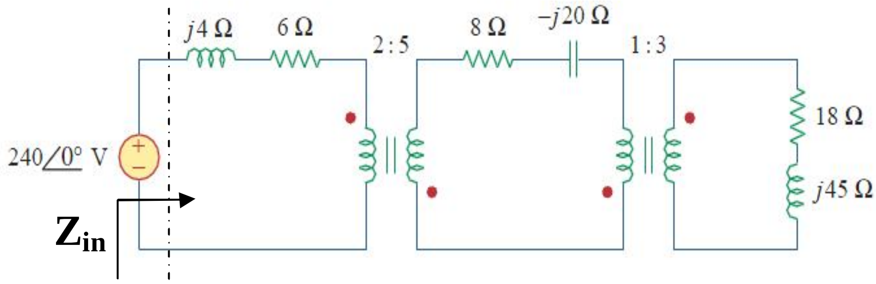
Midterm Solutions

Prob. 1 **For the following circuit (transformers are ideal), find**

a) Input Impedance Z_{in}

b) Complex power supplied by the source?

c) Average power delivered to the 18 Ohm resistor



Solution:

(a) Reflect the load to the middle circuit.

$Z_L' = 8 - j20 + (18 + j45)/3^2 = 10 - j15$

We now reflect this to the primary circuit so that

$Z_{in} = 6 + j4 + (10 - j15)/n^2 = 7.6 + j1.6 = 7.767\angle 11.89°$, where $n = 5/2 = 2.5$

(b) $I_1 = 240/Z_{in} = 240/7.767\angle 11.89° = 30.9\angle{-11.89°}$

$S = 0.5 v_s I_1^* = (120\angle 0°)(30.9\angle 11.89°)$ = **<u>3708 ∠11.89° VA</u>**

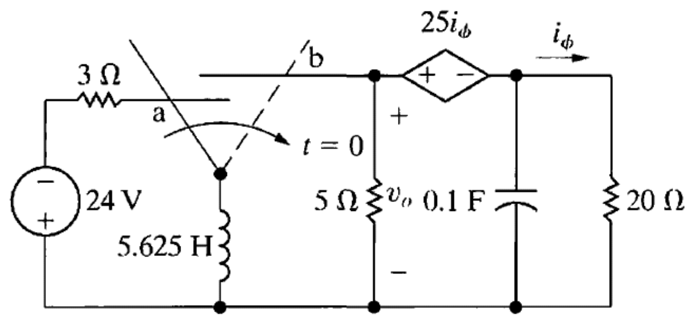
(c) $I_2 = -I_1/n, \quad n = 2.5$

$I_3 = -I_2/n', \quad n' = 3$

$I_3 = I_1/(nn') = 30.9\angle{-11.89°}/(2.5x3) = 4.12 \quad \angle{-11.89°}$

$p = 0.5|I_3|^2(18) = 9(4.12 \quad)^2$ = **<u>152.77 watts</u>**

Prob. 2 **The switch in this circuit has been in position a for a long time. At t = 0, it moves instantaneously to position b.**



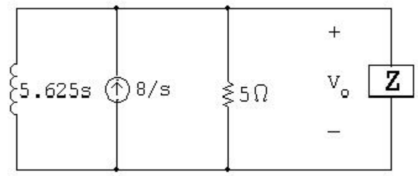
a) Construct S-domain circuit for t > 0

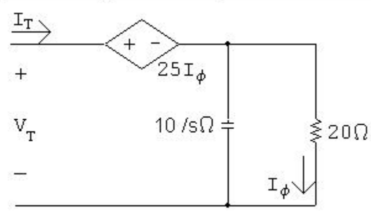
b) Find $V_o(s)$?

c) Find $v_o(t)$?

Solution:

[a] $i_L(0^-) = i_L(0^+) = \frac{24}{3} = 8\,\text{A}$ directed upward





[b]

$$V_T = 25I_\phi + \left[\frac{20(10/s)}{20 + (10/s)}\right] I_T = \frac{25I_T(10/s)}{20 + (10/s)} + \left(\frac{200}{10 + 20s}\right) I_T$$

$$\frac{V_T}{I_T} = Z = \frac{250 + 200}{20s + 10} = \frac{45}{2s + 1}$$

$$\frac{V_o}{5} + \frac{V_o(2s + 1)}{45} + \frac{V_o}{5.625s} = \frac{8}{s}$$

$$\frac{[9s + (2s + 1)s + 8]V_o}{45s} = \frac{8}{s}$$

$$V_o[2s^2 + 10s + 8] = 360$$

$$V_o = \frac{360}{2s^2 + 10s + 8} = \frac{180}{s^2 + 5s + 4}$$

[c]

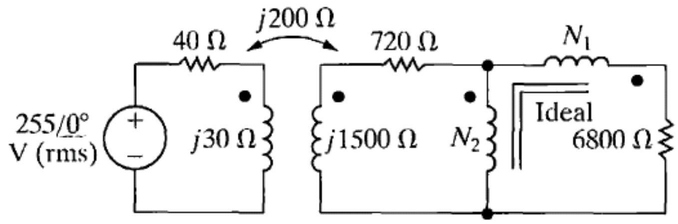
$$V_o = \frac{180}{(s + 1)(s + 4)} = \frac{K_1}{s + 1} + \frac{K_2}{s + 4}$$

$$K_1 = \frac{180}{3} = 60; \qquad K_2 = \frac{180}{-3} = -60$$

$$V_o = \frac{60}{s + 1} - \frac{60}{s + 4}$$

$$v_o(t) = [60e^{-t} - 60e^{-4t}]u(t)\,\text{V}$$

Prob. 3 **In the following circuit,**



a) If N1 = 1000 turns, how many turns should be placed on the N2 winding of the ideal transformer so that Maximum Average Power is delivered to the 6800 ohm load ?

b) Find the Average Power delivered to the 6800 ohm load

c) What percentage of average power generated by voltage source is delivered to 6800 ohm load?

Solution:

[a] $Z_{\text{Th}} = 720 + j1500 + \left(\frac{200}{50}\right)^2 (40 - j30) = 1360 + j1020 = 1700\underline{/36.87^\circ}\,\Omega$

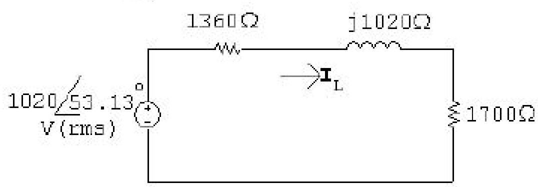
$\therefore \quad Z_{\text{ab}} = 1700\,\Omega$

$$Z_{\text{ab}} = \frac{Z_{\text{L}}}{(1 + N_1/N_2)^2}$$

$(1 + N_1/N_2)^2 = 6800/1700 = 4$

$\therefore \quad N_1/N_2 = 1 \quad \text{or} \quad N_2 = N_1 = 1000 \text{ turns}$

[b] $\mathbf{V}_{\text{Th}} = \frac{255\underline{/0^\circ}}{40 + j30}(j200) = 1020\underline{/53.13^\circ}\,\text{V}$

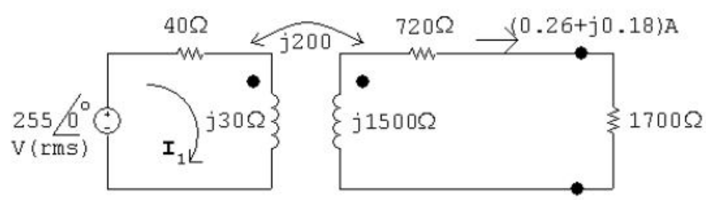


$$\mathbf{I}_L = \frac{1020\underline{/53.13^\circ}}{3060 + j1020} = 0.316\underline{/34.7^\circ}\,\text{A(rms)}$$

Since the transformer is ideal, $P_{6800} = P_{1700}$.

$P = |\mathbf{I}|^2(1700) = 170\,\text{W}$

[c]



$255\underline{/0^\circ} = (40 + j30)\mathbf{I}_1 - j200(0.26 + j0.18)$

$\therefore \quad \mathbf{I}_1 = 4.13 - j1.80\,\text{A(rms)}$

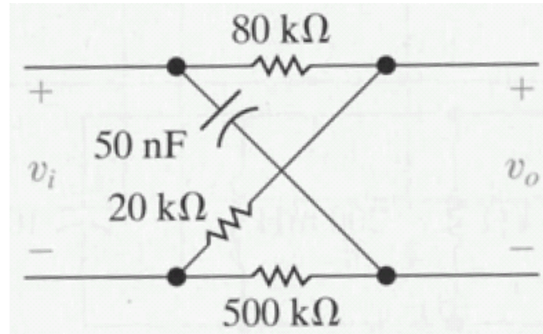
$P_{\text{gen}} = (255)(4.13) = 1053\,\text{W}$

100 (170/1053) = 16.14 %

Prob. 4

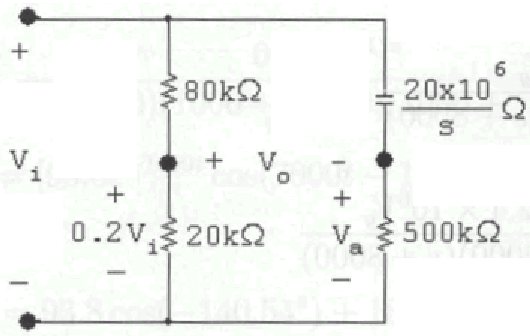
a) Find the numerical expression for the transfer function $H(s) = V_o/V_i$ for the circuit

b) Give the numerical value of each pole and zero of $H(s)$.



Solution:

[a]



$$V_a = \frac{V_i}{500{,}000 + [(20\times10^6)/s]}(500{,}000) = \frac{s}{s+40}V_i$$

$$0.2V_i = V_o + V_a$$

$$\therefore \quad V_o = 0.2V_i - \frac{s}{s+40}V_i$$

$$\frac{V_o}{V_i} = \frac{0.2(s+40)-s}{s+40} = \frac{-0.8s+8}{s+40} = \frac{-0.8(s-10)}{s+40}$$

[b] $-z_1 = 10\,\text{rad/s}$

$-p_1 = -40\,\text{rad/s}$