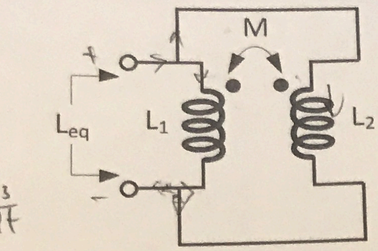
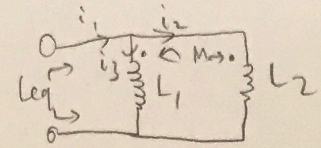
1. Find the equivalent inductance of the circuit below. $L_1 = 5H$, $L_2 = 2H$, and $M = 3H$.



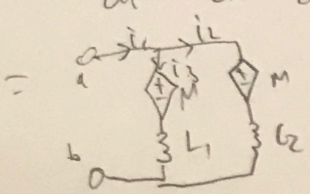
$L_{eq} = \frac{V_{ab}}{\frac{di_1}{dt}}$

$V = L\frac{di}{dt}$

inductors are in parallel



$\frac{di_1}{dt} = \frac{di_2}{dt} + \frac{di_3}{dt}$



$V_{ab} = M\frac{di_2}{dt} + L_1\frac{di_3}{dt}$

$V_{ab} = M\frac{di_3}{dt} + L_2\frac{di_2}{dt}$

$V_{ab} = M\left(\frac{di_1}{dt}\right)\left(\frac{M-L_1}{M-L_2}\right)\left(\frac{M-L_2}{M-L_2+M-L_1}\right) + L_1\left(\frac{di_1}{dt}\right)\left(\frac{M-L_2}{M-L_2+M-L_1}\right)$

$\frac{di_2}{dt}(M - L_2) = \frac{di_3}{dt}(M - L_1)$

$\frac{di_2}{dt} = \frac{di_3}{dt}\left(\frac{M-L_1}{M-L_2}\right)$

$V_{ab} = \frac{di_1}{dt}\left(M\left(\frac{M-L_1}{2M-L_2-L_1}\right) + L_1\left(\frac{M-L_2}{2M-L_2-L_1}\right)\right)$

$\frac{di_1}{dt} = \frac{di_3}{dt}\left(1 + \frac{M-L_1}{M-L_2}\right)$

$L_{eq} = \frac{V_{ab}}{\frac{di_1}{dt}} = \frac{M^2 - L_1M}{2M-L_2-L_1} + \frac{L_1M - L_1L_2}{2M-L_2-L_1}$

$\frac{di_1}{dt} = \frac{di_2}{dt}\left(\frac{M-L_2}{M-L_1}\right)\left(1 + \frac{M-L_1}{M-L_2}\right)$

$= \frac{M^2 - L_1L_2}{2M-L_2-L_1}$

$M = 3H$
$L_1 = 5H$ $L_2 = 2H$

$\frac{9-10}{6-5-2} = \frac{-1}{-1} = \boxed{1}$

$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}$

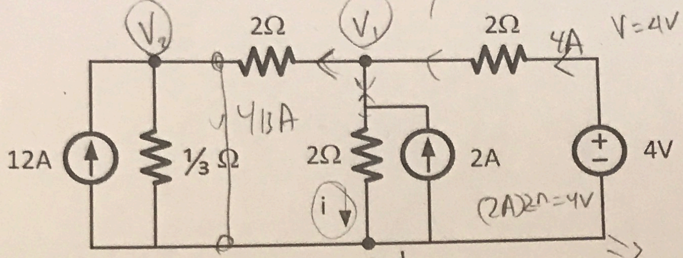
Leq = 1H

$= \frac{L_1L_2 - M^2}{L_1+L_2-2M}$ ✓

$\frac{10-9}{7-6} = \frac{1}{1}$ ✓

+30

2. In the circuit below, using node analysis,
   a. Find the current i.
   b. Find the power dissipated in the $\frac{1}{3}\,\Omega$ resistor.



a) V_1: $\frac{V_1 - V_2}{2} + \frac{V_1 - 4}{2} - 2 + \frac{V_1}{2} = 0$

V_2: $\frac{V_2 - V_1}{2} - 12 + \frac{V_2}{1/3} = 0$

$V_1 - V_2 + V_1 - 4 + V_1 = 4$

$3V_1 - V_2 = 8$

$\frac{V_2 - V_1}{2} - 12 + 3V_2 = 0 \Rightarrow V_2 - V_1 - 24 + 6V_2 = 0$

$7V_2 = V_1 + 24$

$V_1 = 7V_2 - 24$

$3(7V_2 - 24) - V_2 = 8$

$21V_2 - 72 - V_2 = 8$

$20V_2 = 80$

$V_2 = 4V$

$V_1 = (4)7 - 24$

$V_1 = 4V$

Find $R_{th} = 3\,\Omega$
$I_{SC} = 4/3\,A$
$V_{OC} = 4V$
from LHS

from RHS

$4A - 4/3A = 8/3A$

but, $\frac{V_1}{2} = \frac{4}{2} = 2A = i$

$i = 2A$

b) $V_2 = 4V$ $P = \frac{V^2}{R} = \frac{(4)^2}{\frac{1}{3}\Omega} = \frac{16}{\frac{1}{3}} = 48W$

P = 48W in $\frac{1}{3}\Omega$ resistor

+30

$$i_1 = \frac{100 + 10 i_L(t)}{25}$$

3. The circuit below has been idle for a long time (switch is open). At $t = 0$, the switch is closed.

   a. Find the inductor current right before and after the switch is closed ($i_L(0^-)$, and $i_L(0^+)$).

   b. Find v_o right before and after the switch is closed ($v_o(0^-)$, and $v_o(0^+)$).

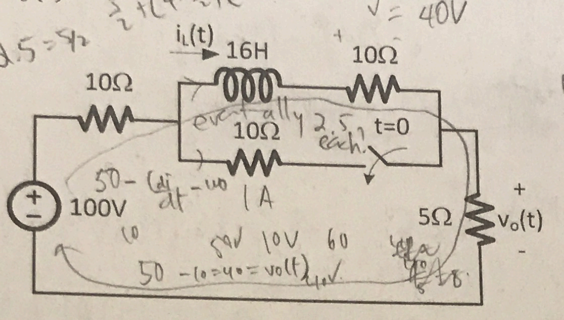
   c. Find and plot $v_o(t)$.

$16\frac{di}{dt} + 10(i_L) = (10\Omega)$

$L\frac{di}{dt} = 40V$

$R_{eq} = 10 + 5 + 5 = 20$

$\tau = \frac{L}{R}$, $\frac{16}{20} = \frac{4}{5}$



at infinity, shorts again, we get

a) KVL at $t = 0^-$. Also, inductor is a short if idle for a long time b/c no change in current

$$100V = 10(i_L(t)) + 10(i_L(t)) + 5(i_L(t)) \Rightarrow 25 i_L(t) = 100V$$

$i_L(0^-) = 4A$

at $t = 0^+$, switch closes. Inductor doesn't want change in current.

$i_1 = i_L(t) + i_2(t)$

$i_2(t) = i_1 - i_L(t)$

$$100V = 10(i_1) + 16\frac{di_L(t)}{dt} + 10 i_L(t) + 5(i_1)$$

$i_L(0^+) = i_1 - i_2(t)$

$$100V = 10(i_1) + 10(i_1 - i_L(t)) + 5(i_1)$$

$$25 i_1 - 10 i_L(t) = 100V \qquad i_1 = \frac{100 + 10 i_L(t)}{25}$$

$$100V = 10\left(\frac{100 + 10 i_L(t)}{25}\right) + 16\frac{di_L(t)}{dt} + 10 i_L(t) + 5\left(\frac{100 + 10 i_L(t)}{25}\right)$$

$$40 = 16 i_L(t) + 16\frac{di_L(t)}{dt}$$

$$\frac{di_L(t)}{dt} + i_L(t) = \frac{10}{4} = \frac{5}{2}$$

$$\int (i_L(t) \cdot e^t)' = \int \frac{5}{2} e^t + C$$

$u = e^{\int dt} = e^t$

$i_L(t) = \frac{5}{2} + Ce^{-t}$

$i_L(0) = 4$

$i_L(0^+) = 4A$

$i_L(0^+) = \frac{5}{2} + \frac{3}{2}e^{-t}$ for $t = 0$, 4A

140/5 = 28

b) before switch closes, V_o?

$V_o(t)$

$i(0^-) = 4A$

$100 - 4(10) - 4(10) - 4(5)$

$= 0V$

$V_o(0^-) = 20V$

+2

after switch closes?

if $i_L(0^+) = \frac{3}{2}A$

$i_1 = \frac{100 + 10(i_L(t))}{25} = \frac{100 + 10(\frac{9}{2})}{25} = \frac{100 + 45}{25} = \frac{140}{25}$

For $v_o(0^+)$

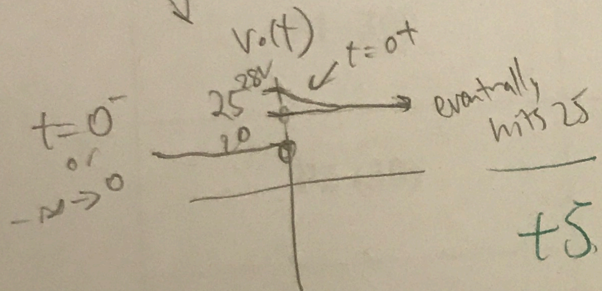
$V_o(t) = (i_1) R_{5\Omega} = 23V = V_o(t)$ at 0^+

$\frac{115}{25} = \frac{23}{5} = i_1$

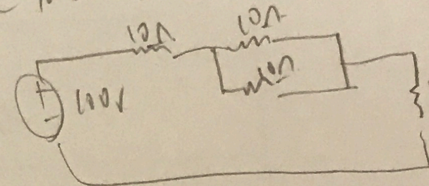
$V_o(t) = 5\frac{(100 + 10(\frac{5}{2} + \frac{3}{2}e^{-t}))}{25} = \frac{1}{5}(100 + 10(\frac{8}{2})) = \frac{140}{5} = 28V = (V_o(0^+) = 28V)$

+5

c) at infinity, we know inductor shorts and we get



+5



$= R_{eq} = 20$

$I_{out} = 5A$

$(5) 5\Omega = V_o(\infty)$

$V_o(t) = ?$