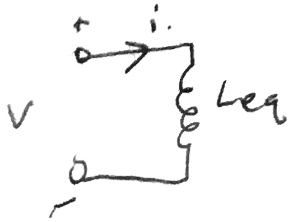
Name: _

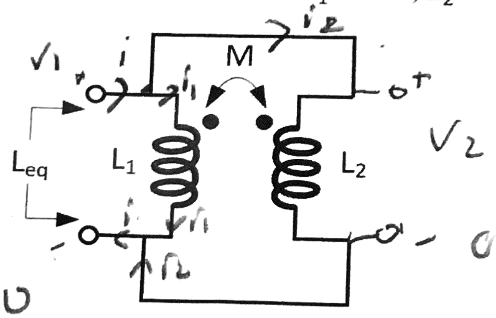
UID: _ ____

Total of 3 questions, 100 minutes.

P1 (30)	30
P2 (30)	30
P3 (40)	26
Total (100)	

1. Find the equivalent inductance of the circuit below. $L_1 = 5H$, $L_2 = 2H$, and $M = 3H$.





M should be added since currents enter inductors through dots.

By KVL: $-v_1 + v_2 = 0$

$v_1 = v_2 = v$ where v is some voltage.

$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = v$ $\qquad v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = v$

$L_2 \left(v = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}\right)$ $\Rightarrow$ $L_2 v = L_1 L_2 \frac{di_1}{dt} + M L_2 \frac{di_2}{dt}$

$-M \left(v = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}\right)$ $\qquad + \left(-Mv = -ML_2 \frac{di_2}{dt} - M^2 \frac{di_1}{dt}\right)$

$(L_2 - M) v = (L_1 L_2 - M^2) \frac{di_1}{dt}$

$$\frac{di_1}{dt} = \frac{(L_2 - M) v}{L_1 L_2 - M^2}$$

$-M \left(v = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}\right)$ $\Rightarrow$ $-Mv = -M L_1 \frac{di_1}{dt} + M^2 \frac{di_2}{dt}$

$L_1 \left(v = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}\right)$ $\qquad + \ L_1 v = L_1 L_2 \frac{di_2}{dt} + M L_1 \frac{di_1}{dt}$

$(L_1 - M) v = (L_1 L_2 - M^2) \frac{di_2}{dt}$

$$\frac{di_2}{dt} = \frac{(L_1 - M) v}{L_1 L_2 - M^2}$$

By KCL: $-i + i_1 + i_2 = 0$ $\quad i = i_1 + i_2$

$\frac{d}{dt} (i = i_1 + i_2)$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$= \frac{(L_1 - M) v}{L_1 L_2 - M^2} + \frac{(L_2 - M) v}{L_1 L_2 - M^2} = \frac{(L_1 + L_2 - 2M) v}{L_1 L_2 - M^2}$$

$$V = L_{eq} \frac{di}{dt}$$

$$V = L_{eq} \left(\frac{(L_1 + L_2 - 2M)\, \dot{x}}{L_1 L_2 - M^2} \right)$$

$$L_{eq} = \frac{1}{\frac{L_1 + L_2 - 2M}{L_1 L_2 - M^2}}$$

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$L_1 = 5\,H$ $L_2 = 2\,H$ and $L_3 = 3\,H$

so

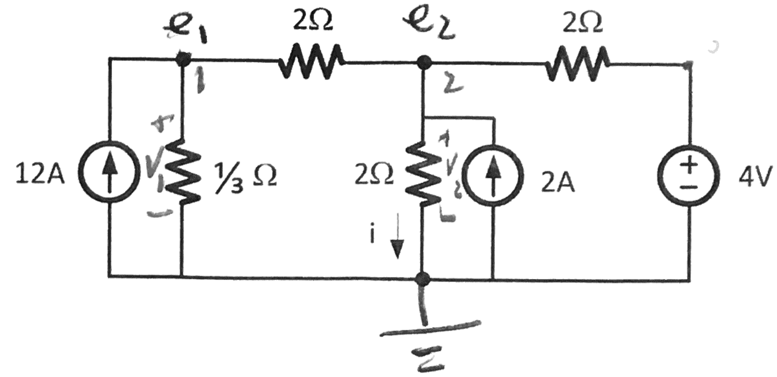
$$L_{eq} = \frac{(5\,H)(2\,H) - (3\,H)^2}{5\,H + 2\,H - 2(3\,H)}$$

$$= \frac{10\,H^2 - 9\,H^2}{7\,H - 6\,H} = \frac{1\,H^2}{1\,H} = 1\,H$$

$$\boxed{L_{eq} = 1\,H}$$

+30

2. In the circuit below, using node analysis,
   a. Find the current i.
   b. Find the power dissipated in the $\frac{1}{3}\Omega$ resistor.



a. Node-voltage analysis

Node 1: $-12A + \frac{e_1 - 0}{1/3\,\Omega} + \frac{e_1 - e_2}{2\,\Omega} = 0$

$$3e_1 + \frac{e_1}{2} - \frac{e_2}{2} = 12$$

$$\frac{7}{2}e_1 - \frac{e_2}{2} = 12$$

Node 2: $\frac{e_2 - e_1}{2\,\Omega} + \frac{e_2 - 0}{2\,\Omega} - 2A + \frac{e_2 - 4}{2\,\Omega} = 0$

$$\frac{3e_2}{2} - \frac{e_1}{2} - 2 - 2 = 0$$

$$-\frac{e_1}{2} + 3\frac{e_2}{2} = 4$$

system of equations:

$3\left(\frac{7}{2}e_1 - \frac{e_2}{2} = 12\right) \Rightarrow$ $\left(\frac{21}{2}e_1 - \frac{3e_2}{2} = 36\right)$

$-\frac{e_1}{2} + 3\frac{e_2}{2} = 4$ $\quad + \left(-\frac{e_1}{2} + \frac{3e_2}{2} = 4\right)$

$\frac{20}{2}e_1 = 40 \qquad 10e_1 = 40$

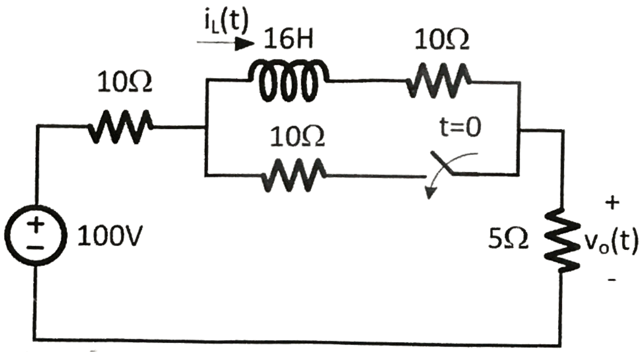
$e_1 = 4\text{ V}$

3. The circuit below has been idle for a long time (switch is open). At $t = 0$, the switch is closed.

a. Find the inductor current right before and after the switch is closed ($i_L(0^-)$, and $i_L(0^+)$).

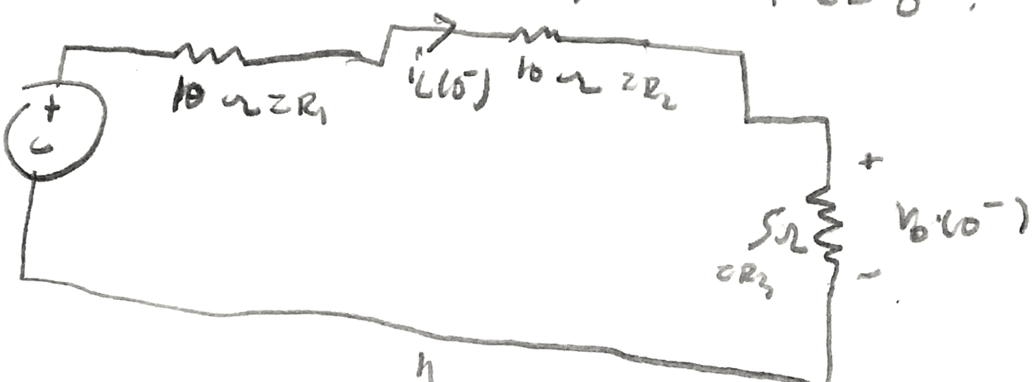
b. Find v_o right before and after the switch is closed ($v_o(0^-)$, and $v_o(0^+)$).

c. Find and plot $v_o(t)$.



a. If circuit is idle for long time by $t=0^-$, then inductor acts as a short at $t=0^-$.

equivalent circuit at $t=0^-$



$R_{eq} = R_1 + R_2 + R_3 = 10\Omega + 10\Omega + 5\Omega$
$= 25\ \Omega$

$i_L(0^-) = \frac{100\ V}{R_{eq}} = \frac{100\ V}{25\ \Omega} = 4A$

By KVL, voltage appeared across the resistor = 100V

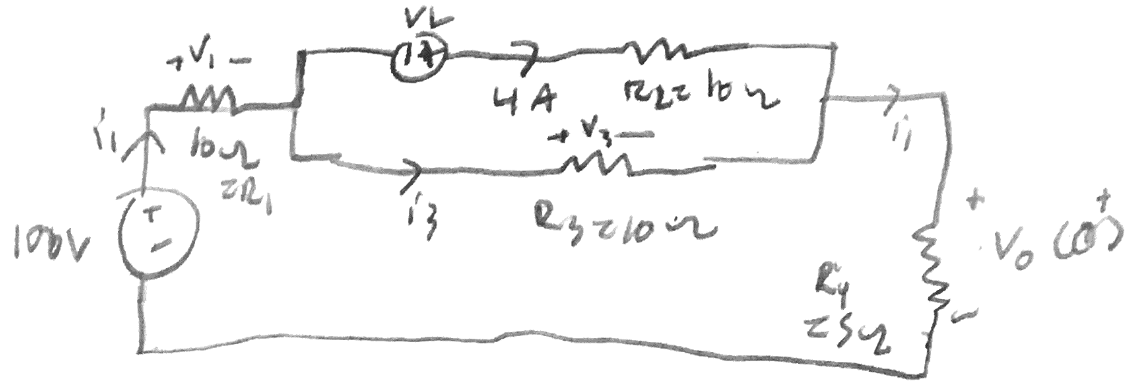
$\boxed{i_L(0^-) = 4A}$ +5

At $t=0^+$, an infinite voltage can't be supplied to the inductor as the voltage supply is only 100 V, so $i_L(0^+) = i_L(0^-)$ $\boxed{i_L(0^+) = 4A}$ +2

b. Look at equivalent circuit at $t=0^-$, since $i_L(0^-) = 0$ +2

$V_o(0^-) = i_L(0^-) R_3 = (4\ A)(5\ \Omega) = 20\ V$ $\boxed{V_o(0^-) = 20V}$

equivalent circuit at $t = 0^+$



By KCL: $-i_1 + i_3 + 4A = 0$ $\quad i_1 = i_3 + 4A$

By KVL: $-100V + v_1 + v_3 + v_0(0^+) = 0$

$v_1 + v_3 + v_0(0^+) = 100V$

$i_1 R_1 + i_3 R_3 + i_1 R_4 = 100V$

$(i_3 + 4) R_1 + i_3 R_3 + (i_3 + 4) R_4 = 100$

$i_3 (R_1 + R_3 + R_4) = 100 - 4R_1 - 4R_4$

$i_3 = \frac{100 - 4R_1 - 4R_4}{R_1 + R_3 + R_4} A = \frac{100 - 4(10) - 4(5)}{10 + 10 + 5} A$

$= \frac{100 - 60}{25} A = \frac{40}{25} A = \frac{8}{5} A$

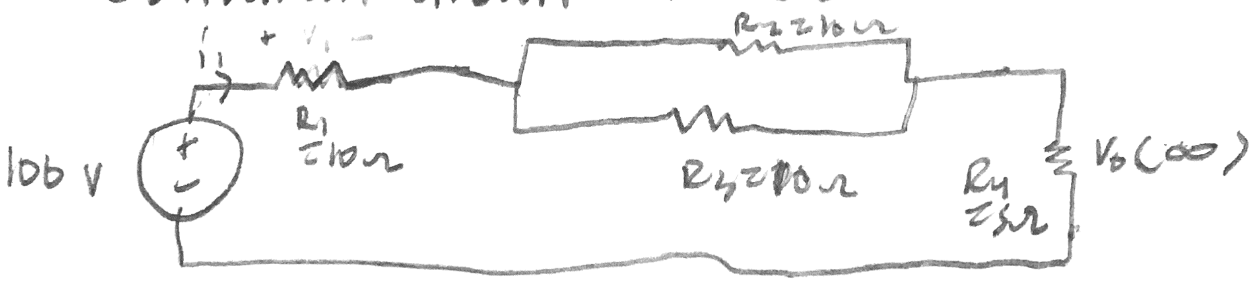
$v_0(0^+) = i_1 R_4 = (i_3 + 4) R_4 = (\frac{8}{5} A + 4A)(5\Omega)$

$= 28V$ $\quad \boxed{v_0(0^+) = 28V}$

+5

c. at $t = \infty$, inductor acts as short again

equivalent circuit at $t = \infty$



equivalent to

$R_{eq} = R_1 + R_2 \parallel R_3 + R_4$

$= R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4 =$

$(10\Omega) + \frac{(10\Omega)(10\Omega)}{10\Omega + 10\Omega} + 5\Omega$

$= 10\Omega + 5\Omega + 5\Omega = 20\Omega$

By KVL voltage across $R_{eq} = 100\text{ V}$

$$i_1 = \frac{100\text{ V}}{R_{eq}} = \frac{100\text{ V}}{20\,\Omega} = 5\text{ A}$$

$$V_o(\infty) = i_1 R_4 = (5\text{ A})(5\,\Omega) = 25\text{ V}$$ +5

Since this is a DC circuit: for $t < 0$: $V_o(t) = V_o(0^-) =$

for $t \geq 0$: $V_o(t) = V_o(\infty) + (V_o(0^+) - V_o(\infty))\, e^{-\frac{t}{\tau}}$

in this case $\tau = \frac{L_{eq}}{R_{eq}(t)}$ Note: $R_{eq}(t) = R_{eq}(\infty) = 20\,\Omega$ recall

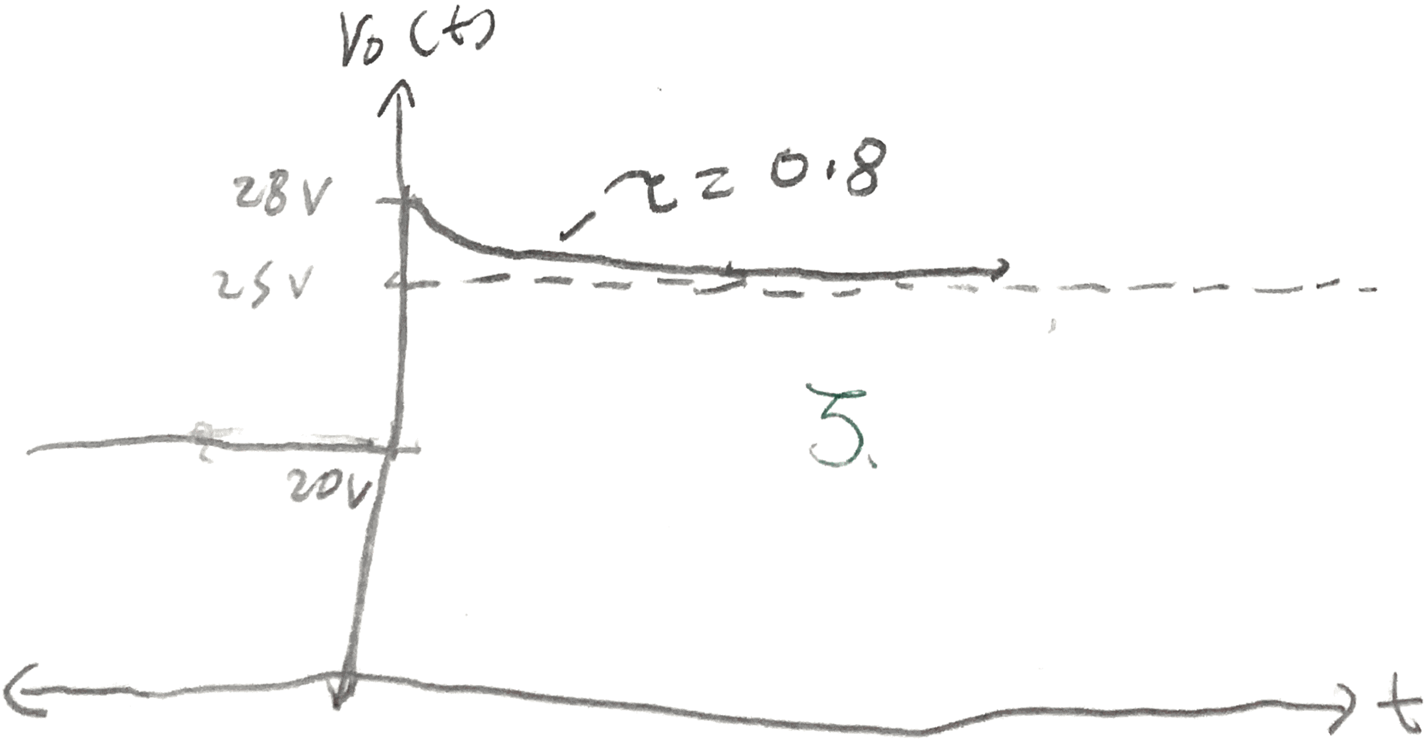
$$\tau = \frac{16\text{ H}}{20\,\Omega} = 0.8$$ X

so $V_o(t) = 25\text{ V} + (28\text{ V} - 25\text{ V})\, e^{-\frac{t}{0.8}}$

$= 25\text{ V} + 3\text{ V}\, e^{-1.25t}$ for $t \geq 0$

so

$$\boxed{V_o(t) = \begin{cases} 20\text{ V} & \text{for } t < 0 \\ 25\text{ V} + 3\text{ V}\, e^{-1.25t} & \text{for } t \geq 0 \end{cases}}$$



$\frac{7}{2}(4V) - \frac{e_2}{2} = 12V$

$14V - \frac{e_2}{2} = 12V$

$-\frac{e_2}{2} = -2V \quad e_2 = 4V$

$i = \frac{v_2}{2\Omega} = \frac{e_2 - 0}{2\Omega} = \frac{4V - 0}{2\Omega} = 2A$

$\boxed{i = 2A}$

b. $P_{2\Omega_3} = \frac{(v_1)^2}{1/3\,\Omega} = \frac{(e_1 - 0)^2}{1/3\,\Omega} = \frac{(4V - 0)^2}{1/3\,\Omega}$

$= \frac{16V^2}{1/3\,\Omega} = 48W$

$\boxed{P_{2\Omega_3} = 48W}$

+30