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Quiz 1 04/13/10

Allotted time: 1 hour (1pt/min delayed submission penalty). Total 20 points.

Calculator and one handwritten double-sided sheet of paper allowed

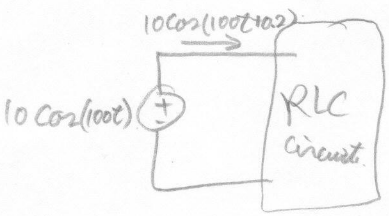
Name:

Student on right:

Student on left:

Student in front:

Q1. A complicated RLC network has steady state current response of 10cos(100t + 0.2) when a voltage source 10cos(100t) is applied to it. Now a voltage source 20cos(100t + 0.1) is applied to the circuit instead of the original source. Can you find the new current response ? Is the information enough for you to find it ? **5 points**



$\underline{V = ZI}$

If we find the equivalent impedence of the circuit, we can find the new current response.

$$Z = \frac{V}{I} = \frac{10\angle 0}{10\angle 0.2} = 1\angle 0-0.2 = \boxed{1\angle -0.2}$$

(Assuming that phase is in radians)

$$= 1\cos(-0.2) + j\sin(-0.2)$$

$$= 0.98 - j\,0.1987$$

Now if the voltage source is

$$20\cos(100t + 0.1) = 20\angle 0.1$$

$$I = \frac{V}{Z} = \frac{20\angle 0.1}{1\angle -0.2} = 20\angle 0.1-(-0.2)$$

$$= 20\angle 0.3$$

$$= \boxed{20\cos(100t + 0.3)}$$

is the new steady state current response of the new source.

5

Q2. You found a rather strange piece of circuitry which three exposed terminals (x,y,z) and it has exactly one resistor, on e inductor, and one capacitor, but you don't know how they are connected. The capacitor has a capacitance of 1 μF. However, the other labels are not readable. The resistances (measured by applying a 1V DC source) between the terminals of the mystery circuit are as follows:

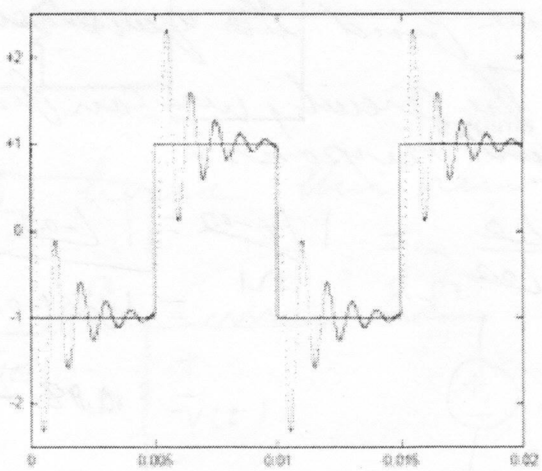
x – y: infinity

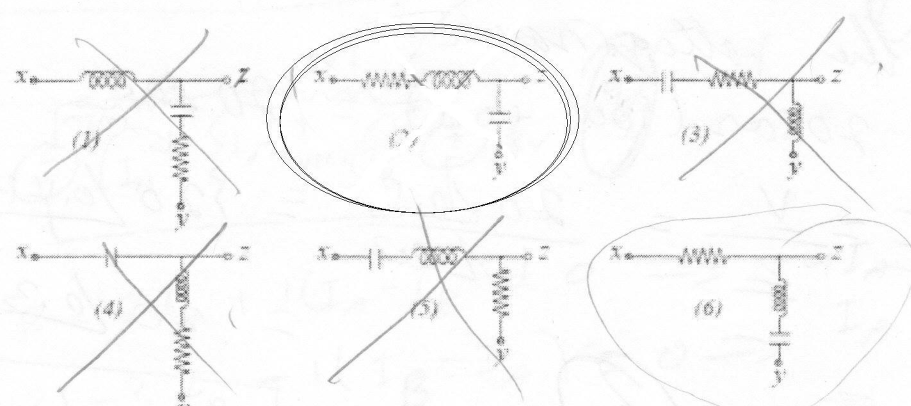
y – z: infinity

z – x: 10

Next, you apply a 1 Volt, 100 Hz, square-wave signal from the x terminal to the y terminal. The plot below displays the voltage from z to y superimposed on the square wave, at the same scale as the square wave. As you can see, waveform from z to y follows the square wave, but there is ringing with a cycle time of about 1 ms.

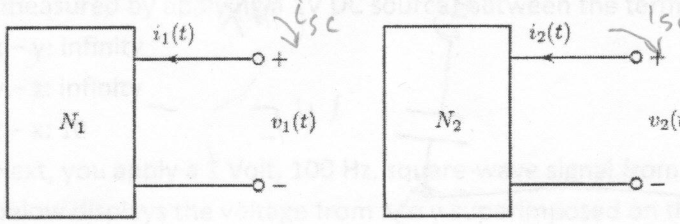
Choose the likely circuit implementation from one of the following options. Also approximately find out the values of R and L . **10 points**



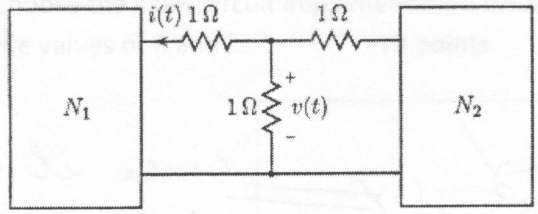


Q3. Consider a two terminal network which have v-i relations as follows

N1: $v1(t) = 4i1(t) - 8$ N2: $v2(t) = 2i2(t) + 3$



Determine v(t) and i(t) if the two networks are connected as below.



5 points

EE-110 Quiz 1

Q1

$V_{s_1}(t) = 10\cos(100t)$

$\Rightarrow P\{V_{s_1}(t)\} = 10\angle 0$

$I_1(t) = 10\cos(100t+0.2)$

$\Rightarrow P\{I(t)\} = 10\angle 0.2$

$V_{s_2}(t) = 20\cos(100t+0.1)$

$\Rightarrow P\{V_{s_2}(t)\} = 20\angle 0.1$

$I_2(t) = ?$

Note

$P\{V(t)\}$ = phasor of $V(t)$

Z = resultant impedance of the network

We have

$$\frac{P\{V_s(t)\}}{P\{I(t)\}} = Z \quad \underline{or} \quad P\{I(t)\} = \frac{P\{V_s(t)\}}{Z}$$

$$\Rightarrow Z = \frac{P\{V_s(t)\}}{P\{I(t)\}}$$

$$\Rightarrow \frac{10\angle 0}{10\angle 0.2} = \frac{20\angle 0.1}{I_{s_2}(t)}$$

$$\Rightarrow \boxed{P\{I_{s_2}(t)\} = 20\angle 0.3}$$

$$\Rightarrow I_{s_2}(t) = 20\cos(100t+0.3)$$

Question 1: Alternate Solution

Q1. A RLC network is a "Linear" System.

When

$$v_1(t) = 10\cos(100t) \longrightarrow \boxed{RLC} \longrightarrow i_1(t) = 10\cos(100t + 0.2)$$

and if $v_2(t) = 20\cos(100t + 0.1)$,

$$v_2(t) = 2 \cdot v_1(t + 1/1000)$$ (← time shifted by 1/1000 & scaled by a factor of ×2).

Applying $v_2(t)$ to RLC network.

$$v_2(t) \longrightarrow \boxed{RLC} \longrightarrow i_2(t).$$

$$\begin{aligned} i_2(t) &= 2 \cdot i_1(t + 1/1000) \\ &= 20\cos(100(t + 1/1000) + 0.2) \\ &= 20\cos(100t + 0.3) \end{aligned}$$

Q2 We have

$R(z-x) = 10$

$\Rightarrow$ (1), (4) and (5) are ruled out.

$R(x-y) = \infty$

$R(y-z) = \infty$

$\Rightarrow$ (3) is ruled out as it just has an inductor along $(z-y)$.

$\Rightarrow$ Possible choices :- (2) or (6)

Consider circuit (6)

Consider the time interval when input voltage goes from $-1V$ to $+1V$.

$\Rightarrow$ $V(x \to y)$ goes from $-1V$ to $+1V$ at $t = t_0$

In the series RLC circuit, current across the inductor cannot change instantaneously.

$\Rightarrow I_L(t = t_0^+) - I_L(t = t_0^-) = 0$

$\Rightarrow V_R(t = t_0^+) - V_R(t = t_0^-) = 0$

Also, voltage across the capacitor cannot change instantaneously

$\Rightarrow$ $V_C(t = t_0^+) - V_C(t = t_0^-) = 0$

$\Rightarrow$ from $t = t_0^-$ to $t = t_0^+$

$\Delta V_R = 0$

$\Delta V_C = 0$

$\Rightarrow$ $\Delta V_L = 2V$ {KVL}

$\Rightarrow$ $\Delta V(y-z) = \Delta V_L + \Delta V_C$

$= 2V$

Therefore in circuit 6, from $t = t_0^-$ to $t = t_0^+$, voltage across $(Y-Z)$ should bump up suddenly by 2V. However, in the waveform, it is seen that $V(y-z)$ lags a little and does not bump up instantaneously.

$\Rightarrow$ (6) is not a solution

Consider (2).

Again, from $t = t_0^-$ to $t = t_0^+$

$$\left.\begin{aligned} \Delta V_R &= 0 \\ \Delta V_C &= 0 \\ \Delta V_L &= 2V \end{aligned}\right\} \quad t_0^- \text{ ~~to~~ } t_0^+$$

$\Rightarrow \Delta V(y-z) = \Delta V_C = 0$

This is verified from the waveform where $V(y-z)$ lags a little compared to the squared input.

$\Rightarrow$ Circuit implementation is (2).

<u>We have</u>

$R(\cancel{z}-x) = 10\,\Omega$

$\Rightarrow \boxed{R = 10\,\Omega}$

$T = 1 \times 10^{-3} = \frac{2\pi}{\omega_d}$

$\Rightarrow \omega_d = 2\pi \times 10^3$

$\sqrt{\omega_0^2 - \alpha^2} = \omega_d$

$\Rightarrow \frac{1}{LC} - \frac{R^2}{4L^2} = 4\pi^2 \times 10^6$

$$\Rightarrow \quad 16\pi^2 L^2 - 4L + 10^{-4} = 0$$

$$\Rightarrow \quad L = \frac{4 \pm \sqrt{16 - 4 \times 16\pi^2 \times 10^{-4}}}{2 \times 16\pi^2}$$

$$\Rightarrow \quad \boxed{L = 0.025\,H}$$

Q3 We will find the Thevenin Equivalent circuits for blocks N_1 and N_2.

I N_1: $v_1(t) = 4i_1(t) - 8$

$i_1(t) = 0 \Rightarrow v_1(t) = -8$

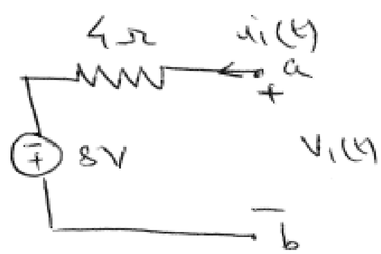
$\Rightarrow V_{oc}(t) = -8V = V_{Th_1}$

$v_1(t) = 0 \Rightarrow i_1(t) = 2A$

$\Rightarrow i_{sc}(t) = -i_1(t)\big|_{v_1(t)=0}$

$= -2A$

$\Rightarrow R_{Th_1} = \dfrac{V_{Th_1}}{i_{sc}\big|_{v_1=0}} = 4\Omega$



(II) N_2: $v_2(t) = 2i_2(t) + 3$

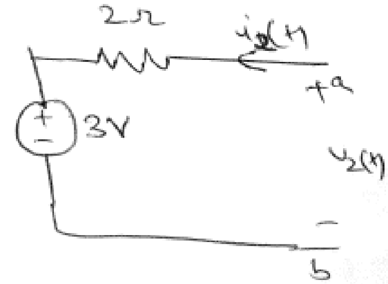
$V_{oc}(t) = V_2(t)\big|_{i_2(t)=0}$

$= 3V = V_{Th_2}$

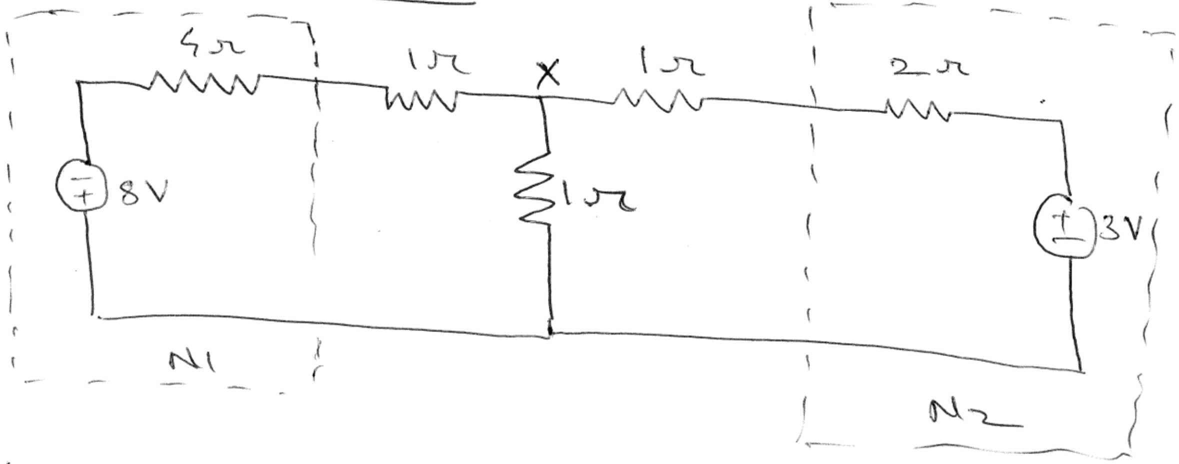
$i_2(t)\big|_{v_2(t)=0} = -\frac{3}{2}A$

$\Rightarrow i_{sc} = -i_2(t)\big|_{v_2(t)=0} = \frac{3}{2}A$

$\Rightarrow R_{Th_2} = \dfrac{V_{Th_2}}{i_{sc}\big|_{v_2=0}} = 2\Omega$



Then, we have



KCL at node X

$$\frac{V_x-(-8)}{4+1} + \frac{V_x}{1} + \frac{V_x-3}{2+1} = 0$$

$$\Rightarrow \frac{V_x+8}{5} + V_x + \frac{V_x-3}{3} = 0$$

$$\Rightarrow 3V_x + 24 + 15V_x + 5V_x - 15 = 0$$

$$\Rightarrow 23V_x + 9 = 0$$

$$\Rightarrow \boxed{V_x = -\frac{9}{23}V} = v(t).$$

$$i(t) = \frac{V_x+8}{5} = \frac{8-\frac{9}{23}}{5} = \boxed{\frac{35}{23}\ A}$$