EE110 – Circuit Analysis II
Spring 2017 – Dr. Shervin Moloudi
Mid-Term Exam – Thursday May 4, 2017

<u>Instructions</u>

1- You have 1 hour and 50 minutes.

2- Do not attach your own paper. Ask the proctors for extra paper

5- The exam is closed book. No formula sheets, electronic devices, tablets, smart phones/watches allowed. Regular wrist watches allowed.

Name: ____________________

UID: ____________________

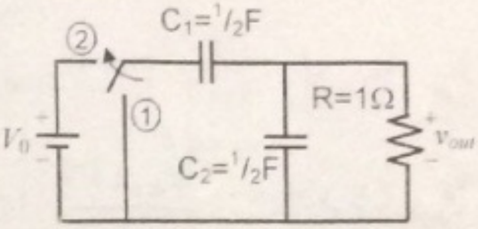
Discussion Session: ____________________

Question	
1	7.5 / 10
2	30 / 30
3	30 / 30
4	30 / 30
Grade	97.5 / 100

Excellent!

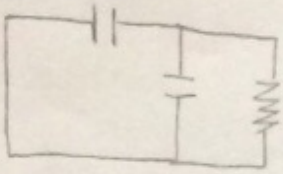
Name:

2- Find $v_{out}(t)$ for all t if the switch goes from position 1 to position 2 to at $t = 0$ and back to position 1 at $t = 1$ sec. The circuit had been at rest for a long time before $t = 0$.



(30 points)

① For $t<0$

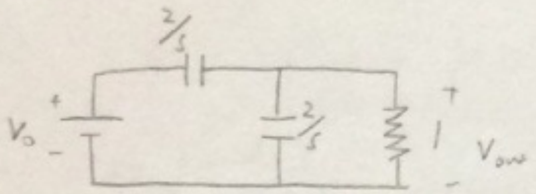


$V_{c_1}(0^-) = 0$

$V_{c_2}(0^-) = 0$

$V_{out}(0^-) = 0$

② For $0<t<1$



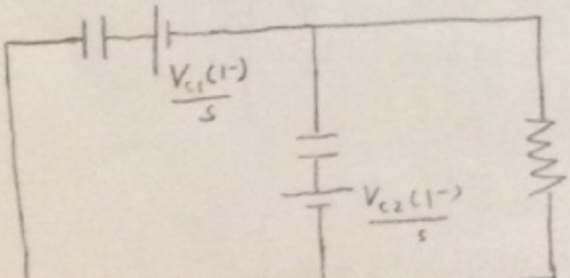
$$\frac{V_{out}(s)}{V_o(s)} = \frac{1 // \frac{2}{s}}{1 // \frac{2}{s} + \frac{2}{s}} = \frac{\frac{\frac{2}{s}}{1+\frac{2}{s}}}{\frac{\frac{2}{s}}{1+\frac{2}{s}} + \frac{2}{s}}$$

$$= \frac{\frac{2}{s}}{\frac{2}{s} + \frac{2}{s}\frac{s+2}{s}} = \frac{2s}{2s+2s+4} = \frac{s}{2s+2} = \frac{1}{2}\frac{s}{s+1}$$

$\therefore V_o(s) = \frac{1}{s}V_o \Rightarrow V_{out}(s) = \frac{1}{2}\frac{1}{s+1}V_o$

$\therefore V_{out}(t) = V_o \frac{1}{2} e^{-t} u(t)$ for $0<t<1$

③ For $t>1$



Name:

Using law of super position

$$V_{out}(s) = -\frac{V_{c1}(1^-)}{s} \cdot \left(\frac{1}{2}\frac{s}{s+1}\right) + \frac{V_{c2}(1^-)}{s} \cdot \left(\frac{1}{2}\frac{s}{s+1}\right)$$

$$V_{c2}(1^-) = V_{out}(1^-) = \frac{1}{2} V_0 e^{-1}$$

$$V_{c1}(1^-) = V_0 - V_{c2}(1^-) = V_0\left(1 - \frac{1}{2}e^{-1}\right)$$

$$\therefore V_{out}(s) = V_0(-1+e^{-1})\frac{1}{2}\frac{1}{s+1}$$

$$\therefore V_{out}(t) = \left[V_0(-1+e^{-1})\frac{1}{2}e^{-t}\right]u(t)$$

However, for $t>1$, time starts at $t=1$

$$\therefore V_{out}(t) = \left[V_0(-1+e^{-1})\frac{1}{2}e^{-(t-1)}\right]u(t-1) \quad , \text{ for } t>1$$

$\therefore$ overall.

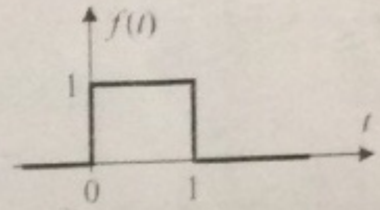
$$V_{out}(t) = \begin{cases} V_0 \frac{1}{2} e^{-t} & , \ 0<t<1 \\ V_0 \frac{1}{2}(-1+e^{-1}) e^{-(t-1)} & , \ t>1 \\ 0 & , \ t<0 \end{cases}$$

$$= \begin{cases} V_0 \frac{1}{2} e^{-t} & , \ 0<t<1 \\ V_0 \frac{1}{2}(1-e) e^{-t} & , \ t>1 \\ 0 & , \ t<0 \end{cases}$$

Name:

3- A) Without using the Laplace Transform, find the impulse response of a circuit, with input $f(t)$ and output $y(t)$, governed by this equation:

$$y' = f'' + f$$



B) Without using the Laplace Transform, find the complete response of this circuit to the input $f(t)$ shown here, if $y(0^-)=1$.

(10+20=30 points)

A) $y = A\,u(t) + B\,\delta(t) + C\,\delta'(t)$

$y' = A\,\delta(t) + B\,\delta'(t) + C\,\delta''(t)$

$y' = \delta''(t) + \delta(t)$

$\Rightarrow \begin{cases} A = 1 \\ B = 0 \\ C = 1 \end{cases}$

$\therefore\ y = u(t) + \delta'(t)$ is the impulse response

$= h(t)$

B) $f(t) = u(t) - u(t-1)$

$$y_{zs}(t) = \int_{-\infty}^{\infty} f(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_0^1 u(t-\tau) + \delta'(t-\tau)\,d\tau$$

$$= \int_0^1 u(t-\tau)\,d\tau + \int_0^1 \delta'(t-\tau)\,d\tau$$

$$= \begin{cases} 0, & t<0 \\ t, & 0\le t\le 1 \\ 1, & t>1 \end{cases} + \big(\delta(t) - \delta(t-1)\big)$$

$$= \begin{cases} 0, & t<0 \\ t+\delta(t)-\delta(t-1), & 0\le t\le 1 \\ 1, & t>1 \end{cases}$$

$\longrightarrow$

3- A) Without using the Laplace Transform, find the impulse response of a circuit, with input $f(t)$ and output $y(t)$, governed by this equation:

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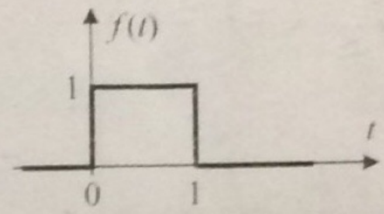

B) Without using the Laplace Transform, find the complete response of this circuit to the input $f(t)$ shown here, if $y(0^-)=1$.

(10+20=30 points)

A) $y = A\,u(t) + B\,\delta(t) + C\,\delta'(t)$

$y' = A\,\delta(t) + B\,\delta'(t) + C\,\delta''(t)$

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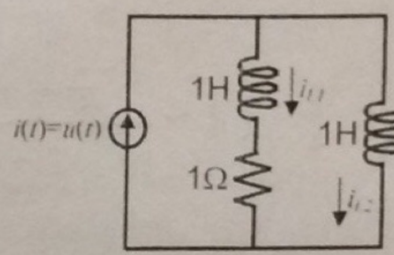
$$= \begin{cases} 0, & t<0 \\ t, & 0 \le t \le 1 \\ 1, & t>1 \end{cases} + \left(\delta(t) - \delta(t-1)\right)$$

$$= \begin{cases} 0, & t<0 \\ t + \delta(t) - \delta(t-1), & 0 \le t \le 1 \\ 1, & t>1 \end{cases}$$

$\longrightarrow$

Name:

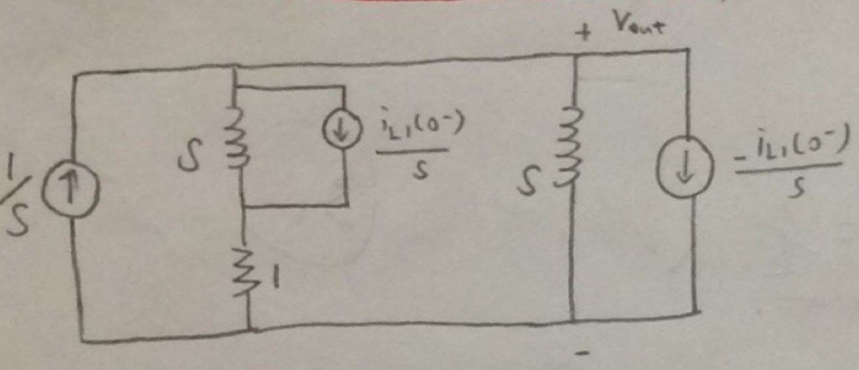
4- A) Find the complete response for $v_{out}(t)$ when the inductors have initial currents $i_{L1}(0^-)$ and $i_{L2}(0^-)$ at $t=0^-$. You can use the Laplace Transform if you choose to.
B) Determine the condition for $i_{L1}(0^-)$ and $i_{L2}(0^-)$ for which the exponential part of the response becomes equal to 0.



(20+10=30 points)

A) at $t=0^-$, $i(t)=0$

$\therefore \; i_{L1}(0^-) = -i_{L2}(0^-)$



$$\frac{1}{S} = I_{L1}(s) + \frac{i_{L1}(0^-)}{S} + I_{L2}(s) + \frac{-i_{L1}(0^-)}{S}$$

$$= I_{L1}(s) + I_{L2}(s)$$

$$I_{L1}(s)\, S + \left[I_{L1}(s) + \frac{i_{L1}(0^-)}{S}\right]\cdot 1 = I_{L2}(s)\cdot S$$

$$\Rightarrow I_{L1}(s)\,(s+1) = I_{L2}(s)\, s - \frac{i_{L1}(0^-)}{S}$$

$$\Rightarrow \frac{1}{S} = \frac{S}{S+1} I_{L2} - \frac{i_{L1}(0^-)}{S(S+1)} + I_{L2}$$

$$\therefore I_{L2} = \left[\frac{1}{S} + \frac{i_{L1}(0^-)}{S(S+1)}\right] \Big/ \left(\frac{2S+1}{S+1}\right)$$

$$V_{out}(S) = I_{L2}(S)\cdot S = \left(1 + \frac{i_{L1}(0^-)}{S+1}\right) \Big/ \left(\frac{2S+1}{S+1}\right)$$

$$= \frac{S+1+i_{L1}(0^-)}{S+1} \cdot \frac{S+1}{2S+1} = \frac{1}{2} \, \frac{S+1+i_{L1}(0^-)}{S+\frac{1}{2}} \rightarrow$$

Name:

$$= \frac{1}{2}\left(1 + \frac{\frac{1}{2} + i_{L1}(0^-)}{s + \frac{1}{2}}\right)$$

$$\therefore V_{out}(t) = \frac{1}{2}\delta(t) + \frac{1}{2}\left(\frac{1}{2} + i_{L1}(0^-)\right)e^{-\frac{1}{2}t}$$ ✓

B) For the exponential part to be 0,

$$i_{L1}(0^-) = -\frac{1}{2}$$

$$\therefore i_{L2}(0^-) = -i_{L1}(0^-) = \frac{1}{2}$$ ✓

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