x(n-1)$

$6Y(e^{jw}) - 5e^{-jw}Y(e^{jw}) + e^{-j2w}Y(e^{jw}) = 2e^{-jw}X(e^{jw})$

$Y(e^{jw})(2-e^{-jw})(3-e^{-jw}) = X(e^{jw})(2e^{-jw})$

$$\boxed{H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{2e^{-jw}}{(2-e^{-jw})(3-e^{-jw})}}$$

(b) $H(e^{jw}) = \dfrac{2e^{-jw}}{(2-e^{-jw})(3-e^{-jw})} = \dfrac{A}{2-e^{-jw}} + \dfrac{B}{3-e^{-jw}}$

$A = \left.\dfrac{2e^{-jw}}{3-e^{-jw}}\right|_{e^{-jw}=2} = 4 \quad B = \left.\dfrac{2e^{-jw}}{2-e^{-jw}}\right|_{e^{-jw}=3} = -6 \quad H(e^{jw}) = \dfrac{2e^{-jw}}{(2-e^{-jw})(3-e^{-jw})} = \dfrac{4}{2-e^{-jw}} - \dfrac{6}{3-e^{-jw}}$

$H(e^{jw}) = \dfrac{2}{1-\frac{1}{2}e^{-jw}} - \dfrac{2}{1-\frac{1}{3}e^{-jw}}$

$$\boxed{h(n) = 2\left(\tfrac{1}{2}\right)^n u(n) - 2\left(\tfrac{1}{3}\right)^n u(n)}$$

(c) $x(n) = \cos(\frac{\pi}{2}n + \frac{\pi}{4}) = \frac{1}{2}\left(e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + e^{-j(\frac{\pi}{2}n+\frac{\pi}{4})}\right) = \frac{1}{2}e^{j\frac{\pi}{4}}e^{j\frac{\pi}{2}n} + \frac{1}{2}e^{-j\frac{\pi}{4}}e^{-j\frac{\pi}{2}n}$

$X(e^{jw}) = \frac{1}{2}e^{j\frac{\pi}{4}}(2\pi\delta(w-\frac{\pi}{2})) + \frac{1}{2}e^{-j\frac{\pi}{4}}(2\pi\delta(w+\frac{\pi}{2})) = \pi e^{j\frac{\pi}{4}}\delta(w-\frac{\pi}{2}) + \pi e^{-j\frac{\pi}{4}}\delta(w+\frac{\pi}{2})$

$Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \dfrac{2e^{-jw}}{(2-e^{-jw})(3-e^{-jw})}\left(\pi e^{j\frac{\pi}{4}}\delta(w-\frac{\pi}{2}) + \pi e^{-j\frac{\pi}{4}}\delta(w+\frac{\pi}{2})\right) = \dfrac{2\pi e^{j\frac{\pi}{4}}e^{-j\frac{\pi}{2}}}{(2-e^{-j\frac{\pi}{2}})(3-e^{-j\frac{\pi}{2}})} + \dfrac{2\pi e^{-j\frac{\pi}{4}}e^{j\frac{\pi}{2}}}{(2-e^{j\frac{\pi}{2}})(3-e^{j\frac{\pi}{2}})}$

$Y(e^{jw}) = \dfrac{2\pi e^{-j\frac{\pi}{4}}}{(2-e^{-j\frac{\pi}{2}})(3-e^{-j\frac{\pi}{2}})} + \dfrac{2\pi e^{j\frac{\pi}{4}}}{(2-e^{-j\frac{\pi}{2}})(3-e^{-j\frac{\pi}{2}})} = \dfrac{4\pi\cos(\frac{\pi}{4})}{(2-e^{-j\frac{\pi}{2}})(3-e^{-j\frac{\pi}{2}})} = \dfrac{\frac{4\pi}{\sqrt{2}}}{12} = \dfrac{\pi}{3}\left(\dfrac{1}{\sqrt{2}}\right) = \boxed{\dfrac{\pi\sqrt{2}}{6}}$

Parts (b) (c) (d) (e) on back!

5. (20 PTS) A casual system is composed of the **series cascade** of two LTI systems with impulse response sequences given by $h_1(n) = \left(\frac{1}{2}\right)^n u(2n)$ and $h_2(n) = \left(\frac{1}{4}\right)^n u(n-1)$.

   (a) Use the z-transform to determine the impulse response sequence and transfer function of the overall system.

   (b) Determine a description for the overall system in terms of a constant-coefficient difference equation. Denote its input and output sequences by $x(n)$ and $y(n)$, respectively.

   (c) Decide the stability of the system from the transfer function and its ROC.

   (d) What are the system's modes? Determine an input sequence such that only the largest mode appears at the corresponding output sequence.

   (e) Consider the difference sequence in part (a). Use the unilateral z-transform to determine initial conditions $y(-1)$ and $y(-2)$ such that only the smallest mode appears at the output of the system when the input is $x(n) = \delta(n)$.

(a) $h_1(n) = \left(\frac{1}{2}\right)^n u(2n) = \left(\frac{1}{4}\right)^{2n} u(2n) \Longleftrightarrow \frac{1}{2}\sum_{k=0}^{1} X(e^{-j\frac{2\pi k}{2}} z^{1/2}) = \frac{1}{2}\left(X(z^{\frac12}) + X(-z^{\frac12})\right)$

$H_1(z) = \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}z^{-\frac12}} + \frac{1}{1+\frac{1}{4}z^{-\frac12}}\right) = \frac{1}{2}\left(\frac{1+\frac{1}{4}z^{-\frac12}+1-\frac{1}{4}z^{-\frac12}}{1-\frac{1}{16}z^{-1}}\right) = \frac{1}{1-\frac{1}{16}z^{-1}}$

$h_2(n) = \left(\frac{1}{4}\right)^n u(n-1) = \frac{1}{4}\left(\frac{1}{4}\right)^{n-1} u(n-1) \Longleftrightarrow H_2(z) = \frac{1}{4}\frac{z^{-1}}{1-\frac{1}{4}z^{-1}}$

$H(z) = H_1(z)H_2(z) = \frac{1}{4}\frac{z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{16}z^{-1})}$, ROC, $|z| > \frac{1}{4}$

$H(z) = \frac{\frac{1}{4}z}{(z-\frac{1}{4})(z-\frac{1}{16})}$, ROC, $|z| > \frac{1}{4}$

Remaining parts on back

$H(z) = \frac{A}{z-\frac{1}{4}} + \frac{B}{z-\frac{1}{16}} \Rightarrow A = \left.\frac{\frac{1}{4}z}{z-\frac{1}{16}}\right|_{z=\frac{1}{4}} = \frac{\frac{1}{16}}{\frac{3}{16}} = \frac{1}{3}$, $B = \left.\frac{\frac{1}{4}z}{z-\frac{1}{4}}\right|_{z=\frac{1}{16}} = \frac{\frac{1}{64}}{-\frac{3}{16}} = -\frac{1}{12}$

$H(z) = \frac{\frac{1}{3}}{z-\frac{1}{4}} - \frac{\frac{1}{12}}{z-\frac{1}{16}}$, ROC, $|z| > \frac{1}{4} \Longleftrightarrow h(n) = \frac{1}{3}\left(\frac{1}{4}\right)^{n-1} u(n-1) - \frac{1}{12}\left(\frac{1}{16}\right)^{n-1} u(n-1)$

(b) $H(z) = \dfrac{\frac{1}{4}z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{16}z^{-1})}$, ROC: $|z| > \frac{1}{4}$

$$\frac{Y(z)}{X(z)} = \frac{\frac{1}{4}z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{16}z^{-1})}$$

$$Y(z)\left(1-\tfrac{1}{4}z^{-1}\right)\left(1-\tfrac{1}{16}z^{-1}\right) = \tfrac{1}{4}z^{-1}X(z)$$

$$Y(z) - \tfrac{5}{16}z^{-1}Y(z) + \tfrac{1}{64}z^{-2}Y(z) = \tfrac{1}{4}z^{-1}X(z)$$

$$\boxed{y(n) - \tfrac{5}{16}y(n-1) + \tfrac{1}{64}y(n-2) = \tfrac{1}{4}x(n-1)}$$

(c) The ~~function~~ system is stable because $|z|=1$ is in the ROC. In other words the unit circle is in the ROC, so we know the system is stable.

(d) The modes are $\frac{1}{4}$ and $\frac{1}{16}$.

To make only the largest mode appear we need to make the smallest disappear.
Need input with z transform $1-\frac{1}{16}z^{-1}$ so it will be cancelled out when multiplied by $H(z)$

$$1-\tfrac{1}{16}z^{-1} \Longleftrightarrow \boxed{\delta(n) - \tfrac{1}{16}\delta(n-1)}$$

(e) $y(n) - \frac{5}{16}y(n-1) + \frac{1}{64}y(n-2) = \frac{1}{4}x(n-1)$

$$Y^+(z) - \tfrac{5}{16}z^{-1}Y^+(z)\ \tfrac{5}{16}y(-1) + \tfrac{1}{64}z^{-2}Y^+(z) + \tfrac{1}{64}y(-1)z^{-1} + \tfrac{1}{64}y(-2) = \tfrac{1}{4}z^{-1}X^+(z) + \tfrac{1}{4}x(-1)$$

~~$x(n) = \delta(n) \Leftrightarrow X^+(z) = 1 \quad x(-1) = 0$~~ $x(n) = \delta(n) \Longleftrightarrow X^+(z) = 1$

$$Y^+(z) - \tfrac{5}{16}z^{-1}Y^+(z) - \tfrac{5}{16}y(-1) + \tfrac{1}{64}z^{-2}Y^+(z) + \tfrac{1}{64}y(-1)z^{-1} + \tfrac{1}{64}y(-2) = \tfrac{1}{4}z^{-1}$$

$$\Rightarrow Y^+(z)\left(1-\tfrac{1}{4}z^{-1}\right)\left(1-\tfrac{1}{16}z^{-1}\right) - \tfrac{5}{16}y(-1) + \tfrac{1}{64}y(-1)z^{-1} + \tfrac{1}{64}y(-2) = \tfrac{1}{4}z^{-1}$$

we want $Y^+(z) = 1-\frac{1}{16}z^{-1} \Rightarrow \left(1-\frac{1}{4}z^{-1}\right) - \frac{5}{16}y(-1) + \frac{1}{64}y(-1)z^{-1} + \frac{1}{64}y(-2) = \frac{1}{4}z^{-1}$

$$-\tfrac{5}{16}y(-1) + 1 + \tfrac{1}{64}y(-2) = 0$$

$$-\tfrac{1}{4}z^{-1} + \tfrac{1}{64}y(-1)z^{-1} = \tfrac{1}{4}z^{-1}$$

$$\boxed{y(-1) = 32,\ y(-2) = 576}$$

6. (15 PTS) The figure shows the DTFT of a real-valued sequence $x(n)$, which is fed into a cascade of two filters with modulators. The first filter is an ideal lowpass with cutoff frequency $w_\ell = \frac{\pi}{2}$ radians/sample. The second filter is an ideal highpass with cutoff frequency $w_h = \frac{3\pi}{4}$ radians/sample.

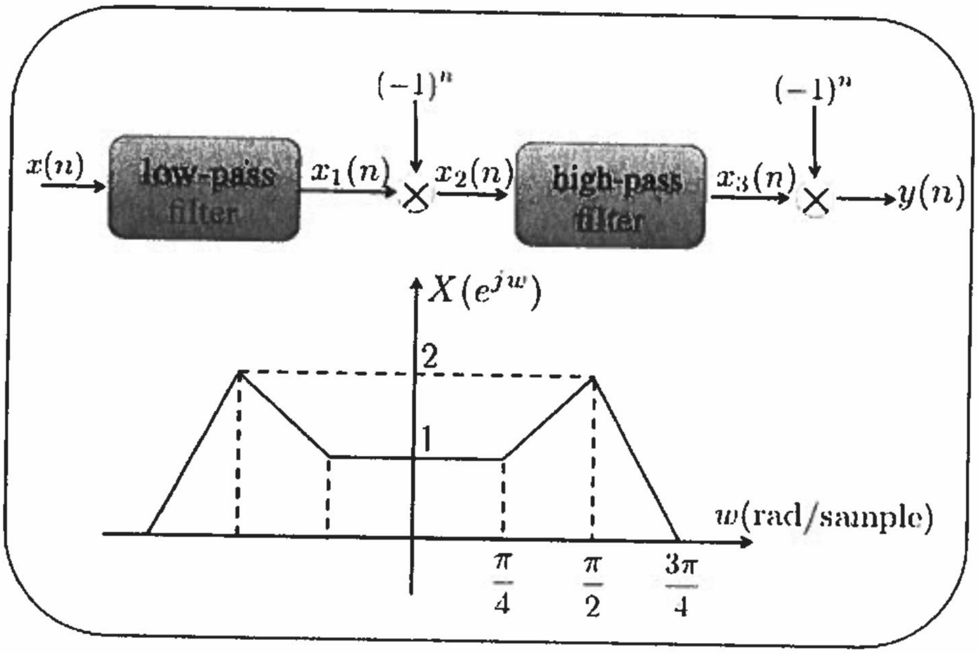
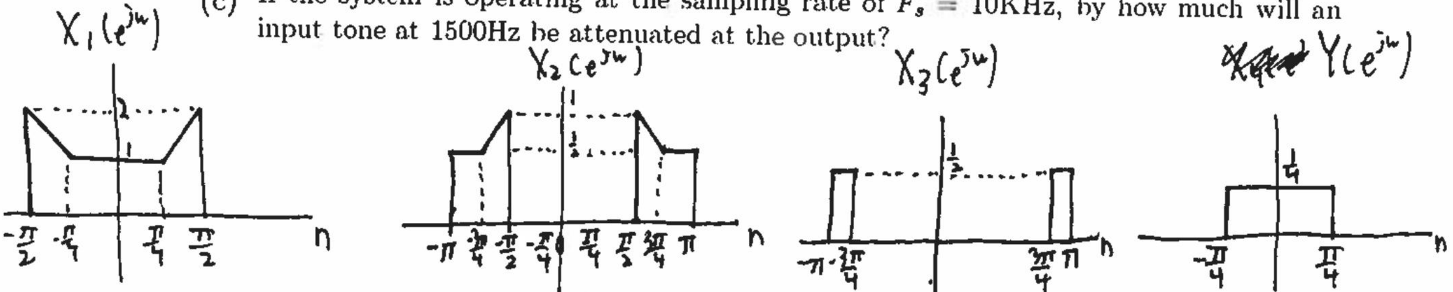


Figure 1: Block diagram for problem 6.

(a) Draw the DTFT of the sequences $x_1(n), x_2(n), x_3(n)$ and $y(n)$.

(b) Write the expression for the sequence $y(n)$?

(c) If the system is operating at the sampling rate of $F_s = 10$KHz, by how much will an input tone at 1500Hz be attenuated at the output?



(b) $Y(e^{jw}) = \frac{1}{4}\begin{cases} 1, & |w| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} \le |w| \le \pi \end{cases} \Leftrightarrow y(n) = \frac{1}{4}\left(\begin{cases} \frac{1}{4}, & n=0 \\ \frac{1}{4}\,\text{sinc}(\frac{\pi}{4}n), & n \neq 0 \end{cases}\right)$

$y(n) = \begin{cases} \frac{1}{16}, & n=0 \\ \frac{1}{16}\,\text{sinc}(\frac{\pi}{4}n), & n \neq 0 \end{cases}$

(c) $w = \frac{1500}{10000}(2\pi) = \frac{3\pi}{10}$ $\quad \frac{3\pi}{10} > \frac{\pi}{4}$, so the input tone will be fully attenuated by the lowpass filter

This input yields no output

7. (10 PTS) Three finite-length discrete-time signals $x(n)$, $x_1(n)$ and $x_2(n)$ are shown in Figure 2. In signal $x(n)$, the value of $x(3)$ is an unknown constant c. The sample with amplitude c is not necessarily drawn to scale.

(a) Let $X(k)$ and $X_1(k)$ be the 5-point DFTs of $x(n)$ and $x_1(n)$. Suppose $X(k)$ and $X_1(k)$ satisfy the following relation

$$X_1(k) = X(k)e^{j2\pi 3k/5}.$$

With the plots of $x(n)$ and $x_1(n)$ and the above relation, derive the value of c.

(b) Let $X_1(k)$ and $X_2(k)$ be the N-point DFTs of $x_1(n)$ and $x_2(n)$. Suppose $X_1(k)$ and $X_2(k)$ satisfy the following relation

$$X_1(k) = X_2(k)e^{-j2\pi 3k/N}.$$

With the plots of $x_1(n)$ and $x_2(n)$ and the above relation, derive the value of N.

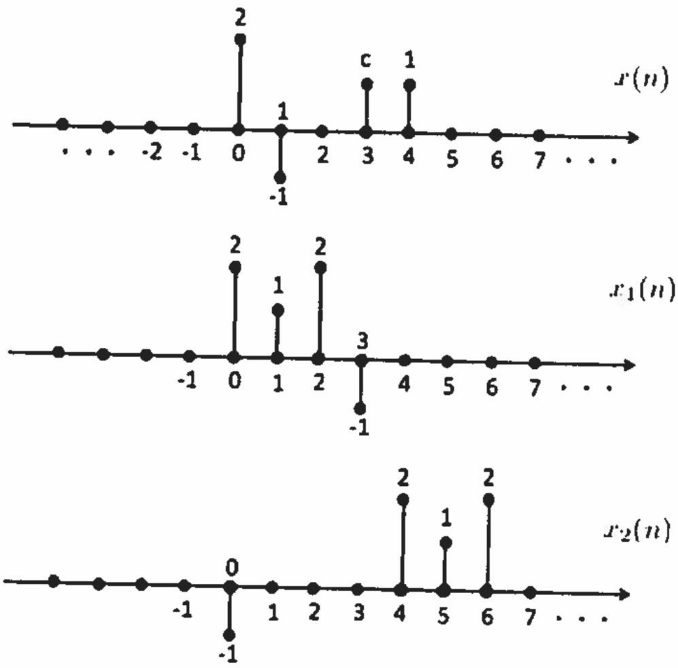


Figure 2: Plots of $x(n)$, $x_1(n)$ and $x_2(n)$.

(a) $X_1(k) = e^{-j2\pi(\frac{3k}{5})} X(k) \Leftrightarrow x_1(n) = x((n+3) \bmod 5)$ $x_1(0) = x(3 \bmod 5)$

$x_1(0) = x(3)$

$c = 2$

$\boxed{c = 2}$ ←

(b) $X_1(k) = e^{-j2\pi(\frac{3k}{N})} X_2(k) \Leftrightarrow x_1(n) = x_2((n-3) \bmod N)$

$x_1(0) = x_2(-3 \bmod N)$

$2 = x_2(-3 \bmod N)$

need $-3 \bmod N = 4 \Rightarrow N = 7$

$\boxed{N = 7}$