EE10 Midterm 2

Department of Electrical Engineering, UCLA

Winter 2016

Instructor: Prof. Gupta

1. Exam is closed book. Calculator and one double sided cheat-sheet is allowed.
2. Cross out *everything* that you don't want me to see. Points will be deducted for everything wrong!
3. No points will be given without proper explanations
4. Time allotted: 75 minutes

Name: [redacted]

Student ID: [redacted]

Student on Left: [redacted]

Student on Right: [redacted]

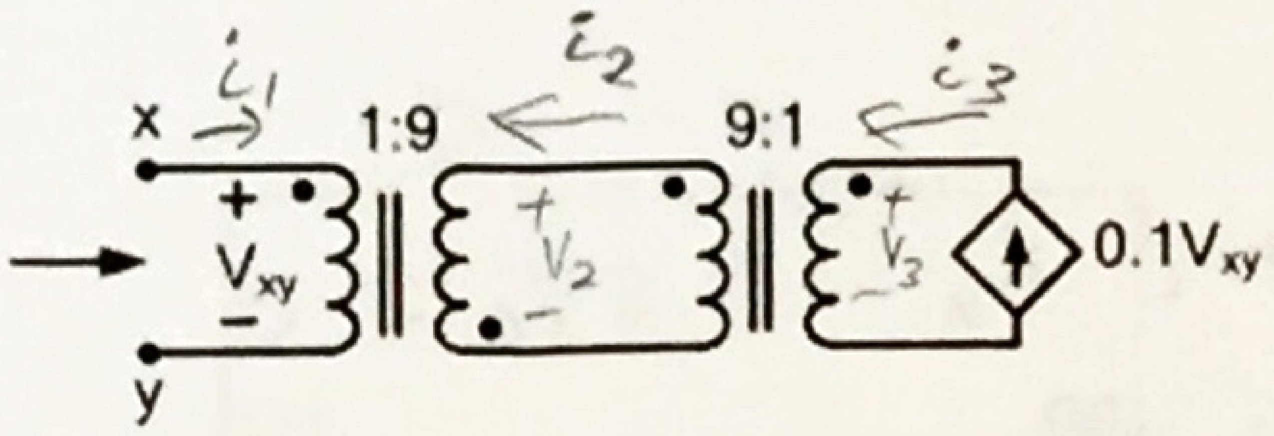
Student in Front: [redacted]

Problem	Maximum Score	Your Score
1	10	2
2	10	5
3	10	10
Total	30	17

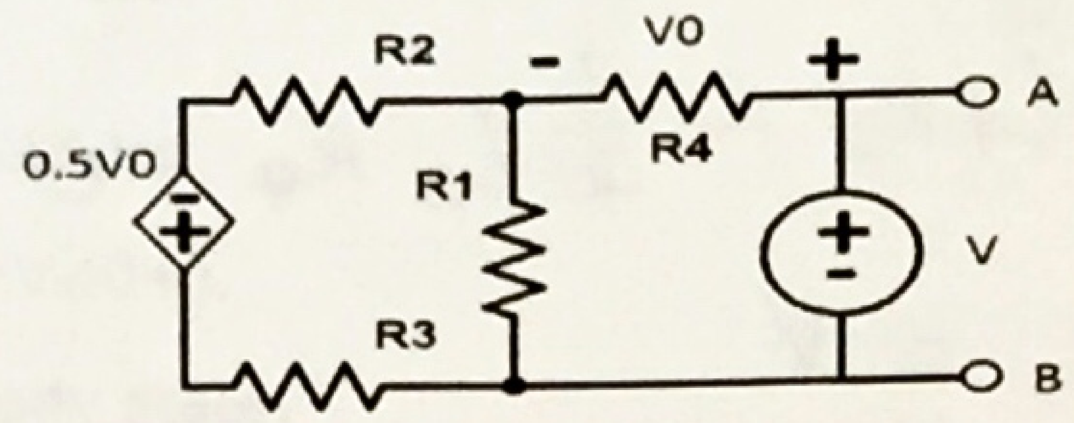
Q1. 10 points (5+5)

Find the Thevenin equivalent circuit of each network at terminals x-y/A-B.

(a)



(b)



a) $i_2 = \frac{1}{n} i_1 \qquad V_2 = -n V_{xy} \rightarrow i_2 = \frac{1}{9} i_1 \qquad V_2 = -9 V_{xy}$

$-i_3 = -\frac{1}{n} i_2 \qquad V_3 = n V_2 \rightarrow i_3 = 9 i_2 \qquad V_3 = \frac{1}{9} V_2$

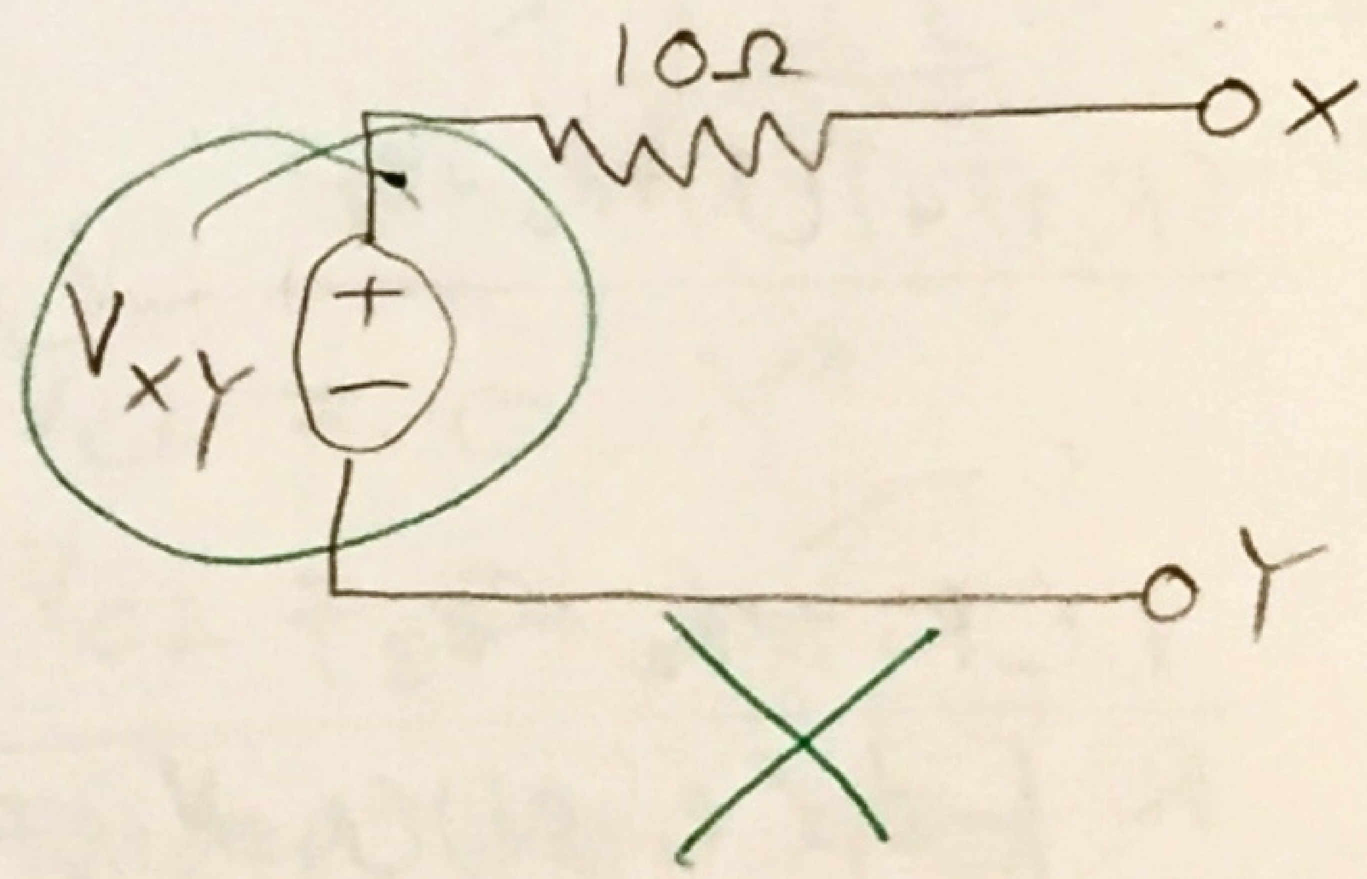
$i_3 = 0.1 V_{xy} \qquad V_3 = \frac{1}{9} V_2$

$0.1 V_{xy} = 9 i_2 = 9(\frac{1}{9} i_1) \qquad V_3 = -V_{xy}$

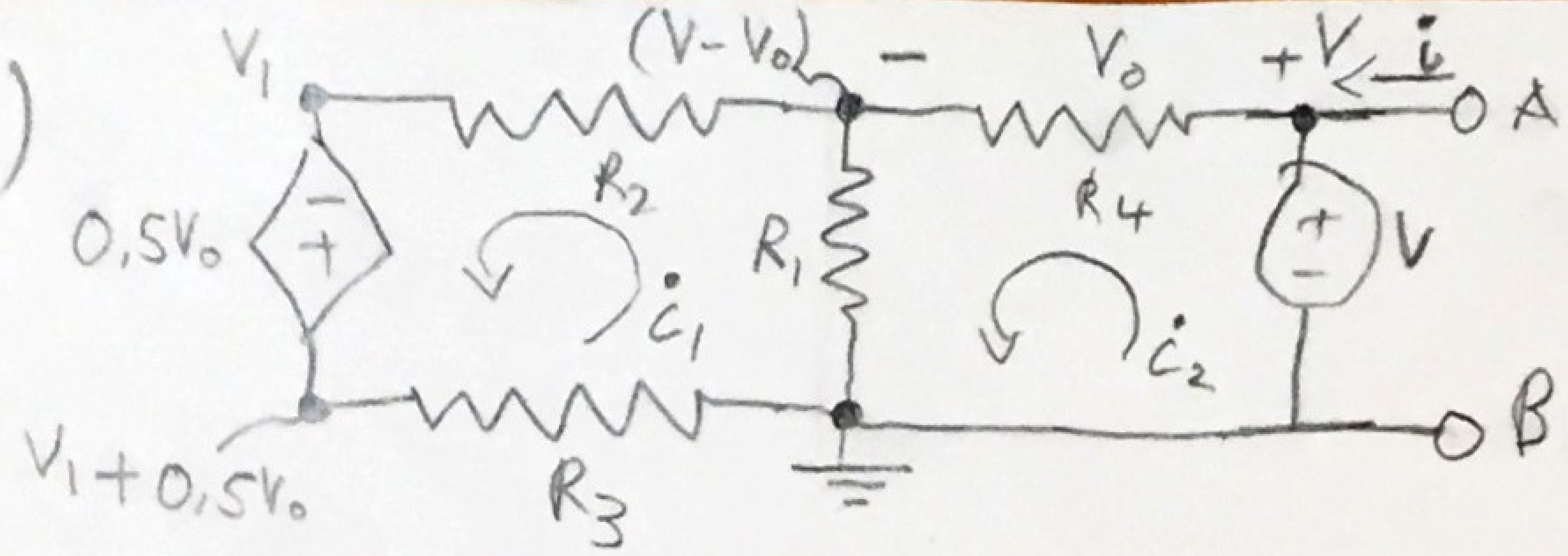
$0.1 V_{xy} = i_1$

$\frac{V_{xy}}{i_1} = 10\Omega$ ✓

$R_{TH} = 10\Omega$



b)



$V_{OC} = V$ ✓

① $i_1R_3 + (i_1-i_2)R_1 + i_1R_2 - 0.5V_0 = 0$ $\qquad V_0 = i_2R_4$

② $-V + i_2R_4 + (i_2-i_1)R_1 = 0$

① $i_1R_3 + i_1R_1 - i_2R_1 + i_1R_2 - \frac{1}{2}i_2R_4 = 0$

② $i_2R_4 + i_2R_1 - i_1R_1 = V$

① $i_1(R_1+R_2+R_3) - i_2(R_1+\frac{1}{2}R_4) = 0$

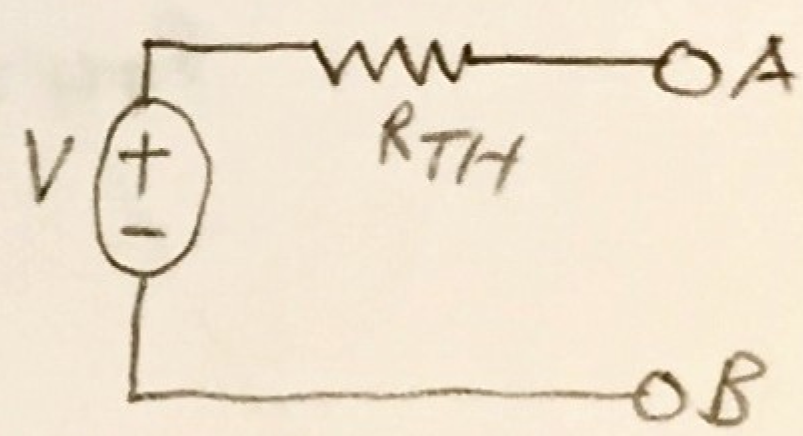
② $-i_1R_1 + i_2(R_1+R_4) = V$

$$i_1 = -\frac{V - i_2(R_1+R_4)}{R_1} = \frac{i_2(R_1+R_4)}{R_1} - \frac{V}{R_1}$$

① $\left[\frac{i_2(R_1+R_4)}{R_1} - \frac{V}{R_1}\right](R_1+R_2+R_3) - i_2(R_1+\frac{1}{2}R_4) = 0$

$$i_2\left[\frac{(R_1+R_4)(R_1+R_2+R_3)}{R_1} - (R_1+\tfrac{1}{2}R_4)\right] = \frac{V(R_1+R_2+R_3)}{R_1}$$

$$i_2 = \frac{V(R_1+R_2+R_3)}{R_1\left[\frac{1}{R_1}(R_1+R_4)(R_1+R_2+R_3) - (R_1+\frac{1}{2}R_4)\right]}$$

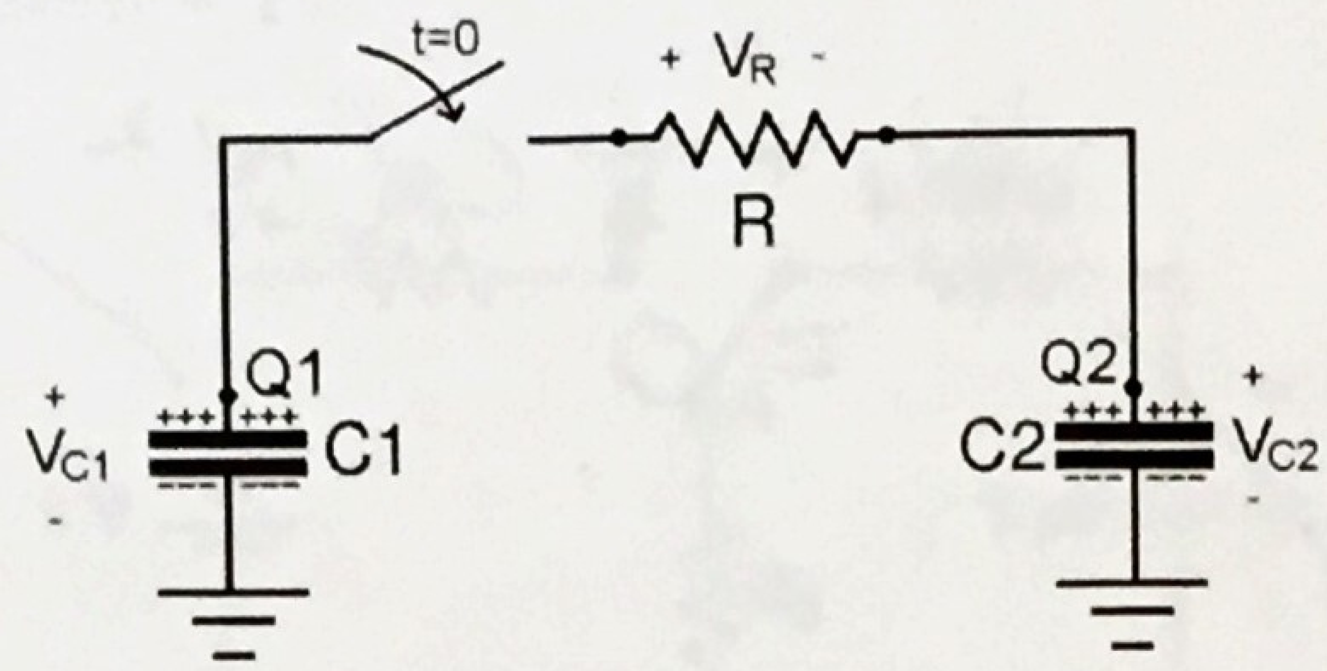


$i = i_2$

$$R_{TH} = \frac{V_{OC}}{i} = \frac{V}{i_2} = \frac{R_1\left[\frac{1}{R_1}(R_1+R_4)(R_1+R_2+R_3) - (R_1+\frac{1}{2}R_4)\right]}{(R_1+R_2+R_3)}$$ ✗

Q2. 10 points

Capacitors C1 and C2 are initially charged with Q1 and ~~Q1~~ Q2 Coulombs respectively. At t=0 the switch closes.



a) Find $V_{C1}(0-)$, $V_{C2}(0-)$ and $V_R(0-)$.

b) Find $V_{C1}(0+)$, $V_{C2}(0+)$ and $V_R(0+)$.

c) Find V_{C1}, V_{C2} and V_R at steady state.

d) For the case where Q1=Q, Q2=0C and C1=C2=C. Calculate the total amount of energy dissipated by the resistor after the system reached its steady state.

a) $V_{C1}(0^-) = \frac{Q_1}{C_1}$ ✓ $V_{C2}(0^-) = \frac{Q_2}{C_2}$ ✓ $V_R(0^-) = 0V$ ✓

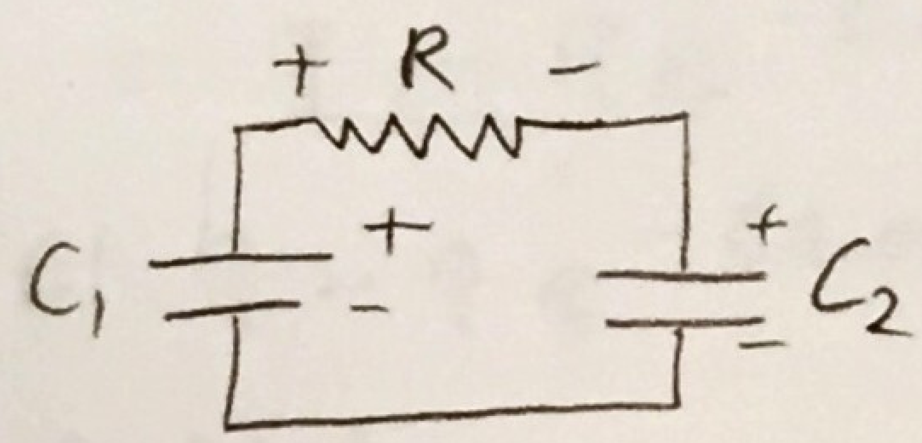
b) $V_{C1}(0^+) = V_{C1}(0^-) = \frac{Q_1}{C_1}$ ✓

$V_{C2}(0^+) = V_{C2}(0^-) = \frac{Q_2}{C_2}$ ✓

$V_R(0^+) = V_{C1}(0^+) - V_{C2}(0^+)$ ✓

$= \frac{Q_1}{C_1} - \frac{Q_2}{C_2}$

c)



$V_{C1} = 0$ ✗

$V_{C2} = 0$ ✗

$V_R = V_{C1} - V_{C2} = 0$ ✓

d) $Q_1 = Q \quad Q_2 = 0 \qquad C_1 = C_2 = C$

$$W_1 = \frac{1}{2} C_1 V_{C1}^2 \qquad W_2 = \frac{1}{2} C_2 V_{C2}^2$$

$$V_{C1} = \frac{Q_1}{C_1} = \frac{Q}{C} \qquad V_{C2} = \frac{Q_2}{C_2} = 0$$

$$W_1 = \frac{1}{2} C \left(\frac{Q}{C}\right)^2 \qquad W_2 = \frac{1}{2} C (0)^2$$

$$= \frac{1}{2} C \frac{Q^2}{C^2} \qquad = 0$$

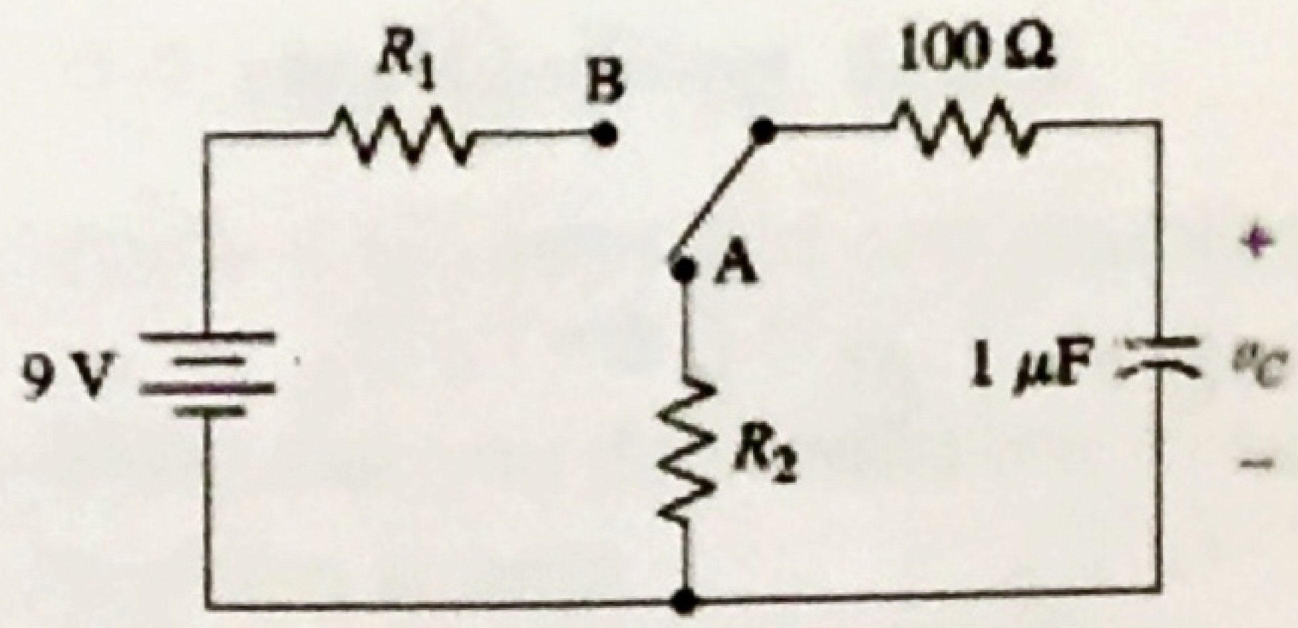
$$= \frac{1}{2} \frac{Q^2}{C}$$

$$E_{tot\ dissipated} = W_1 + W_2$$

$$\boxed{= \frac{1}{2} \frac{Q^2}{C}}$$

Q3. 10 points

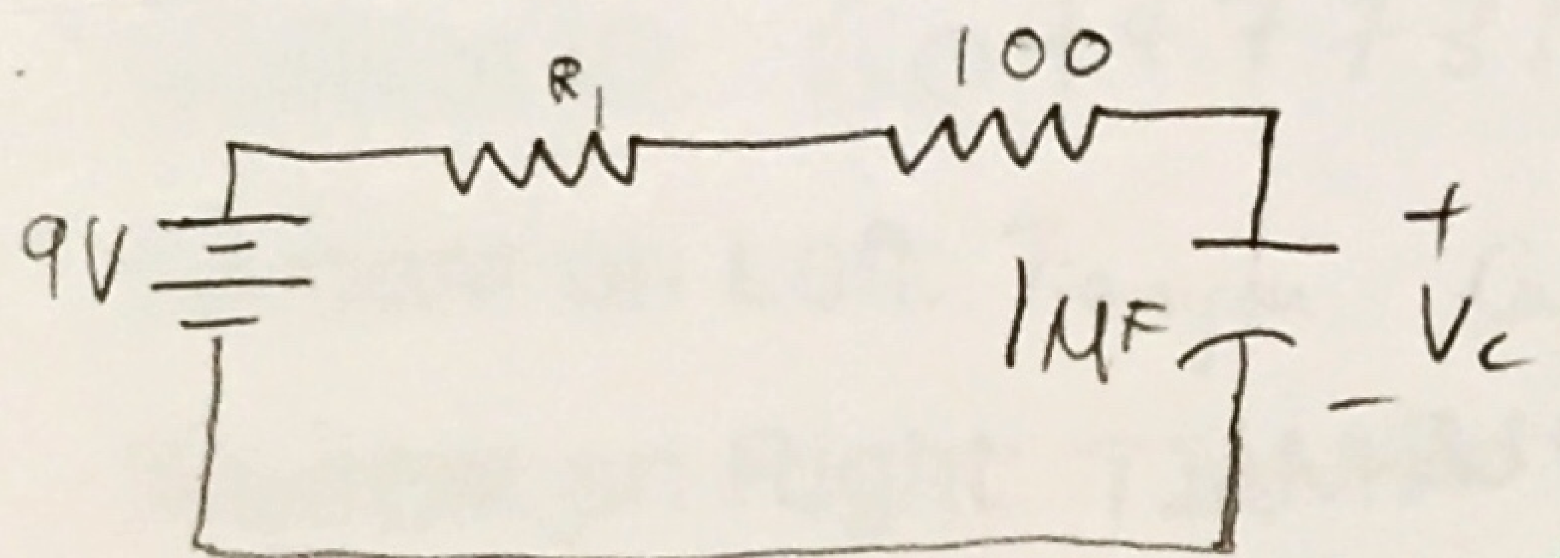
The switch in the circuit below has been in position A for long time. It is switched to position B at t=0 and back to position A at t=1ms. Find R1 and R2 such that Vc(1ms) = 8V and Vc(2ms) = 1V.



$t < 0$:

$V_c(0^-) = V_c(0^+) = 0\,V$

$0 \le t \le 1ms$:



$\tau_1 = RC$

$= (R_1 + 100)(1MF)$

$V_c(0) = 0\,V$

$V_c(\infty) = 9\,V$

$V_c(t) = 9V - [9V - 0]e^{-t/\tau_1}$

$= 9 - 9e^{-t/\tau_1}$

$8V = 9 - 9e^{-(0.001)/\tau_1}$

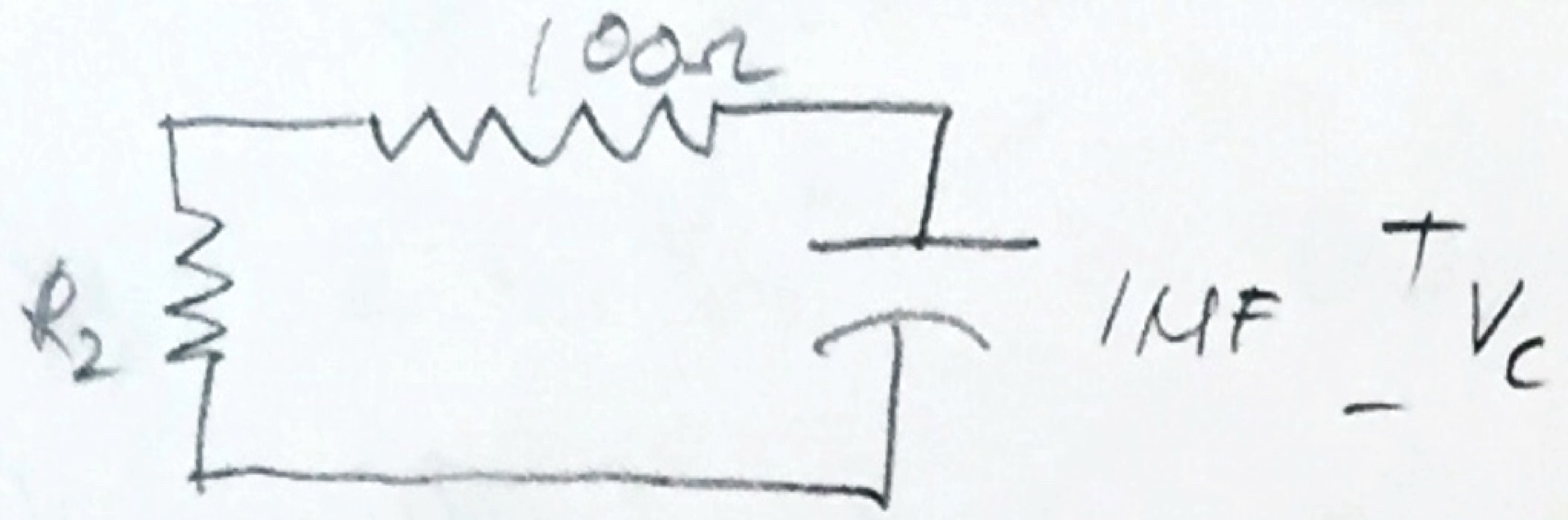
$-\frac{0.001}{\tau_1} = -2\ln(3)$

$\tau_1 \approx 4.55 \times 10^{-4}\,s$

$(4.55 \times 10^{-4}\,s) = (R_1 + 100)(1MF)$

$\boxed{R_1 \approx 355\,\Omega}$

$1 \le t \le \infty$:



$V_c(1ms) = 8V$

$V_c(\infty) = 0V$

$\tau_2 = RC$

$= (R_2 + 100)(1MF)$

$V_c(t) = 0 - [0 - 8V]\, e^{-(t-1)/\tau_2}$

$= 8e^{-(t-1)/\tau_2}$

$1V = 8e^{-(2-1)/\tau_2}$

$\frac{-(1)}{\tau_2} = \ln \frac{1}{8}$

$\tau_2 = -\frac{1}{\ln 1/8} \approx 0.481 ms$

$\tau_2 = (R_2 + 100)(1MF)$

$0.481 \times 10^{-3}\, s = (R_2 + 100)(1MF)$

$\boxed{R_2 \approx 381 \Omega}$