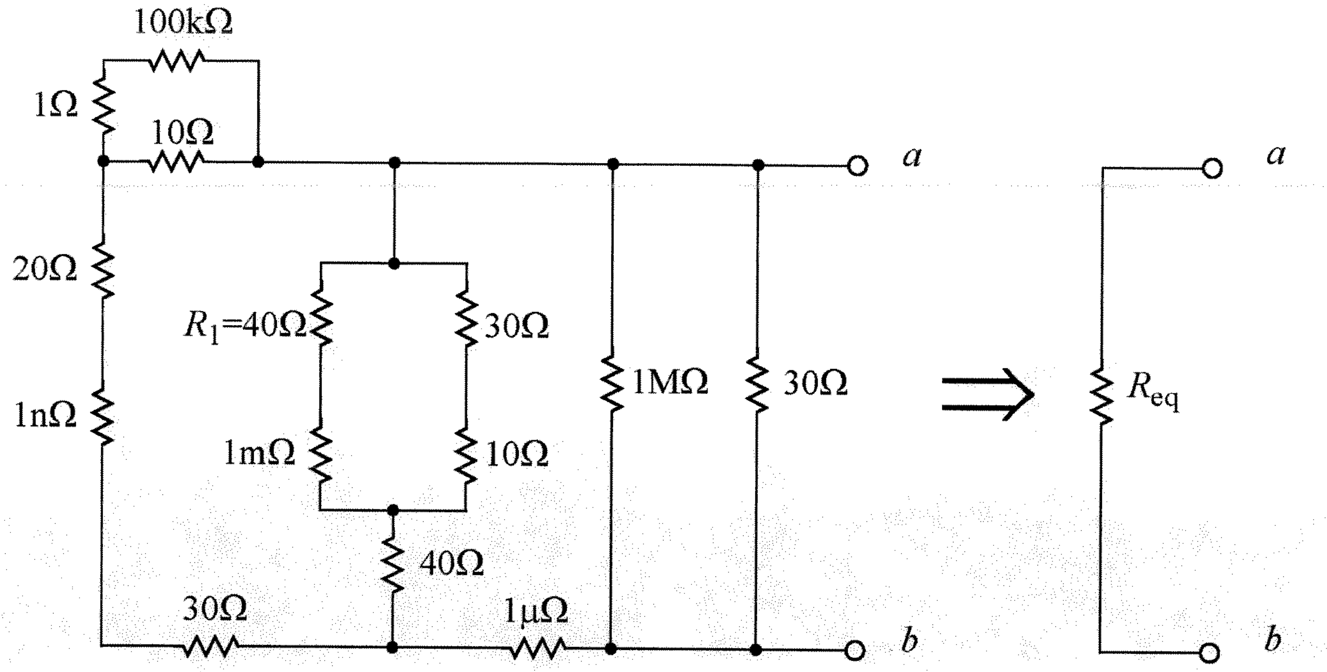
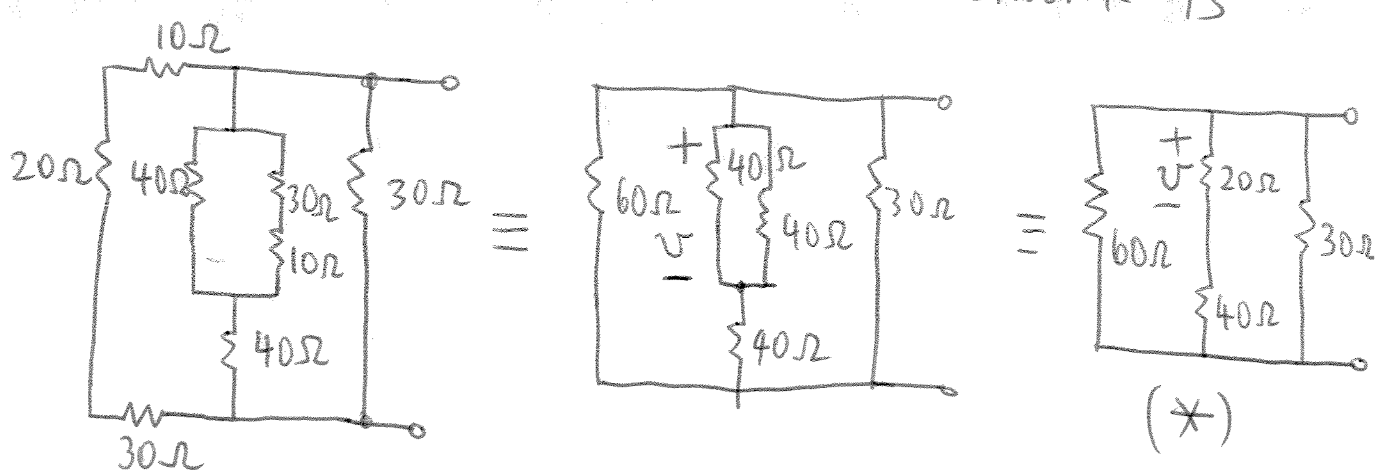
Problem 1: Equivalent Resistance **(15 points)**



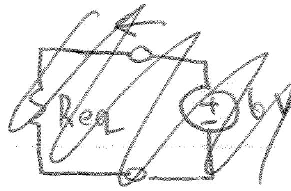
1-1. Reduce the following resistor network to single resistor and find its approximate value (i.e., to two significant digits). Show your work in order to receive full and partial credit. Draw intermediate circuit diagrams as you simplify the resistor network. (5 points)

We remove the large resistors (compared to parallel resistors) to simplify the network. (1MΩ & 100kΩ) The 1Ω resistor also can be neglected when the 100kΩ is removed. The small value resistors (1μΩ, 1nΩ, 1mΩ) can be shorted out. The simplified network is

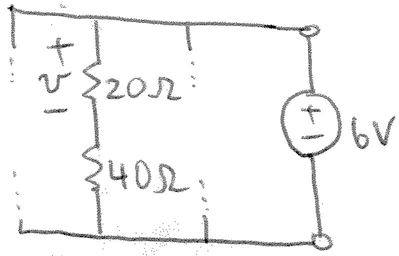


$\equiv (60 \| 60) \| 30 = 30 \| 30 = \boxed{15\,\Omega = R_{eq}}$

1-2. If a 6-V source is attached to nodes a and b (i.e., v_{ab} = 6 V), calculate the approximate voltage across R_1. Show your work in order to receive full and partial credit. This includes re-drawing a portion of the circuit given on the previous page and showing your equations. (5 points)



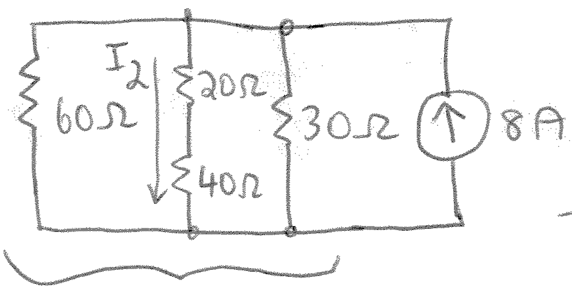
Note the voltage across R_1 is the same voltage across the 20 Ω resistor in the last diagram from Prob 1-1.



$$\boxed{v = 6V \cdot \frac{R_1}{R_{eq}} = 6 \cdot \frac{20}{60} = 2V}$$

1-3. If a 8-A source is attached to nodes a and b (i.e., 8 A flowing from b to a), calculate the current passing through the 1-mΩ resistor. Show your work in order to receive full and partial credit. This includes re-drawing a portion of the circuit given on the previous page and showing your equations. (5 pt)

Again from 1-1, we see the current first splits into 3 branches. By current division,

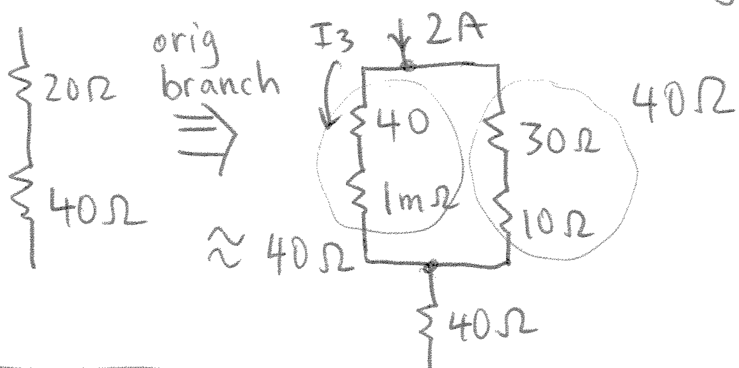


$$I_2 = 8A \cdot \frac{15}{20+40} \quad \left(= \frac{R_{eq} \cdot I_{in}}{R_{branch}}\right)$$

$$= 2A$$

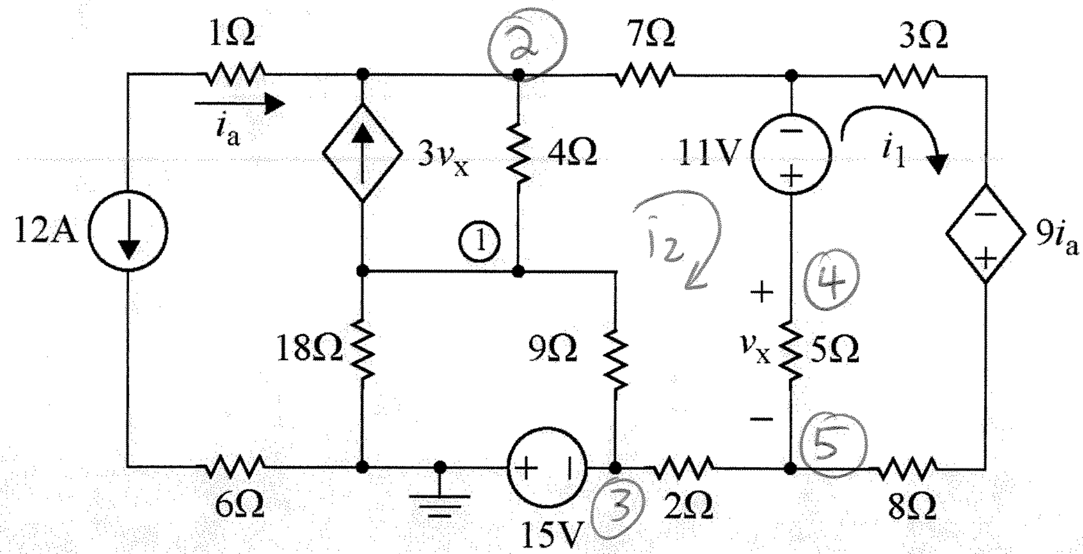
From 1-1, $R_{eq} = 15\,\Omega$

Then this current splits again



$$\boxed{I_3 = 2A \cdot \frac{20\,\Omega}{40\,\Omega} = 1\,A = I \text{ through } 1m\Omega \text{ Res.}}$$

Problem 2: Node-Voltage or Mesh-Current Analysis **(15 points)**



2-1. For the circuit above, would you prefer to use mesh or node-voltage analysis? Explain your choice. (5 points)

Mesh. There are less meshes than node voltages to find.

2-2. Write the node-voltage equation at node (1) in terms of only node voltages you define and numerical values. Simplify the equation by grouping node-voltage terms, but do not numerically evaluate fractions. Put a box around your final answer. (5 points)

Annotate diagram with node names.

$$\frac{V_1}{18} + \frac{V_1 - V_3}{9} + \frac{V_1 - V_2}{4} + 3(V_4 - V_5) = 0$$

But $V_3 = -15$

$$\boxed{V_1\left(\frac{1}{18} + \frac{1}{9} + \frac{1}{4}\right) - \frac{V_2}{4} - \cancel{\frac{V_3}{9}} + 3V_4 - 3V_5 = \frac{-15}{9}}$$

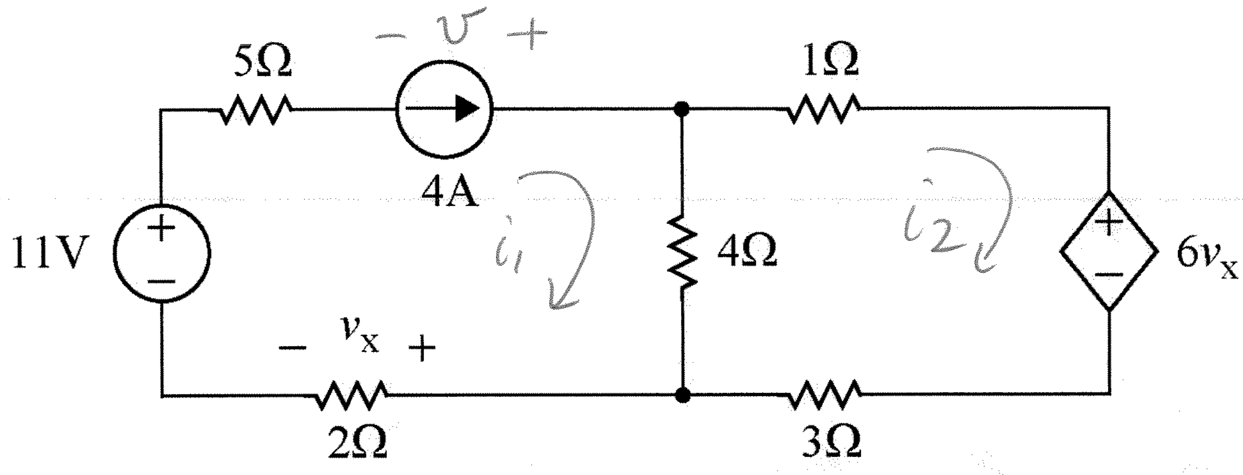
2-3. Write the mesh equation for i_1 in terms of only mesh currents you define and numerical values (i.e., not in terms of dependent-source variables). Simplify the equation by grouping node-voltage terms, but do not numerically evaluate fractions. Put a box around your final answer. (5 points)

Define i_2 mesh. $i_a = -12$ A

$$i_1(3) + (-9i_a) + 8i_1 + (i_1 - i_2)5 + 11 = 0$$

$$\boxed{16i_1 - 5i_2 = -108 - 11 = -119}$$

Problem 3: ***Power Generation and Dissipation*** ***(32 points)***



3-1. Calculate the power in each of the 8 components, and state whether it is generating or absorbing power. Show all intermediate calculations to receive full and partial credit.

$i_1 = 4A \qquad v_x = 8V$

$i_2 + 48 + 3i_2 + 4i_2 - 4i_1 = 0$

$8i_2 = -32$

$i_2 = -4A$

Find v across current source (KVL since we know all I in components)

$32 + 8 - 11 + 20 - v = 0$

$v = 49$ V

Power in $6v_x$

48V ⇒ P = 192 W Gen

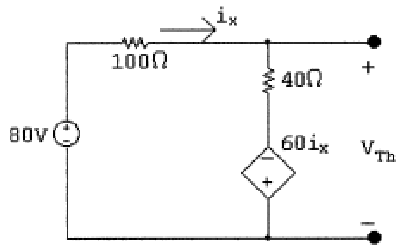
Element (-)	Power (W)	Gen/Dis (-)
11 V	44	Gen
4 A	196	Gen
6 v_x	192	gen
1 Ω	16	Dis
2 Ω	32	Dis
3 Ω	48	Dis
4 Ω	256	Dis
5 Ω	80	Dis

432W

432 W

Q4.

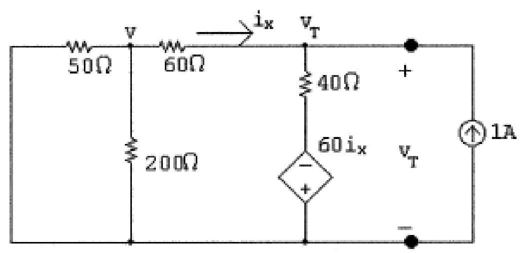
We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_x = \frac{80 + 60i_x}{140}; \qquad i_x = 1 \text{ A}$$

$$V_{\text{Th}} = 40i_x - 60i_x = -20i_x = -20 \text{ V}$$

Using the test-source method to find the Thévenin resistance gives



Use the node voltage method:

$$\frac{v}{50} + \frac{v - v_{\text{T}}}{60} + \frac{v}{200} = 0$$

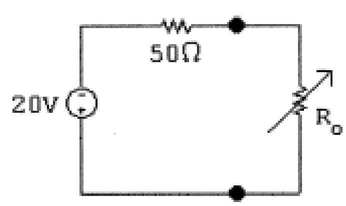
$$\frac{v_{\text{T}} - v}{60} + \frac{v_{\text{T}} + 60i_x}{40} - 1 = 0$$

$$i_x = \frac{v - v_{\text{T}}}{60}$$

Solving, $\qquad v_{\text{T}} = 50$ V.

$$R_{\text{Th}} = \frac{v_{\text{T}}}{1 \text{ A}} = 50\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$\left(\frac{-20}{50 + R_o}\right)^2 R_o = 1.5$$

$$\frac{400R_o}{R_o^2 + 100R_o + 2500} = 1.5$$

$$1.5R_o^2 - 250R_o + 3750 = 0$$

$$\therefore \quad R_o = 16.67\,\Omega; \quad R_o = 150\,\Omega$$