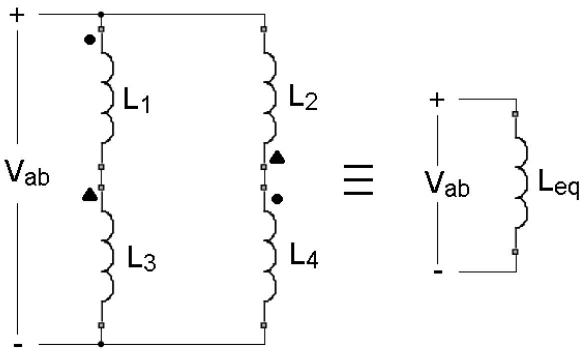
Q1. (10 points)

The four inductors of the figure can be replaced by a single equivalent inductor (Leq). Find Leq as a function of L1, L2, L3 L4 and M.

Assume: $M_{14} = M_{41} = M$

$$M_{23} = M_{32} = \frac{M}{2}$$



i_1 flows from L_1 to L_3

i_2 flows from L_2 to L_4

$$V_{ab} = (L_1 + L_3)\frac{di_1}{dt} + M_{32}\frac{d(-i_2)}{dt} + M_{14}\frac{di_2}{dt} = (L_1 + L_3)\frac{di_1}{dt} + \frac{M}{2}\frac{di_2}{dt}$$

$$V_{ab} = (L_2 + L_4)\frac{di_2}{dt} + M_{14}\frac{di_1}{dt} - M_{32}\frac{di_1}{dt} = (L_2 + L_4)\frac{di_2}{dt} + \frac{M}{2}\frac{di_1}{dt}$$

$$(L_1 + L_3 - \frac{M}{2})\frac{di_1}{dt} = (L_2 + L_4 - \frac{M}{2})\frac{di_2}{dt}$$

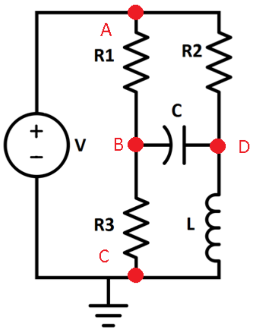
$$\frac{di_2}{dt} = \frac{L_1 + L_3 - \frac{M}{2}}{L_2 + L_4 - \frac{M}{2}}\frac{di_1}{dt}$$

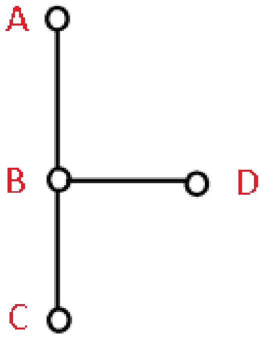
$$L_{eq} = \frac{V_{ab}}{\frac{di_1}{dt} + \frac{di_2}{dt}} = \frac{(L_1 + L_3)\frac{di_1}{dt} + \frac{M}{2}\frac{L_1 + L_3 - \frac{M}{2}}{L_2 + L_4 - \frac{M}{2}}\frac{di_1}{dt}}{\left[1 + \frac{L_1 + L_3 - \frac{M}{2}}{L_2 + L_4 - \frac{M}{2}}\right]\frac{di_1}{dt}} = \frac{(L_1 + L_3)\left(L_2 + L_4 - \frac{M}{2}\right) + \frac{M}{2}\left(L_1 + L_3 - \frac{M}{2}\right)}{L_1 + L_3 + L_2 + L_4 - M}$$

$$L_{eq} = \frac{(L_1 + L_3)(L_2 + L_4) - \frac{M^2}{4}}{L_1 + L_3 + L_2 + L_4 - M}$$

Q2. (12 points)

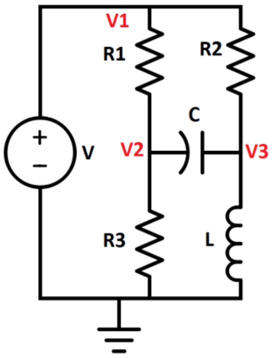
1) Generate a spanning tree of this circuit.





2) Determine the number of nodes and chords.
4 nodes and 3 chords.

3) What is the minimal number of equations to solve all branch voltages? Define node voltages on the figure and write down the equations.
3 equations.

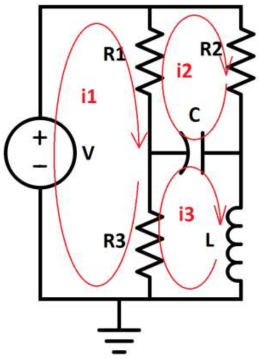


$$V_1 = V$$

$$\frac{V_2}{R_3} + \frac{V_2 - V_1}{R_1} + C\frac{d(V_2 - V_3)}{dt} = 0$$

$$C\frac{d(V_3 - V_2)}{dt} + \frac{V_3 - V_1}{R_2} + \frac{1}{L}\int V_3 dt = 0$$

4) What is the minimal number of equations to solve all branch currents? Define loop currents on the figure and write down the equations.
3 equations.



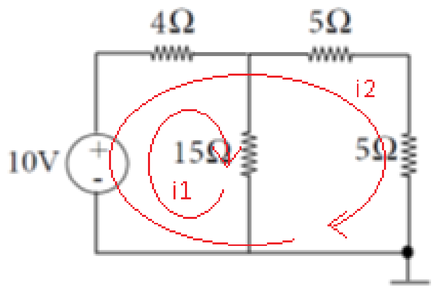
$$-V + (i_1 - i_2)R_1 + (i_1 - i_3)R_3 = 0$$

$$(i_2 - i_1)R_1 + i_2 R_2 + \frac{1}{C}\int (i_2 - i_3)dt = 0$$

$$(i_3 - i_1)R_3 + \frac{1}{C}\int (i_3 - i_2)dt + L\frac{di_3}{dt} = 0$$

Q3. 8 points

First find the branch current at the 15 ohm resistor. Then we can determine power consumption by P=IR^2.



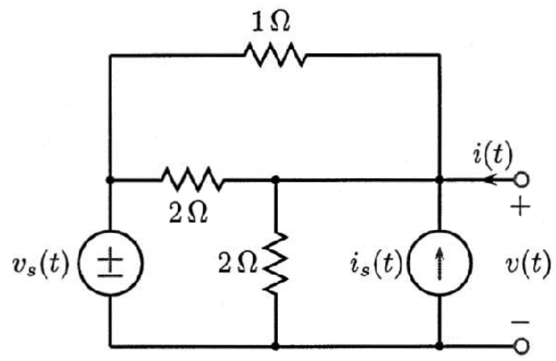
$$10 - 4(i_1 + i_2) - 15i_1 = 0$$

$$15i_1 - 10i_2 = 0$$

$$\Rightarrow i_1 = 0.4\,\text{A},\ i_2 = 0.6\text{A}$$

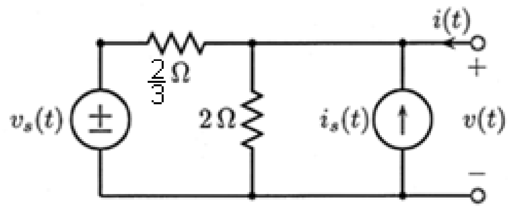
$$P = i_1^{\ 2}R = 0.4^2 \times 15 = 2.4\text{W}$$

Q4. 10 points

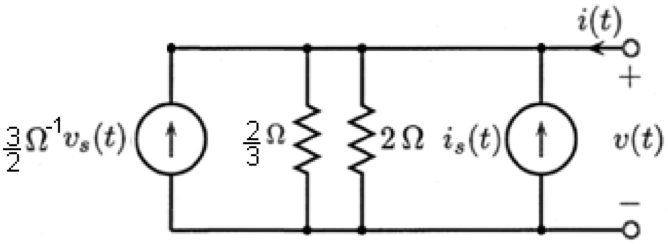


Find v(t)-i(t) relationship using source transformations.

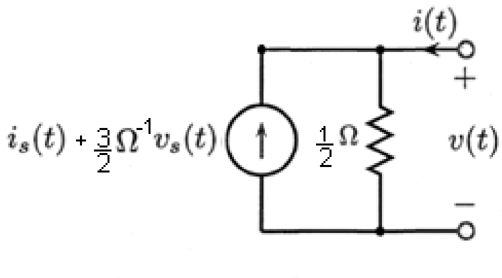
First we combine the 1Ω and the 2Ω resistors which are in parallel:



Then we do a source transformation:



Now we add the two current power supplies and we also combine the two resistors in parallel to get:



The v(t)-i(t) relationship then becomes:

$$v(t) = \frac{1}{2}\Omega\left(i(t) + i_s(t) + \frac{3}{2}\Omega^{-1} v_s(t) \right)$$