EE 110
CIRCUIT ANALYSIS II
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21/30

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MID-TERM EXAMINATION

Do all work in this examination packet. There are three questions. Each counts 10 points. Good luck!

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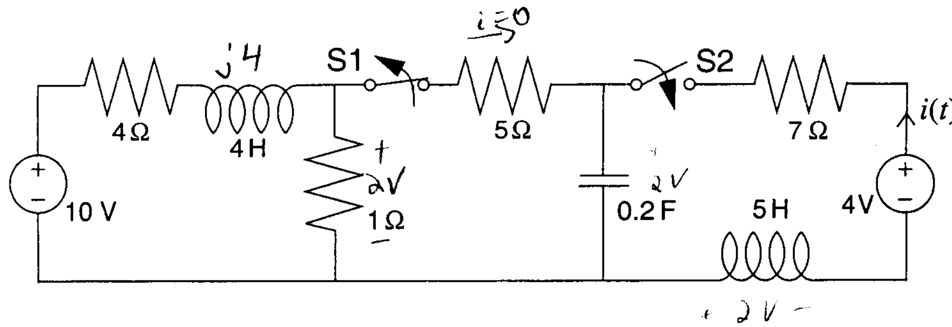
1. Switch S1 has been closed and switch S2 has been open for $t < 0$. At $t = 0$, S1 is opened and S2 is closed. (Both voltage sources are DC voltage sources.)



0 a.) (2 points) Find the characteristic equation for the sub-system governing $i(t)$ for $t > 0$.

0 b.) (1 point) Find the roots of the sub-system's characteristic equation.

1 c.) (1 point) For $t > 0$, the sub-system is (check one box): ☐ undamped; ☒ underdamped; ☐ critically damped; ☐ overdamped.

2 d.) (2 points) Find the quality factor (Q) of the sub-system.

3 e.) (4 points) Obtain $i(t)$ for $t > 0$.

d) $w_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5H \cdot .2F}} = 1$ ✓ $\quad w_o^2 = 1$

$\alpha = \frac{R}{2L} = \frac{7\Omega}{2 \cdot 5H} = .7 \frac{rad}{s}$ ✓ $\quad \alpha^2 = .49$

$w_o^2 > \alpha^2$ means it is underdamped.

$$\boxed{i(t) = I_f + B_1 e^{-\alpha t} \cos w_d t + B_2 e^{-\alpha t} \sin w_d t}$$

where: $\alpha = .7 \frac{rad}{s}$ $\quad w_d = \sqrt{w_o^2 - \alpha^2} = .714 \frac{rad}{s}$

b) $i_f(0) = \emptyset$ b/c no current flowed through the capacitor before the switch was open.

I_f $\quad i(\infty) = 0$ b/c the whole voltage drop will be across the capacitor

$\frac{d\, i_L(0)}{dt} = \frac{V_L}{L} = \frac{-2V}{5H} = -.4$

$i(0) = I_f + B_1(1) + 0$

$\boxed{0 = B_1}$ ✓

$\frac{d\,i(0)}{dt} = -\alpha B_1 e^{-\alpha t} \cos w_d t + B_1 e^{-\alpha t}(-w_d \sin w_d t) + -\alpha B_2 e^{-\alpha t} \sin w_d t + B_2 w_d e^{-\alpha t} \cos w_d t$

$-.4 = B_2 w_d$

$\boxed{-.560 = B_2}$

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - w_o^2}$

$\boxed{\begin{array}{l} s_1 = 1 \angle -134.4° \\ s_2 = 1 \angle 134.4° \end{array}}$

don't need these

d)

$$Q = \frac{w_0}{\beta}$$

$$w_{c_1}, w_{c_2} = \pm\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$R = 7\Omega$
$L = 5H$
$C = .2F$

$w_{c_1} = .521$

$w_{c_2} = 1.92$

$\beta = w_{c_2} - w_{c_1}$

$\beta = 1.40$

$$Q = \frac{1}{1.40}$$

$$\boxed{Q = .714}$$ ✓

e) we have all the parts from part b (see work there) so we can substitute them into our characteristic equation for our final answer

$$\boxed{i(t) = -.560e^{.7t}\sin(.714t)} \quad t > 0$$

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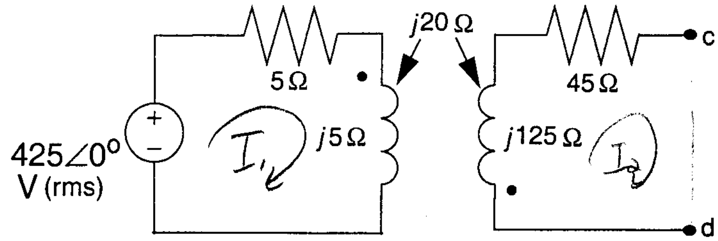
2. For the circuit shown, find the Thévenin equivalent with respect to the terminals c, d.



1.1 I short cct (I_2)   8/10

$$-425 + I_1 5 + I_1 j5 + I_2 j20 = 0$$

$$45 I_2 + I_2 j125 + I_1 j20 = 0$$

$$I_1(5+j5) + I_2(j20) = 425$$

$$I_1(j20) + I_2(45+j125) = 0$$

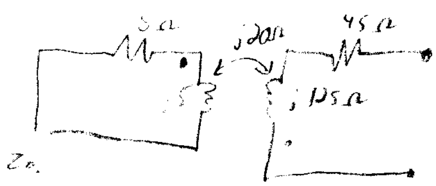
$$I_1 = \frac{425 - I_2(j20)}{5+j5}$$

$$\left(\frac{425 - I_2(j20)}{5+j5}\right)(j20) + I_2(45+j125) = 0$$

$$\frac{425}{5+j5}(j20) = \frac{I_2(j20)}{5+j5}(j20) - I_2(45+j125)$$

$$I_2 = -10 = 10\angle 180$$

find Z_{eq}



$$Z_{LOAD} = 5+5j$$

$$Z_{LOAD}^* = 5-5j$$

$$Z_{eq} = 45+125j + \left(\frac{20}{|5+5j|}\right)^2(5-5j)$$

$$Z_{eq} = 45+125j + \left(\frac{20}{5\sqrt{2}}\right)^2(5-5j)$$

$$Z_{eq} = 45+125j + (+8)(5-5j)$$

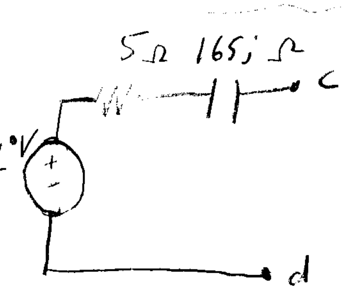
$$Z_{eq} = 45+125j + 40 + 40j$$

$$Z_{eq} = 85 + 165j = \cancel{165\angle 88.3^\circ}\ \Omega$$

85

$$V_{th} = I_2 \cdot Z_{eq} = -850 - j850$$

$$V_{th} = 1650.8 \angle -91.74^\circ\ V$$



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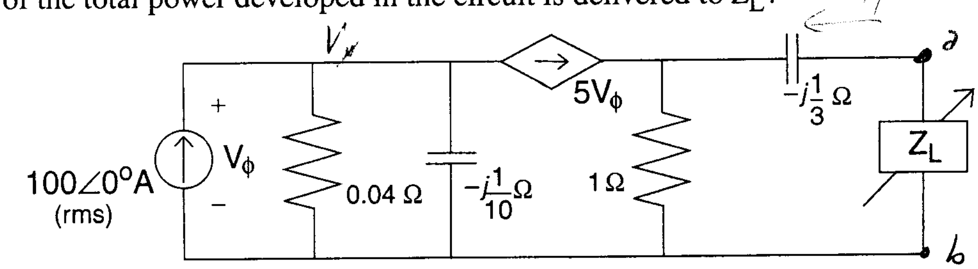
3. The load impedance Z_L for the circuit shown is adjusted until maximum average power is delivered to Z_L.

a.) (8 points) Find the maximum average power delivered to Z_L. 7

b.) (2 points) What percentage of the total power developed in the circuit is delivered to Z_L? 0



first find thevenin equivalent circuit.

$$-100 + \frac{V_\phi}{.04} + \frac{V_\phi}{-j\frac{1}{10}} - 5V_\phi = 0 \quad -1$$

$$V_\phi\left(\frac{1}{.04} - \frac{10}{j} - 5\right) = 100$$

$$V_\phi = 4.47\angle{-26.57^\circ} = 4-2j$$

$$V_{ab} = V_{1\Omega} = V_\phi = 4-2j$$

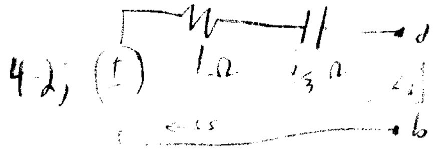
apply a test voltage between a & b & find the test current

$$V_T = i_T \cdot -\frac{j}{3}\Omega + (5V_\phi + i_T)\cdot 1\Omega \qquad \frac{V_\phi}{.04} + \frac{V_\phi}{-j\frac{1}{10}} - 5V_\phi = 0$$

$$V_T = i_T \cdot \frac{j}{3}\Omega + i_T \cdot 1\Omega \qquad V_\phi = 0$$

$$\frac{V_T}{i_T} = Z_{th} = 1 + \frac{j}{3}$$

using V_{th} and Z_{th} we can redraw the circuit:



Thus, Z_L maximizes power at $Z_{th}^* = \left(1 - \frac{j}{3}\right)\Omega$

$$i_s = \frac{4-2j}{1\Omega - \frac{j}{3} + 1\Omega + \frac{j}{3}} = \frac{4-2j}{2} = 2-j \text{ A}$$

$$P = |I_{rms}|^2 R$$

$$P = \left(\sqrt{2^2+1^2}\right)^2 R$$

$$P = (\sqrt{5})^2 \, 1\Omega$$

$$\boxed{P = 5\text{ W}}$$

b) $S_{.04\Omega} = V_\phi^2 \cdot .04\Omega$

$S_{.04\Omega} = .48 - .64j$ VA

$S_{\frac{1}{10\Omega}} = V_\phi^2 \cdot -j\frac{1}{10\Omega}$ X

$S_{j\frac{1}{10\Omega}} = -\frac{8}{5} - \frac{6}{5}j$