EE 10, Fall 2014, Midterm Exam – October 29, 2014

Instructions: This exam booklet consists of three problems, blank sheets for the solutions, reference sheets with mathematical identities, and additional blank sheets. Please follow these instructions while answering your exam:

1. **Write your name and student identification number below.**
2. **Write the names of students to your left and right as well.**
3. **You have 1 hour 45 minutes to finish your exam.**
4. **Write your solutions in the provided blank sheets after each problem.**
5. **The sheets marked "Scratch…" will NOT be graded. These sheets are provided for your rough calculations only.**
6. **Write your solutions clearly. You may box in your final answer. Illegible solutions will NOT be graded.**
7. **Be brief.**
8. **Open Book only. NO homework solutions or lecture notes!**

NAME: Solution Key

STUDENT ID: ____________________

NAMES OF ADJACENT STUDENTS:

LEFT: ____________________

RIGHT: ____________________

Problem 1: Consider the Wheatstone Bridge circuit given.

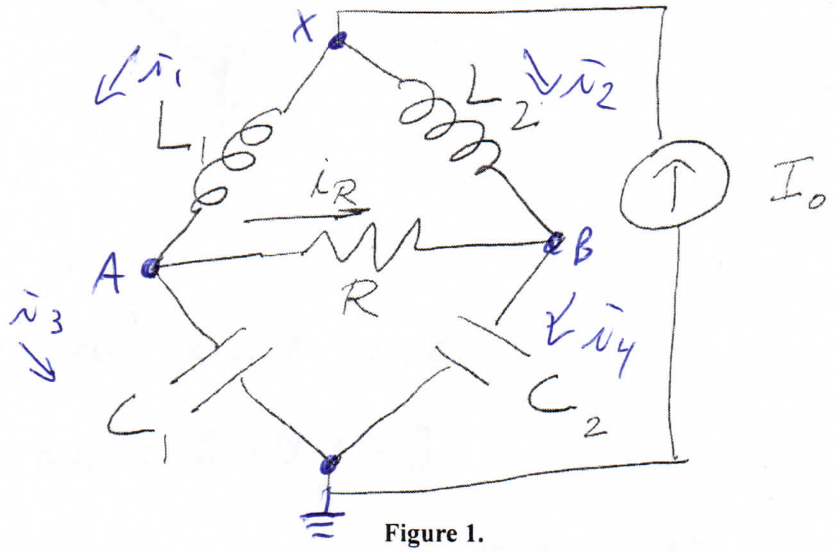


Figure 1.

(a) Draw a graph for this circuit.

(b) Identify a spanning tree.

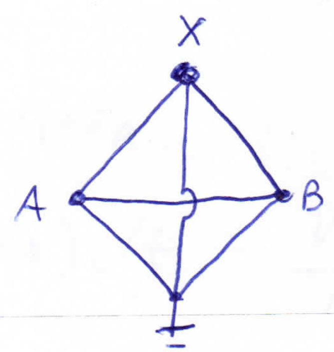
(c) What is the minimum number of unknowns?

(d) Use KCL to write equations for this circuit given the number of minimum unknowns.

(e) If L1 = L2 and C1 = C2 what would be the value of i_R?

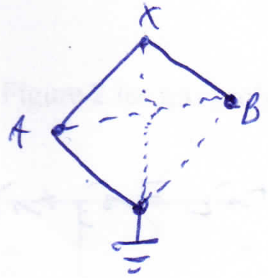
(5 + 5 + 5 + 10 + 5 = 30 points)

a)



Just one example

b)



Just one example

c) Generally you would use

$$b - n + 1 = c = 6 - 4 + 1 = 3$$

Though b/c we have a current source at node X $i_1 + i_2 = I_0$ at node ground $i_3 + i_4 = I_0$ so you would only need to know the voltages at node A & B given that you chose a ground node. So 2 unknowns

d) $I_0 = i_1 + i_2$ & $I_0 = i_3 + i_4$

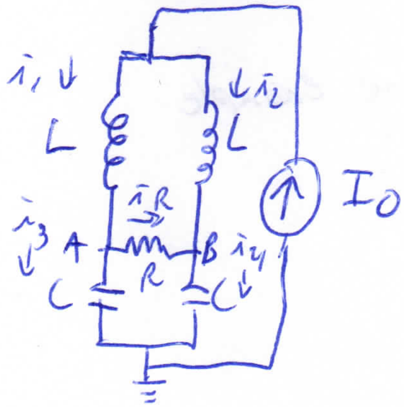
Node A $i_1 = i_3 + i_R$

$$\frac{1}{L_1}\int (V_X - V_A)\,dt = \frac{V_A - V_B}{R} + C_1 \frac{dV_A}{dt}$$

node B $i_R + i_2 = i_4$

$$\frac{1}{L_2}\int (V_X - V_B)\,dt + \frac{V_A - V_B}{R} = C_2 \frac{dV_B}{dt}$$

e)



$L_1 = L_2$

$C_1 = C_2$

$I_0 = i_1 + i_2 = i_3 + i_4$

$i_1 - i_R = i_3$
$i_2 + i_R = i_4$ } subtract these two

$i_1 - i_2 - 2i_R = i_3 - i_4$

$$\frac{1}{L}\int (V_x - V_A)\,dt - \frac{1}{L}\int (V_x - V_B)\,dt$$

$$-2\frac{(V_A - V_B)}{R} = C\frac{dV_A}{dt} - C\frac{dV_B}{dt}$$

b/c $L_1 = L_2$ & $C_1 = C_2$ therefore

$i_1 = i_2$ & $i_3 = i_4$ meaning current splits equally so

$$C\frac{dV_A}{dt} = C\frac{dV_B}{dt} \quad \& \quad \frac{1}{L}\int (V_x - V_A)\,dt = \frac{1}{L}\int (V_x - V_B)\,dt$$

so $-2\frac{(V_A - V_B)}{R} = 0$ current through resistor is <u>zero</u> meaning $V_A = V_B$

Problem 2: Refer to Figure 2 for this problem.

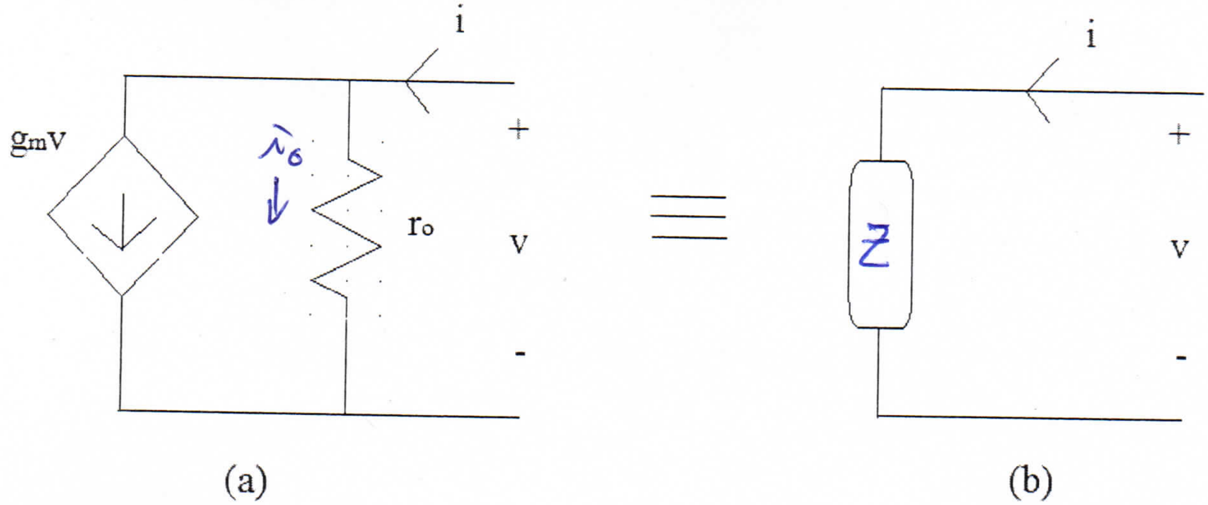


Figure 2.

a. What should the component in Fig.2 (b) be for the two figures to be equivalent?

b. What should be the value of this component?

(10 + 10 = 20 points)

a) & b) $i = i_o + g_m V$ also $V = i_o r_o$

$i = \frac{V}{r_o} + g_m V = V\left(\frac{1}{r_o} + g_m\right)$

$V = i\left(\frac{1}{\frac{1}{r_o} + g_m}\right) = i\underbrace{\left(\frac{r_o}{1 + g_m r_o}\right)}_{Z}$

notice the units of g_m is $\frac{1}{\Omega}$ & units of r_o is Ω

So the term Z has units of Ω's

Z is a resistor (answer for a) with the value of $\frac{r_o}{1 + g_m r_o}$ (answer for b)

Problem 3: Use mesh current analysis method to write down the equations for this circuit.

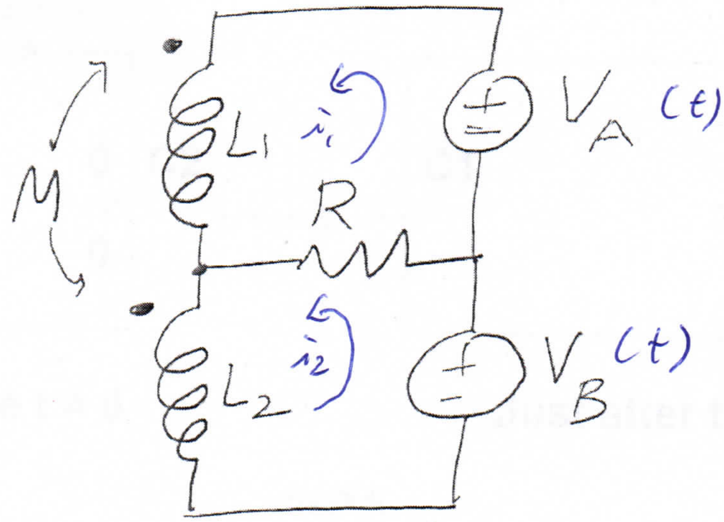


Figure 3.

(20 points)

loop 1

$$V_A(t) = L_1\frac{di_1}{dt} + R(i_1 - i_2) + M\frac{di_2}{dt}$$

loop 2

$$V_B(t) = L_2\frac{di_2}{dt} + R(i_2 - i_1) + M\frac{di_1}{dt}$$

Problem 4: Refer to Figure 4 for this problem. Two capacitors are connected together through a switch that closes at t = 0. C1 has a charge on it prior to the switch closing.

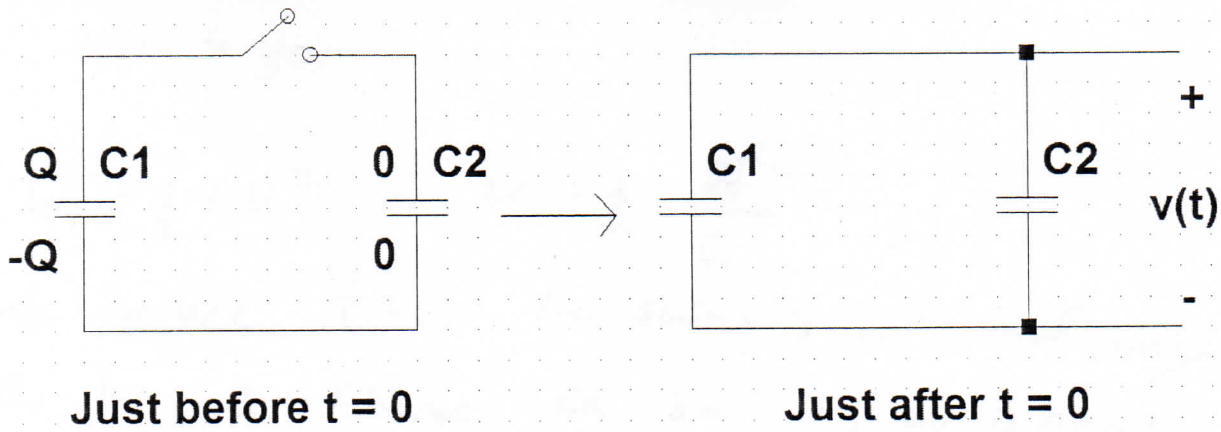


Figure 4.

a. What is the voltage across capacitor C1 just before time t = 0?

b. What is the voltage across capacitor C1 just after time t = 0? [Hint: charge should be conserved].

c. What is the total energy stored in the capacitors before t = 0?

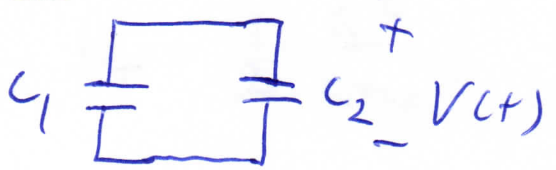
d. What is the total energy stored in the capacitors after t = 0?

(5 + 10 + 5 + 10 = 30 points)

a) $q = CV \rightarrow Q = C_1 V_{c_1} \rightarrow \boxed{V_{c_1} = \frac{Q}{C_1}}$

b) just after time t=0 the switch is already closes so by conservation of charge

Charge before t=0 → $\underline{C_1 V_{c_1} = Q} = V(t)(C_1 + C_2)$



$V(t) = \frac{Q}{C_1 + C_2}$

After the switch is closed the charge must be conserved. Also C_1 & C_2 are now in parallel so V(t) is the voltage across C_1 & C_2. Remember the charge before must equal the charge after (total)

b) The total charge before must equal the total charge after in the circuit. The charge has nowhere else to go.

c) $W_C = \frac{1}{2} C V^2$ or $W_C = \frac{1}{2} \frac{Q^2}{C}$

b/c before $t=0$ the switch is open and only C_1 has a charge Q on it, C_1 is the only capacitor with energy stored

so

$$W_{C_1} = \frac{1}{2} \frac{Q^2}{C_1} \quad , \quad Q = C_1 V_{C_1}$$

d) After $t=0$ the switch is closed and the charge redistributes itself amongst C_1 & C_2. Though remember the total charge is the same ~~which~~ which is Q.

so $$Q = V(t)(C_1 + C_2)$$

$$W_T = \frac{1}{2} \frac{Q^2}{C_1 + C_2}$$