EE10 Midterm 2

Department of Electrical Engineering, UCLA

Winter 2016

Instructor: Prof. Gupta

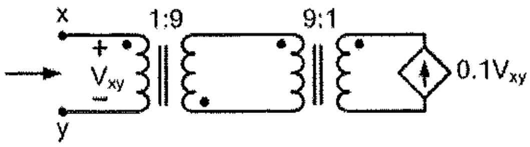
1. Exam is closed book. Calculator and one double sided cheat-sheet is allowed.
2. Cross out *everything* that you don't want me to see. Points will be deducted for everything wrong!
3. No points will be given without proper explanations
4. Time allotted: 75 minutes

	Score	
1	10	10
2	10	10
3	10	10
Total	30	30

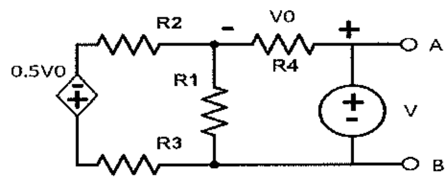
Q1. 10 points (5+5)

Find the Thevenin equivalent circuit of each network at terminals x-y/A-B.

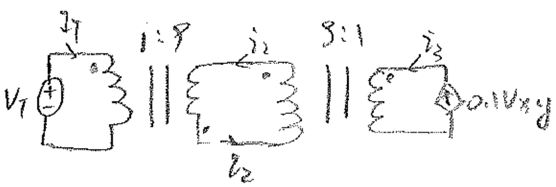
(a)



(b)



(a) We put a test voltage (V_T) at x and y



In this case $R_{TH} = \frac{V_T}{I_T}$

$V_{xy} = V_T$

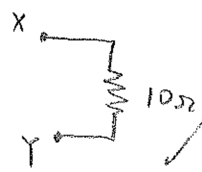
$i_2 = \frac{1}{9} I_T$ $\quad$ $i_2 = \frac{1}{9} i_3$

$i_3 = 0.1 V_T$ $\quad$ $I_T = i_3 = 0.1V_T$

$I_T = 0.1V_T$

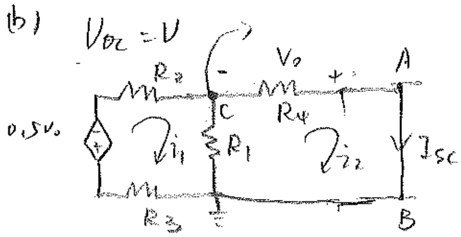
$R_{TH} = \frac{V_T}{I_T} = \frac{V_T}{0.1V_T} = 10\Omega$

thevenin equivalent:



V_{TH} would be zero because there is no independent source.

(b) $V_{oc} = V$



loop1: ① $-R_2 i_1 - R_1 i_1 - R_3 i_1 - 0.5V_0 + i_2 R_1 = 0$ $\quad +\frac{1}{2}R_4 i_2$

loop2: ② $-R_1 i_2 - R_4 i_2 + R_1 i_1 = 0$.

$V_0 = -R_4 i_2$

①

$-i_1(R_1+R_2+R_3) + i_2(R_1 + \frac{1}{2}R_4) = 0$

$i_1(R_1+R_2+R_3) = i_2(R_1+\frac{1}{2}R_4)$ $\quad$ $i_1 = \frac{(R_1+\frac{1}{2}R_4)}{R_1+R_2+R_3} i_2$

②:

$-i_2(R_1+R_4)+R_1 i_1 = 0$

$\left[-(R_1+R_4) + \frac{(R_1+\frac{1}{2}R_4)}{R_1+R_2+R_3} R_1\right] i_2 = 0$

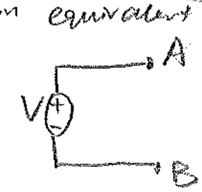
$i_2 = 0 = J_{sc}$

$I_{SC} = \infty$

$V_A = 0 = V_B$

$i_2 = J_{sc} = 0$ $\quad R_{TH} = \frac{V}{0} = \infty$.

thevenin equivalent:



$Q_{1f} + Q_{2f} = Q_1 + Q_2$

$V_{C1}(\infty) = \frac{Q_{1f}}{C_1}$ $\quad$ $\frac{Q_{1f}}{C_1} = \frac{Q_{2f}}{C_2}$

$V_{C2}(\infty) = \frac{Q_{2f}}{C_2}$ $\quad$ $(Q_1 + Q_2 - Q_{2f})C_2 = C_1 Q_{2f}$

$V_{C2} = \frac{1}{C_2} \cdot \frac{C_2Q_1 - C_1Q_2}{\cancel{C_1C_2} R} \cdot \frac{\cancel{C_1C_2}}{-(C_1+C_2)} (0-1)$

$V_{C2} = \frac{C_2Q_1 - C_1Q_2}{C_2(C_1+C_2)} + \frac{Q_2}{C_2}$

$Q_2 \quad V_{C1} = \frac{-(C_2Q_1 - C_1Q_2)}{R C_1 (C_1+C_2)}$

Q2. 10 points

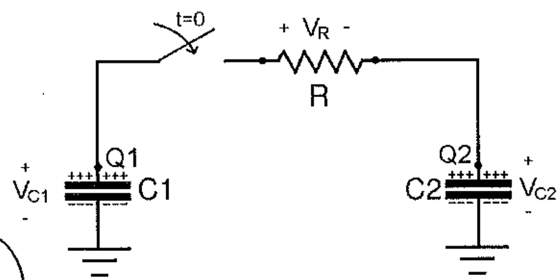
Capacitors C1 and C2 are initially charged with Q1 and ~~Q1~~ Coulombs respectively. At t=0 the switch closes.

$C_2Q_1 + C_2Q_2 - C_2Q_{2f} = C_1Q_{2f}$

$(C_1+C_2)Q_{2f} = C_2Q_1 + C_2Q_2$

$Q_{2f} = \frac{C_2(Q_1+Q_2)}{C_1+C_2}$

$V_{C2} = \frac{Q_1+Q_2}{C_1+C_2} \quad V_{C1} = \frac{Q_1+Q_2}{C_1+C_2}$



$\frac{C_2Q_1 - \cancel{C_1Q_2} + \cancel{Q_2}C_1 + Q_2C_2}{C_2(C_1+C_2)}$

$\frac{\cancel{C_2}(Q_1+Q_2)}{\cancel{C_2}(C_1+C_2)}$

$\frac{Q_1C_1 + \cancel{Q_1C_2} + C_1Q_2 - \cancel{C_2Q_1}}{C_1(C_1+C_2)}$

a) Find $V_{C1}(0-)$, $V_{C2}(0-)$ and $V_R(0-)$.

b) Find $V_{C1}(0+)$, $V_{C2}(0+)$ and $V_R(0+)$.

c) Find V_{C1}, V_{C2} and V_R at steady state.

d) For the case where Q1=Q, Q2=0C and C1=C2=C. Calculate the total amount of energy dissipated by the resistor after the system reached its steady state.

(a) $Q = CV \quad Q_1 = V_{C1} \cdot C_1 \quad Q_2 = C_2 \cdot V_{C2}$

$V_{C1} = \frac{Q_1}{C_1} \quad V_{C2} = \frac{Q_2}{C_2}$ since the switch is open, there is no current

$\boxed{V_{C1}(0-) = \frac{Q_1}{C_1} \quad V_{C2}(0-) = \frac{Q_2}{C_2} \quad V_R(0-) = 0}$

(b) Since the voltage across capacitor could not change instantaneously

$\boxed{V_{C1}(0^+) = \frac{Q_1}{C_1}} \quad \boxed{V_{C2}(0^+) = \frac{Q_2}{C_2}}$

V_{C1}, V_{C2}

$V_R = V_{C1} - V_{C2}$

$\boxed{V_R(0^+) = \frac{Q_1}{C_1} - \frac{Q_2}{C_2}}$

(c) at steady state capacitors work as open circuit no current through the resistor $\boxed{V_R = 0}$

V_{C1}, V_{C2}, i_1

$\left(\frac{1}{C_1} + \frac{1}{C_2}\right) i_1 + R\frac{di_1}{dt} = 0$

$i_1 = K \cdot e^{\frac{-(C_1+C_2)t}{C_1C_2R}}$

$i_1(0^+) = \frac{C_2Q_1 - C_1Q_2}{C_1C_2R} = K$

$i_1 = \frac{C_2Q_1 - C_1Q_2}{C_1C_2R} \cdot e^{\frac{-(C_1+C_2)t}{C_1C_2R}}$

continue on top right

$V_{C2} = \frac{1}{C_2}\int_0^{\infty} i_1\,dt + V_{C2}(0^+)$ at steady state.

$V_{C1} = V_{C1}(0^+) - \frac{1}{C_2}\int_0^{\infty} i_1\,dt$

$\boxed{V_{C2} = \frac{C_2Q_1 - C_1Q_2}{C_2(C_1+C_2)} + \frac{Q_2}{C_2} = \frac{Q_1+Q_2}{C_1+C_2}}$

$\boxed{V_{C1} = \frac{Q_1}{C_1} + \frac{C_1Q_2 - C_2Q_1}{C_1(C_1+C_2)} = \frac{Q_1+Q_2}{C_1+C_2}}$

(d) V_{C1}, $+ V_R -$, V_{C2}, i_1

$Q_1 = Q \quad Q_2 = 0 \quad C_1 = C_2 = C$

$V_R(0^+) = \frac{Q}{C}$

$i(0^+) = \frac{Q}{CR}$

$-\frac{1}{C}\int_{-\infty}^{t} i_1\,dt - i_1 R - \frac{1}{C}\int_{-\infty}^{t} i_1\,dt = 0$

$+\frac{1}{C}i_1 + R\frac{di_1}{dt} + \frac{1}{C}i_1 = 0.$

$+\frac{2}{C}i_1 + R\frac{di_1}{dt} = 0$

$\frac{2}{RC}i_1 + \frac{di_1}{dt} = 0$

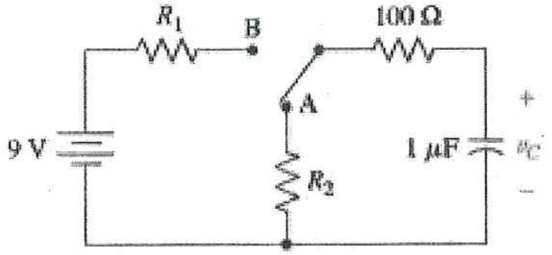
$i_1 = K \cdot e^{-\frac{2t}{RC}}$

$i_1 = \frac{Q}{CR} \cdot e^{\frac{-2t}{RC}}$

(on the back)

Q3. 10 points

The switch in the circuit below has been in position A for long time. It is switched to position B at t=0 and back to position A at t=1ms. Find R1 and R2 such that Vc(1ms) = 8V and Vc(2ms) = 1V.



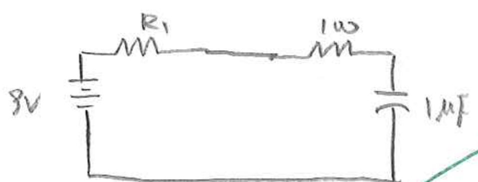
When $t<0$

there is no current.

$V_C(0^-) = V_C(0^+) = 0V$.

Since voltage across capacitor can't change instantaneously

~~t~~ $0<t<1$



$R_{eq} = R_1 + 100$

$\tau_1 = R_{eq} \cdot C$

$V_C = 8(1 - e^{-t/\tau_1})$

$V_C(1ms) = 8 \therefore 9(1 - e^{-1ms/\tau_1})$

$1 - e^{-1ms/\tau_1} = \frac{8}{9}$

$e^{-1ms/\tau_1} = \frac{1}{9}$

$-\frac{1ms}{\tau_1} = \ln\frac{1}{9}$

$\frac{1ms}{\tau_1} = -\ln\frac{1}{9}$

$\tau_1 = \frac{1ms}{-\ln\frac{1}{9}}$

$\tau_1 = (R_1 + 100) \times 10^{-6}$

~~τ~~ $R_1 + 100 = \frac{1 \times 10^{-3} \times 10^{6}}{-\ln\frac{1}{9}}$

$\boxed{R_1 = 355.12\,\Omega}$

$t>1$

$R_{eq} = R_2 + 100$

$\tau_2 = R_{eq} \cdot C = (R_2 + 100) \times 10^{-6}$

$V_C = k \cdot e^{\frac{-(t - 1ms)}{\tau_2}}$

when $t = 1ms$

$V_C = 8V$ so $k = 8$

$V_C = 8 \cdot e^{\frac{-(t-1ms)}{\tau_2}}$

$V_C(2ms) = 8 \cdot e^{\frac{-(1ms)}{\tau_2}} = 1$

$e^{\frac{-(1ms)}{\tau_2}} = \frac{1}{8}$

$\frac{-(1ms)}{\tau_2} = \ln\frac{1}{8}$

~~&~~ $\frac{1ms}{\tau_2} = -\ln\frac{1}{8}$

$\frac{\tau_2}{1ms} = \frac{1}{-\ln\frac{1}{8}}$

$\tau_2 = \frac{1 \times 10^{-3}}{-\ln\frac{1}{8}}$

$R_2 + 100 = \frac{1 \times 10^{3}}{-\ln\frac{1}{8}}$

$\boxed{R_2 = 380.90\,\Omega}$

$\begin{cases} R_1 = 355.12\,\Omega \\ R_2 = 380.9\,\Omega \end{cases}$