EE 110
CIRCUIT ANALYSIS II
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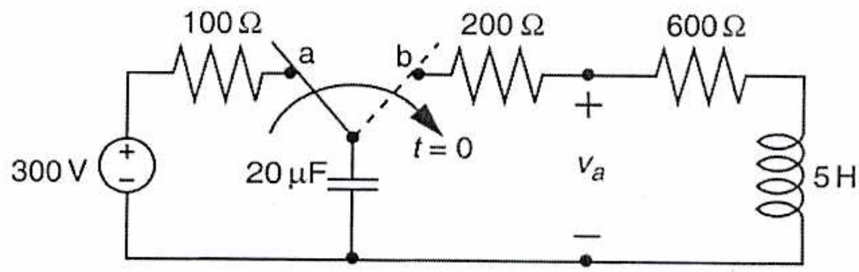
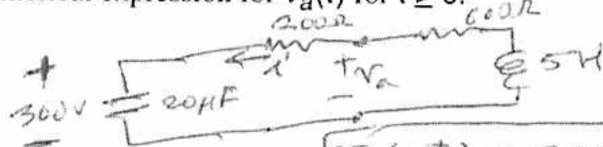
# MID-TERM EXAMINATION

Do all work in this examination packet. There are three questions. Each counts 10 points. Good luck!

1. The switch has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b.
   a.) (1 point) At $t = 0+$, what is the initial value of $v_a$?
   b.) (2 points) What is the initial value of $dv_a/dt$ ?
   c.) (2 points) For $t > 0$, find the roots of the system's characteristic equation.
   d.) (1 point) For $t > 0$, the sub-system is (check one box): ☐ undamped; ☐ critically damped; ☐ underdamped; ☐ overdamped.
   e.) (4 points) Obtain the numerical expression for $v_a(t)$ for $t \geq 0$.




a) For $t > 0$:




Since $i(0^-) = i(0^+) = 0$ ∴ $\boxed{v_a(0^+) = 300\,V}$

(b) $v_a = 200\,i + (5\times10^4)\int_0^t i(x)dx + 300$

$\frac{1}{20\times10^{-6}}$

$\Rightarrow \frac{dv_a}{dt} = 200\frac{di}{dt} + (5\times10^4)\,i$

$\Rightarrow \frac{dv_a(0^+)}{dt} = 200\frac{di(0^+)}{dt} + (5\times10^4)\,i(0^+) = 200\frac{di(0^+)}{dt}$

and also: $-L\frac{di(0^+)}{dt} = 300 \Rightarrow \frac{di(0^+)}{dt} = -0.2\times300 = -60\,A/s$

∴ $\boxed{\frac{dv_a(0^+)}{dt} = -12{,}000\,V/s}$

(c) $L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i(x)dx + 300 = 0$

$\Rightarrow L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0 \Rightarrow 5s^2 + 800s + 5\times10^4 = 0$

$s = \frac{-800 \pm \sqrt{64\times10^4 - 4\times25\times10^4}}{10}$

$= -80 \pm (\sqrt{64-100})10 = -80 \pm j10\sqrt{36}$

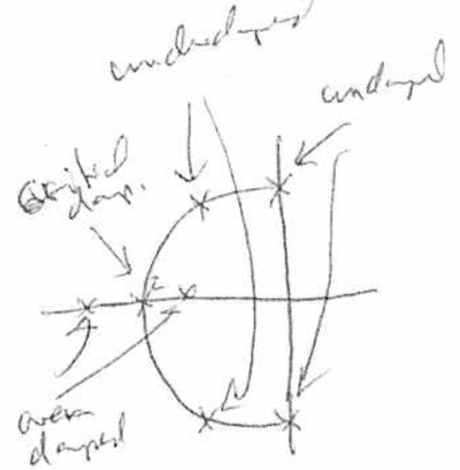
$= \boxed{-80 \pm j60}$

1.) (cont'd.)

Answer to c : $-80 \pm j60$

$\Downarrow$

d.) $\boxed{\text{underdamped}}$




e.) From answers to c) & d):

$$v_a = B_1 e^{-80t} \cos 60t + B_2 e^{-80t} \sin 60t$$

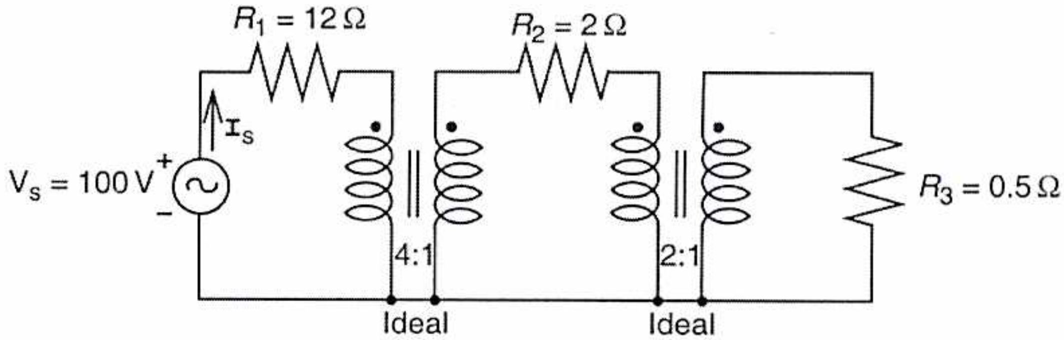
$$\begin{cases} v_a(0) = B_1 = 300V. \\ \frac{dv_a}{dt}(0) = -80B_1 + 60B_2 = -12{,}000 \end{cases}$$

$\Rightarrow B_2 = 200\,v.$

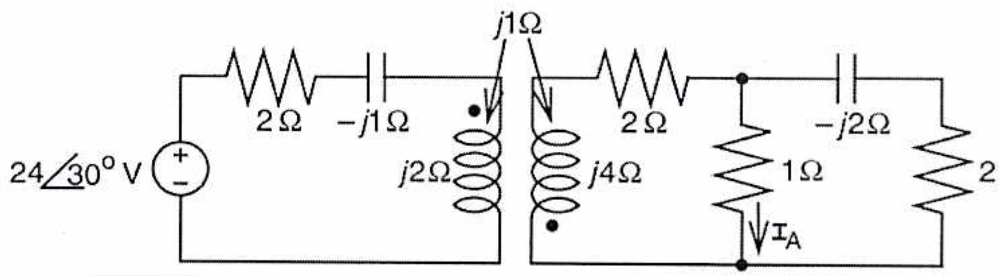
$$\Rightarrow \boxed{v_a(t) = 300e^{-80t} \cos 60t + 200e^{-80t} \sin 60t}$$

for $t \geq 0$

2A. (5 points) Determine the current supplied by the source $V_s$.

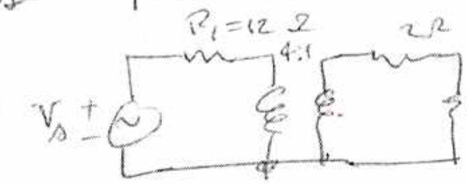



2B. (5 points) Determine $I_A$ in the circuit below.



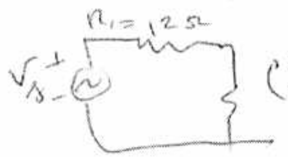

2A: Use impedance-transforming property of ideal transformer:

$\Rightarrow$




$\left(\frac{2}{1}\right)^2 R_3 = 4R_3 = 2\Omega$

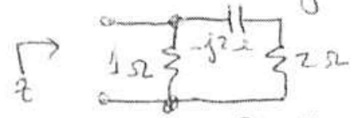
$\Rightarrow$




$\left(\frac{4}{1}\right)^2 \times (2+2) = 16 \times 4 = 64\,\Omega$

$\Rightarrow I_s = V_s/(12+64) = 100/76 = \boxed{1.32\text{ A}}$

2B: The right end of the circuit is:




$Z = \frac{(1)(2-j2)}{3-j2} = 0.77 - j0.15\,\Omega$

Mesh eqns. for the circuit are now: $\begin{cases} (2+j1)\,I_1 + (j1)\,I_2 = 24\angle 30^\circ \\ (j1)\,I_1 + (2+j4+0.77-j0.15)\,I_2 = 0 \end{cases}$

$\begin{bmatrix} (2+j1) & j1 \\ j1 & 2.77+j3.85 \end{bmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 24\angle 30^\circ \\ 0 \end{pmatrix}$ $\Big\{ \Delta = (2+j1)(2.77+j3.85)+1 = 2.69 + j10.47$

$I_2 = -\frac{(j1)(24\angle 30^\circ)}{2.69+j10.47} = 2.22 \angle -135.57^\circ$

using current division: $I_A = I_2(2-j2)/(3-j2) = 1.74 \angle -146.88^\circ$

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3A. (six points).

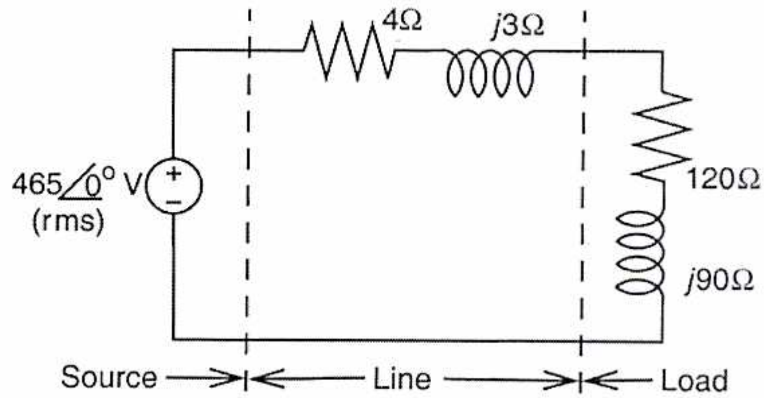
a.) (1 point) Find the average power dissipated in the line for this source/line/load circuit.

b.) (2 points) Find the capacitive reactance that, when connected in parallel with the load, will make the load look purely resistive.

c.) (1 point) What is the equivalent impedance of the load in (b)?

d.) (1 point) Find the average power dissipated in the line when the capacitive reactance is connected across the load.

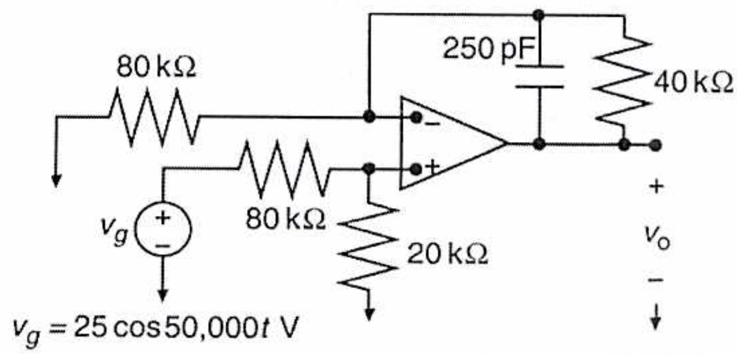
e.) (1 point) Express the power loss in (d) as a percentage of the power loss found in (a).




3B. (four points)

The op amp in this circuit is ideal.

Find the steady-state expression for $v_o(t)$.




3A. (a) $I = \frac{465\angle 0^\circ}{124 + j93} = 2.4 - j1.8 = 3\angle -36.87^\circ$ A(rms) $\Rightarrow P = (3)^2(4) = \boxed{36\text{ W}}$

(b) $Y_L = \frac{1}{120 + j90} = 5.33 - j4$ mS ; $\therefore X_c = \frac{1}{-4\times 10^{-3}} = \boxed{-250\ \Omega}$

(c) $Z_L = \frac{1}{5.33\times 10^{-3}} = \boxed{187.5\ \Omega}$ (d) $I = \frac{465\angle 0^\circ}{191.5 + j3} = 2.43\angle 0.9^\circ$ A $\Rightarrow P = (2.43)^2 4 = \boxed{23.58\text{ W}}$

(e) $\% = \frac{23.58}{36}(100) = \boxed{65.5\%}$ Thus, the power loss after the capacitor is added is 65.5% of the power loss before cap. added.

3B. $V_g = 25\angle 0^\circ$ V. $\Rightarrow V_p = \frac{20}{100} V_g = 5\angle 0^\circ$ & $V_n = V_p = 5\angle 0^\circ$ V.

$$\frac{5}{80{,}000} + \frac{5 - V_o}{Z_p} = 0$$

$$Z_p = -j80{,}000 \| 40{,}000 = 32{,}000 - j16{,}000\ \Omega$$

$$V_o = \frac{5 Z_p}{80{,}000} + 5 = 7 - j1 = 7.07\angle -8.13^\circ \text{ V}$$

$$\Rightarrow \boxed{v_o(t) = 7.07\cos(50{,}000t - 8.13^\circ)\text{ V}}$$