

EE10 Midterm I

Department of Electrical Engineering, UCLA

Winter 2016

Instructor: Prof. Gupta

1. Exam is closed book. Calculator and one double sided cheat-sheet is allowed.
2. Cross out *everything* that you don't want me to see. Points will be deducted for everything wrong!
3. No points will be given without proper explanations
4. Time allotted: 75 minutes

Name:



Student ID:



Student on Left:



Student on Right:



Student in Front:



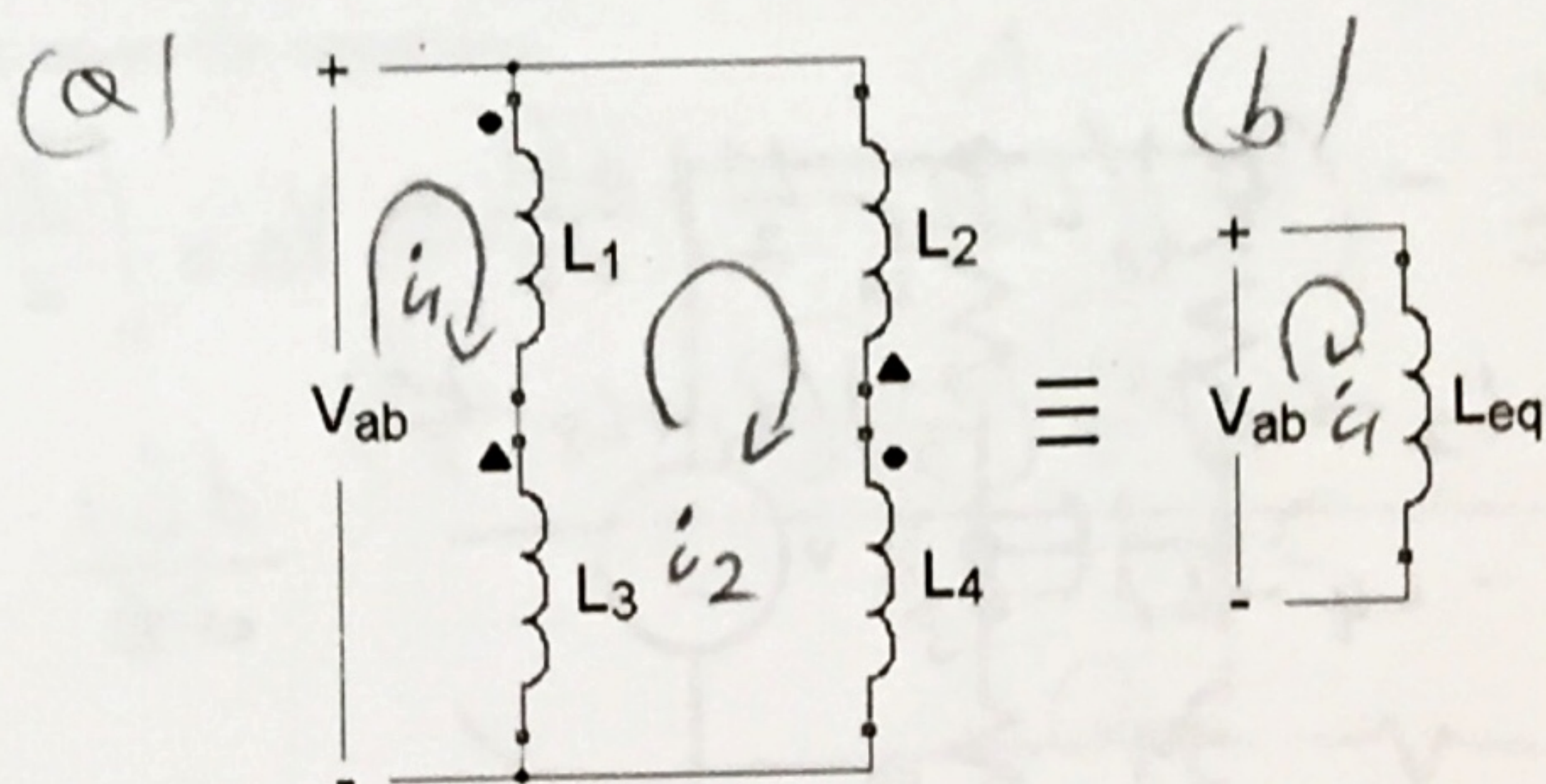
Problem	Maximum Score	Your Score
1	10	10
2	6 12	12
3	4 8	8
4	10	7
Total	30 40	37

Q1. (10 points)

The four inductors of the figure can be replaced by a single equivalent inductor (L_{eq}). Find L_{eq} as a function of L_1, L_2, L_3, L_4 and M .

Assume: $M_{14} = M_{41} = M$

$$M_{23} = M_{32} = \frac{M}{2}$$



(a)

$$\textcircled{1} \quad V_{ab} - L_1 \frac{d(i_1 - i_2)}{dt} - M \frac{di_2}{dt} - L_3 \frac{d(i_1 - i_2)}{dt} + \frac{M}{2} \frac{di_2}{dt} = 0$$

$$\textcircled{2} \quad V_{ab} - L_2 \frac{di_2}{dt} + \frac{M}{2} \frac{d(i_1 - i_2)}{dt} - L_4 \frac{di_2}{dt} - M \frac{d(i_1 - i_2)}{dt} = 0$$

(b) $V_{ab} - L_{eq} \frac{di_1}{dt} = 0 \rightarrow V_{ab} = L_{eq} \frac{di_1}{dt}$

(a) $\textcircled{1} \quad V_{ab} - L_1 \frac{di_1}{dt} + L_1 \frac{di_2}{dt} - M \frac{di_2}{dt} - L_3 \frac{di_1}{dt} + L_3 \frac{di_2}{dt} + \frac{M}{2} \frac{di_2}{dt} = 0$

$$V_{ab} + (-L_1 - L_3) \frac{di_1}{dt} + (L_1 - M + L_3 + \frac{M}{2}) \frac{di_2}{dt} = 0$$

$$V_{ab} - (L_1 + L_3) \frac{di_1}{dt} + (L_1 + L_3 - \frac{M}{2}) \frac{di_2}{dt} = 0$$

$\textcircled{2} \quad V_{ab} - L_2 \frac{di_2}{dt} + \frac{M}{2} \frac{di_1}{dt} - \frac{M}{2} \frac{di_2}{dt} - L_4 \frac{di_2}{dt} - M \frac{di_1}{dt} + M \frac{di_2}{dt} = 0$

$$V_{ab} + (\frac{M}{2} - M) \frac{di_1}{dt} + (-L_2 - \frac{M}{2} - L_4 + M) \frac{di_2}{dt} = 0$$

$$V_{ab} - \frac{M}{2} \frac{di_1}{dt} + (\frac{M}{2} - L_2 - L_4) \frac{di_2}{dt} = 0$$

$$\textcircled{1} V_{ab} - (L_1 + L_3) \frac{di_1}{dt} + (L_1 + L_3 - \frac{M}{2}) \frac{di_2}{dt} = 0$$

$$\textcircled{2} V_{ab} - \frac{M}{2} \frac{di_1}{dt} + (\frac{M}{2} - L_2 - L_4) \frac{di_2}{dt} = 0$$

$$\textcircled{1} \frac{di_2}{dt} = \frac{(L_1 + L_3) \frac{di_1}{dt} - V_{ab}}{L_1 + L_3 - \frac{M}{2}}$$

$$\textcircled{2} \frac{di_2}{dt} = \frac{\frac{M}{2} \frac{di_1}{dt} - V_{ab}}{\frac{M}{2} - L_2 - L_4}$$

$$\left[(L_1 + L_3) \frac{di_1}{dt} - V_{ab} \right] (\frac{M}{2} - L_2 - L_4) = (\frac{M}{2} \frac{di_1}{dt} - V_{ab}) (L_1 + L_3 - \frac{M}{2})$$

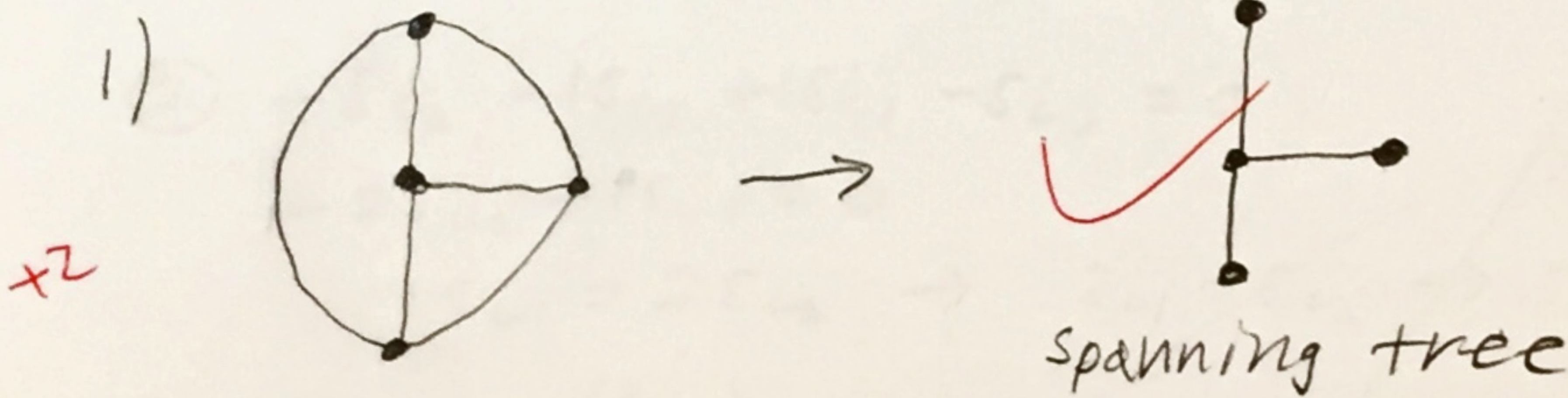
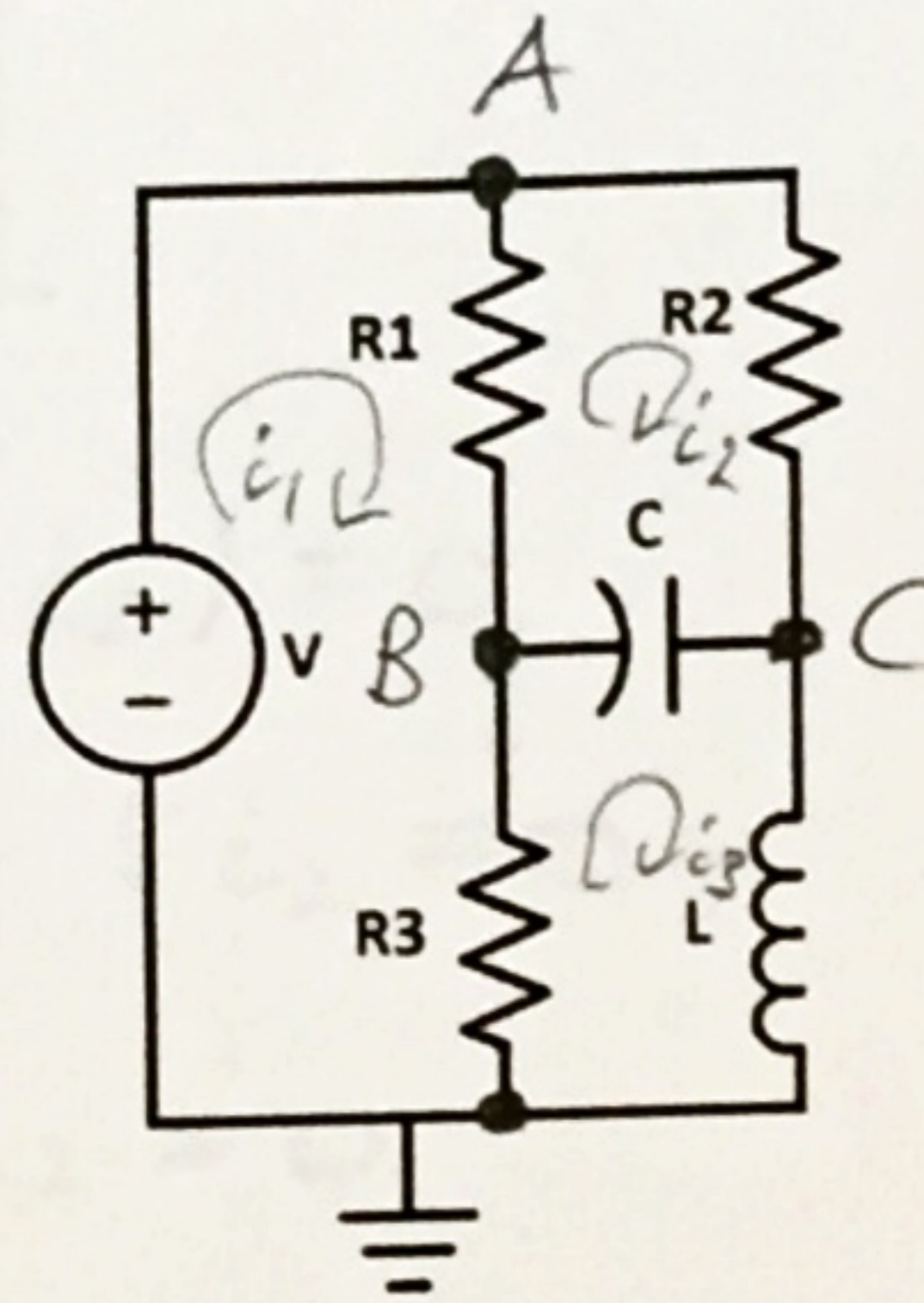
$$(L_1 + L_3) (\frac{M}{2} - L_2 - L_4) \frac{di_1}{dt} - (\frac{M}{2}) (L_1 + L_3 - \frac{M}{2}) \frac{di_1}{dt} = V_{ab} \left[\frac{M}{2} - L_2 - L_4 - L_1 - L_3 + \frac{M}{2} \right]$$

$$V_{ab} = \frac{(L_1 + L_3) (\frac{M}{2} - L_2 - L_4) - (\frac{M}{2}) (L_1 + L_3 - \frac{M}{2})}{M - L_2 - L_4 - L_1 - L_3} \frac{di_1}{dt}$$

$\underbrace{\hspace{15em}}_{L_{eq}}$

Q2. (12 points)

- 1) Generate a spanning tree of this circuit.
- 2) Determine the number of nodes and chords.
- 3) What is the minimal number of equations to solve all branch voltages? Define node voltages on the figure and write down the equations.
- 4) What is the minimal number of equations to solve all branch currents? Define loop currents on the figure and write down the equations.



2) 4 nodes

3 chords

3) $n=4$ $n-1=3$ 3 minimum node voltage equations

$$V_A = V$$

$$+4 \quad \frac{V_B - V_A}{R_1} + C \frac{d(V_B - V_C)}{dt} + \frac{V_B}{R_3} = 0$$

$$C \frac{d(V_C - V_B)}{dt} + \frac{V_C - V_A}{R_2} + \frac{1}{L} \int V_C dt = 0$$

$$4) \quad n=4 \quad b=6 \quad b-(n-1)=3$$

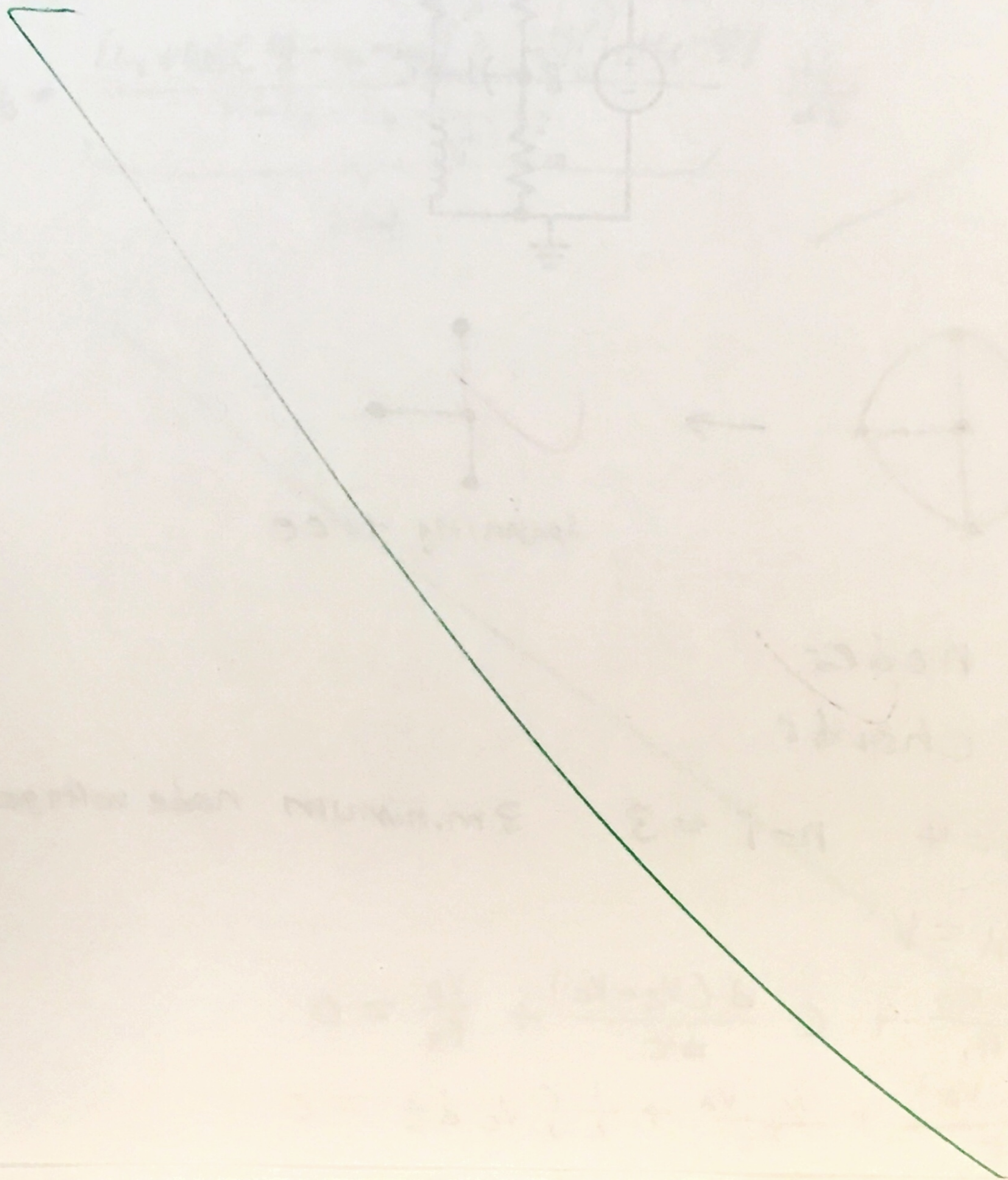
3 minimum loop current equations

$$V - R_1(i_1 - i_2) - R_3(i_1 - i_3) = 0$$

$$-R_2 \dot{i}_2 - \frac{1}{C} \int (i_2 - i_3) dt - R_1(i_2 - i_1) = 0$$

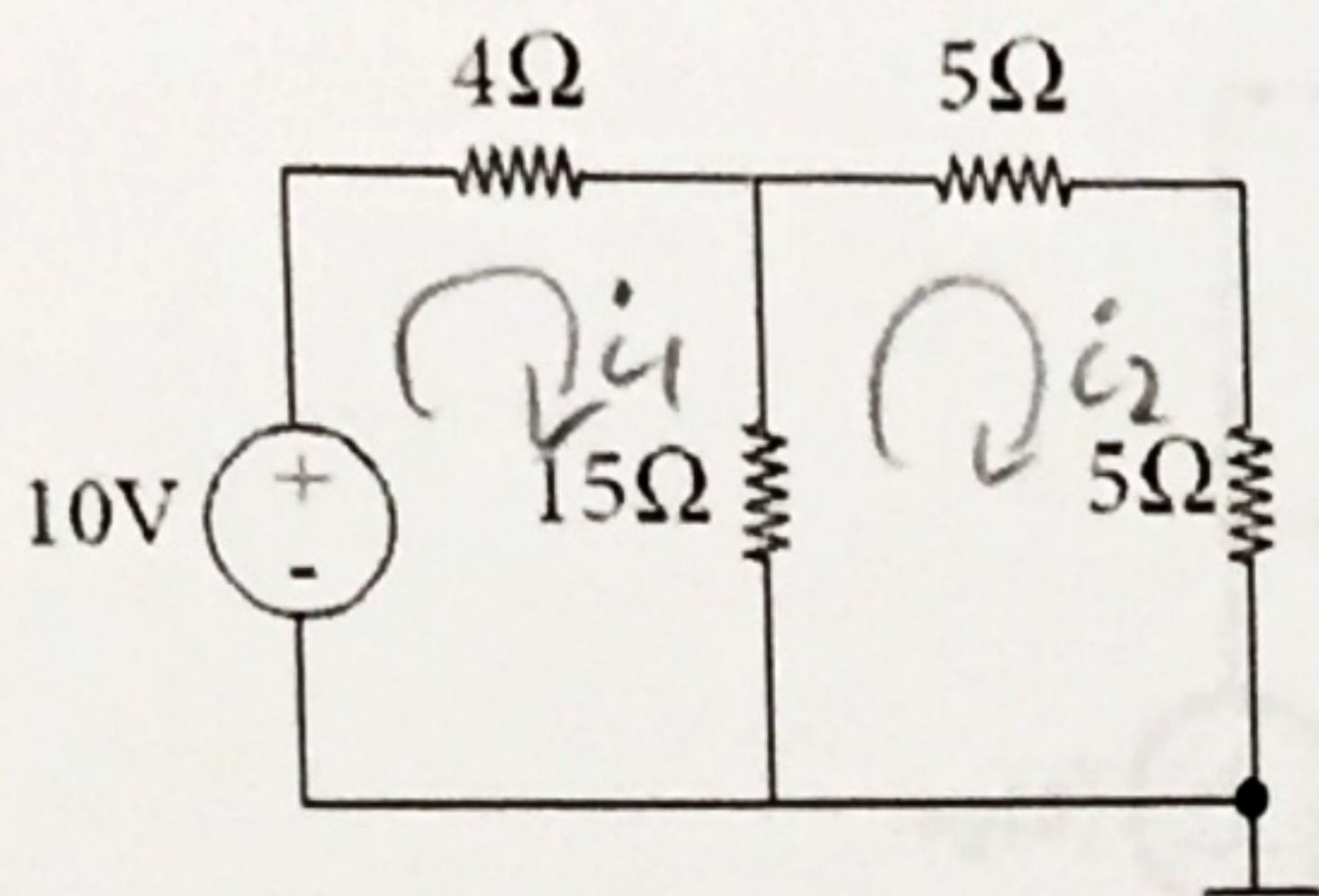
$$-L \frac{di_3}{dt} - R_3(i_3 - i_1) - \frac{1}{C} \int (i_3 - i_2) dt = 0$$

+4



Q3. 8 points

Determine the power consumed by the 15 ohm resistor in the circuit below.



$$\textcircled{1} 10V - 4i_1 - 15(i_1 - i_2) = 0$$

$$\textcircled{2} -5i_2 - 15(i_2 - i_1) - 5i_2 = 0$$

$$\textcircled{1} 10V - 19i_1 + 15i_2 = 0$$
$$19i_1 - 15i_2 = 10$$

$$\textcircled{2} -5i_2 - 15i_2 + 15i_1 - 5i_2 = 0$$
$$-25i_2 + 15i_1 = 0$$

$$15i_1 = 25i_2 \rightarrow 3i_1 = 5i_2 \rightarrow \frac{3}{5}i_1 = i_2$$

$$\textcircled{1} 19i_1 - 15\left(\frac{3}{5}i_1\right) = 10$$
$$19i_1 - 9i_1 = 10$$

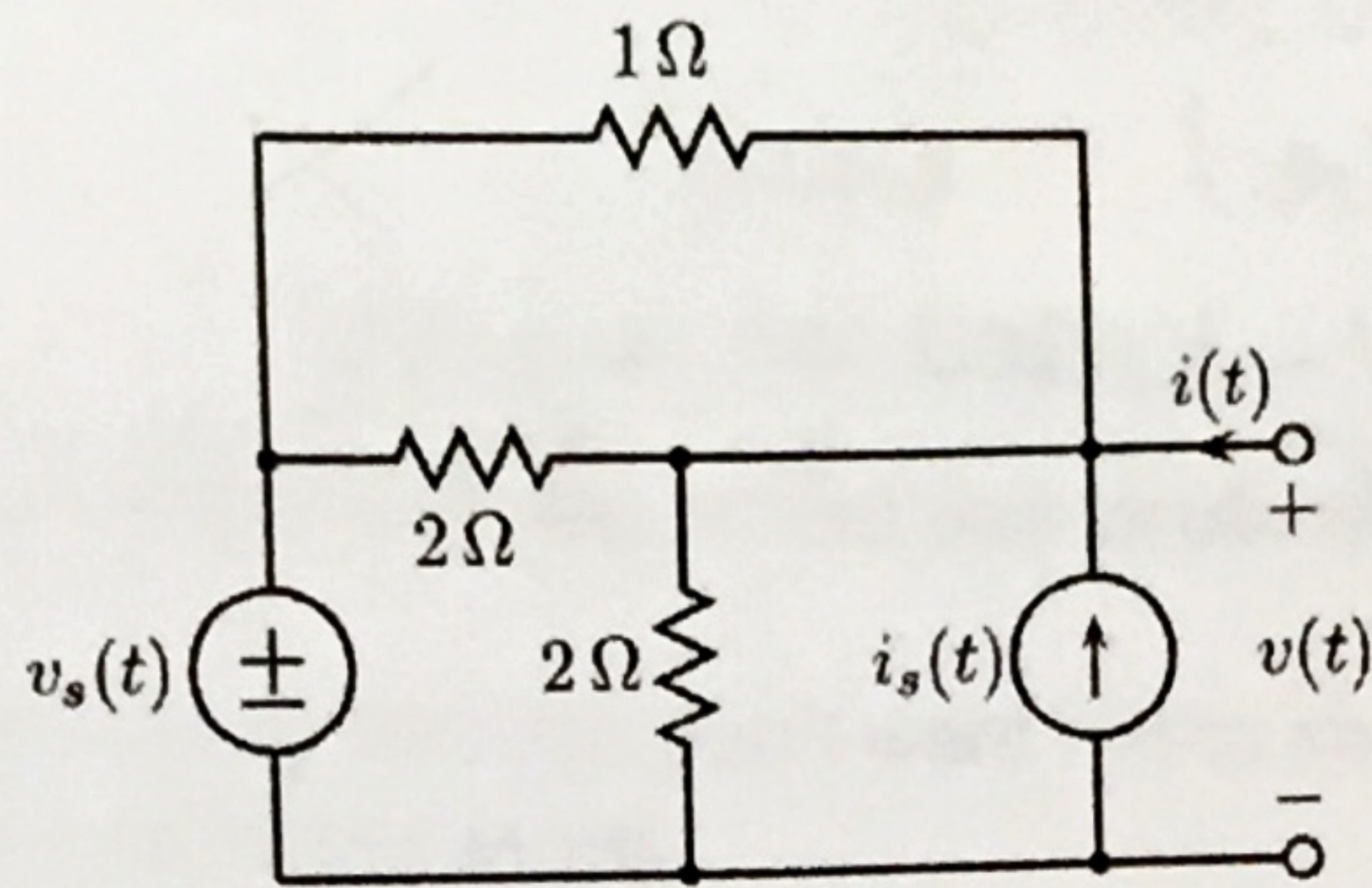
$$10i_1 = 10 \rightarrow i_1 = 1A$$

$$\textcircled{2} i_2 = \frac{3}{5}i_1 = \frac{3}{5}A$$

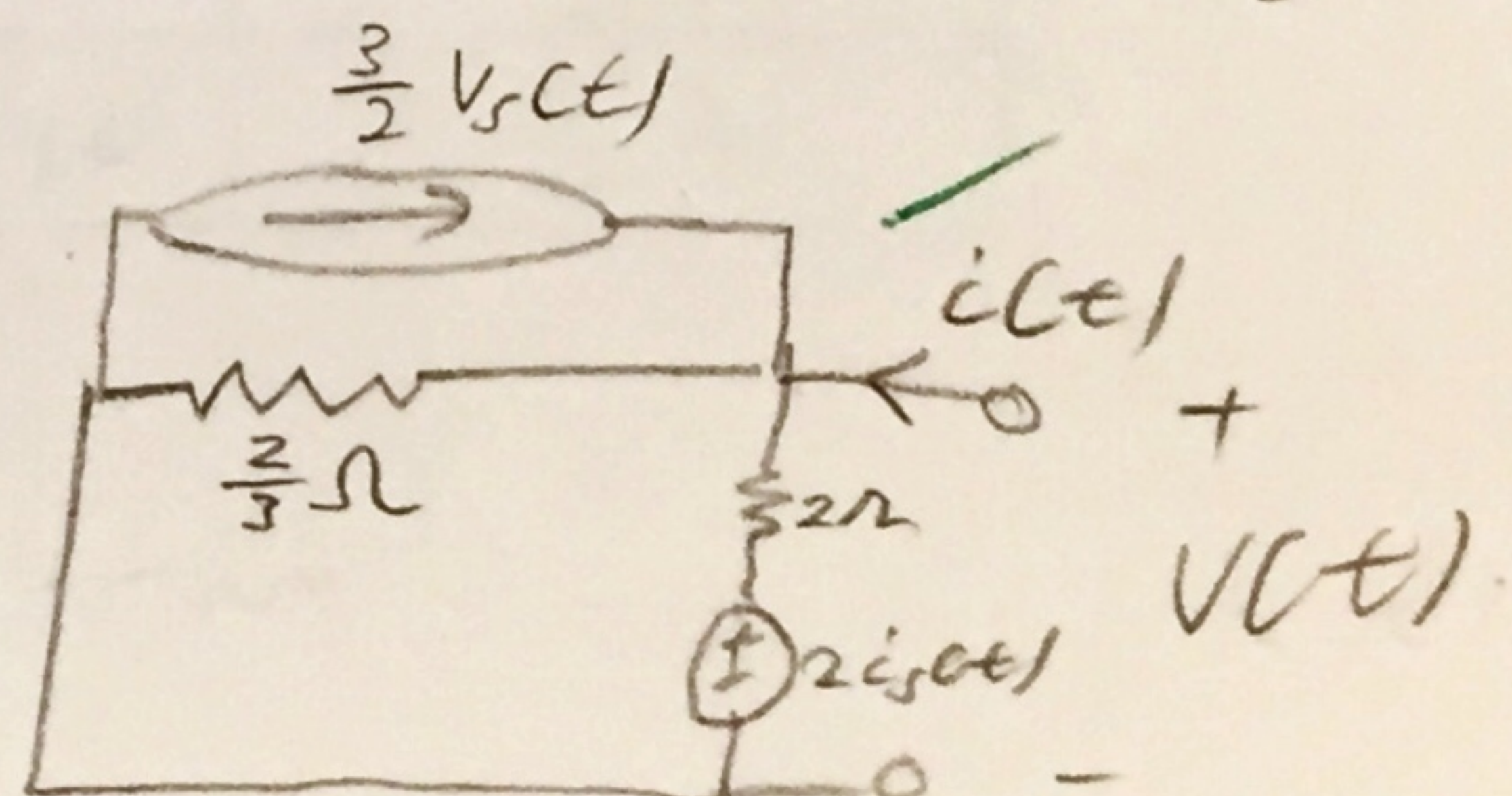
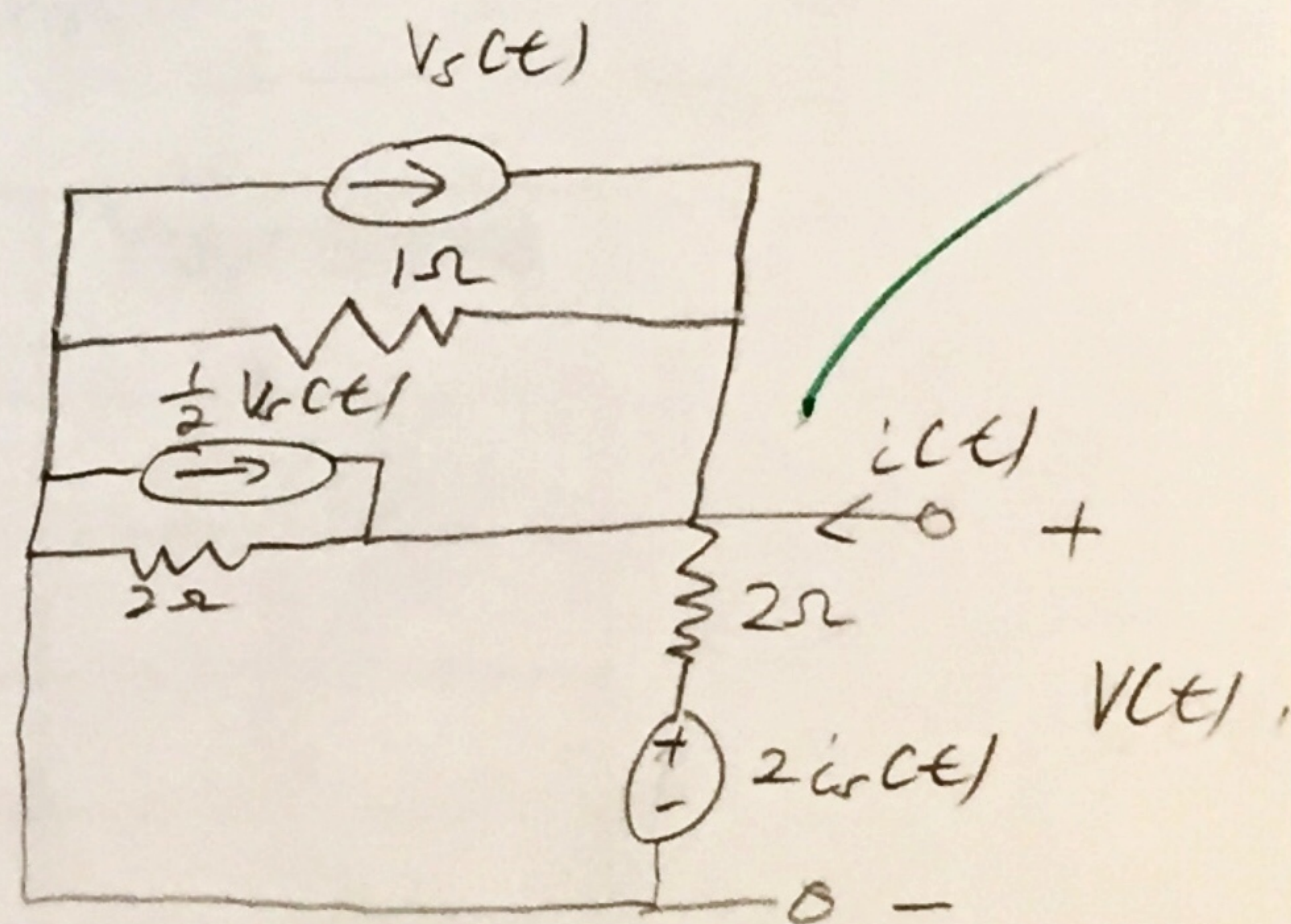
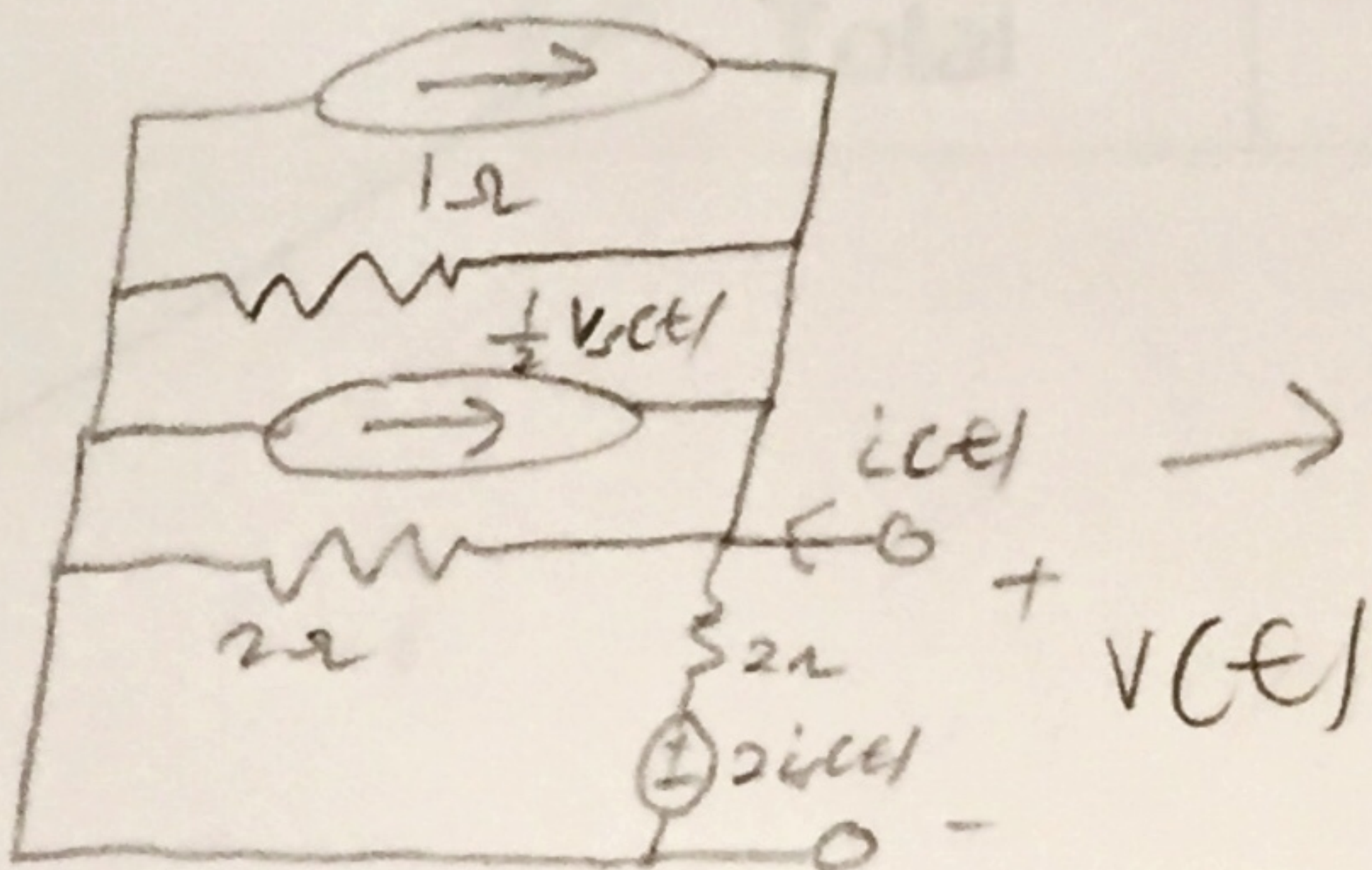
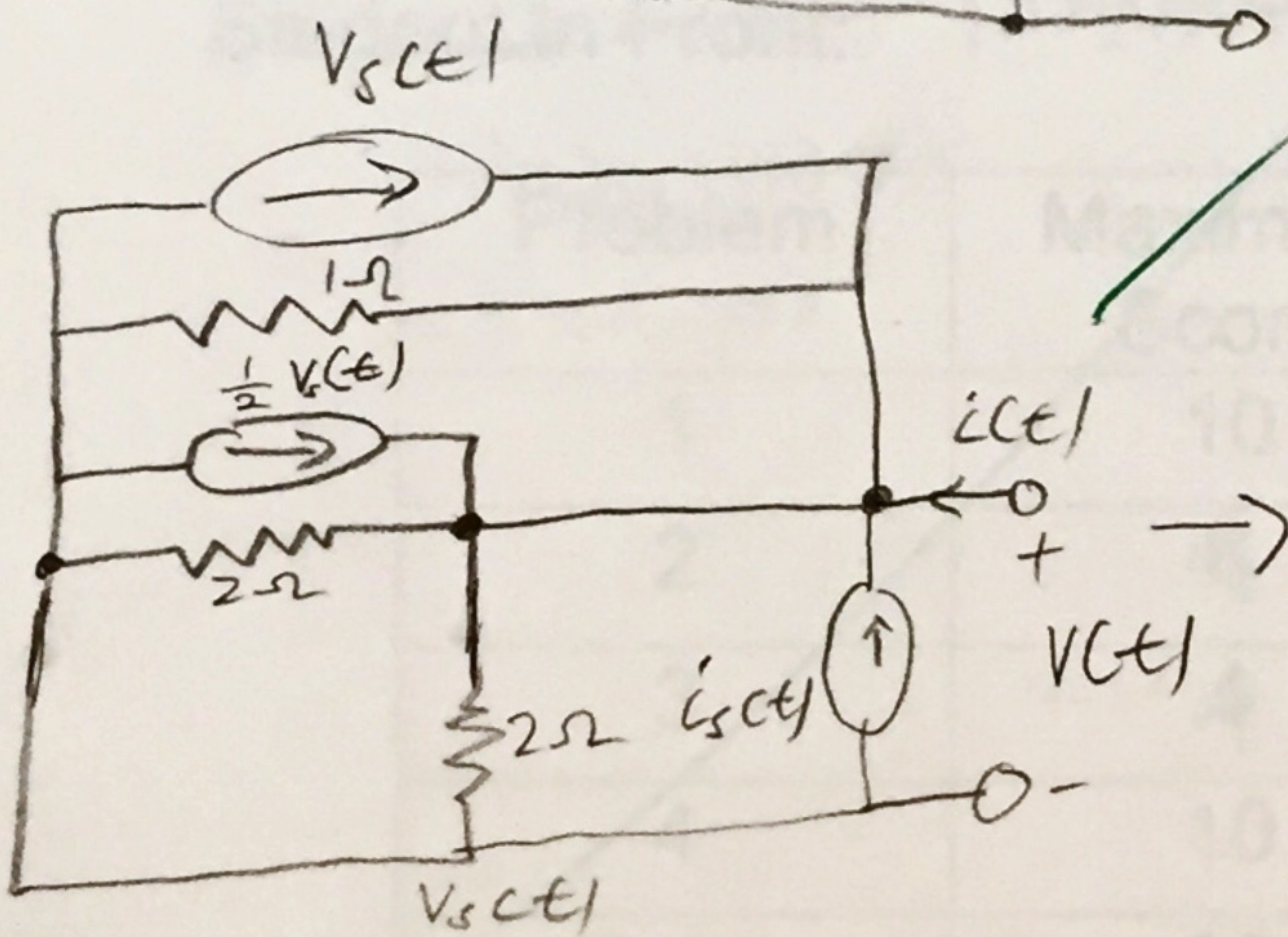
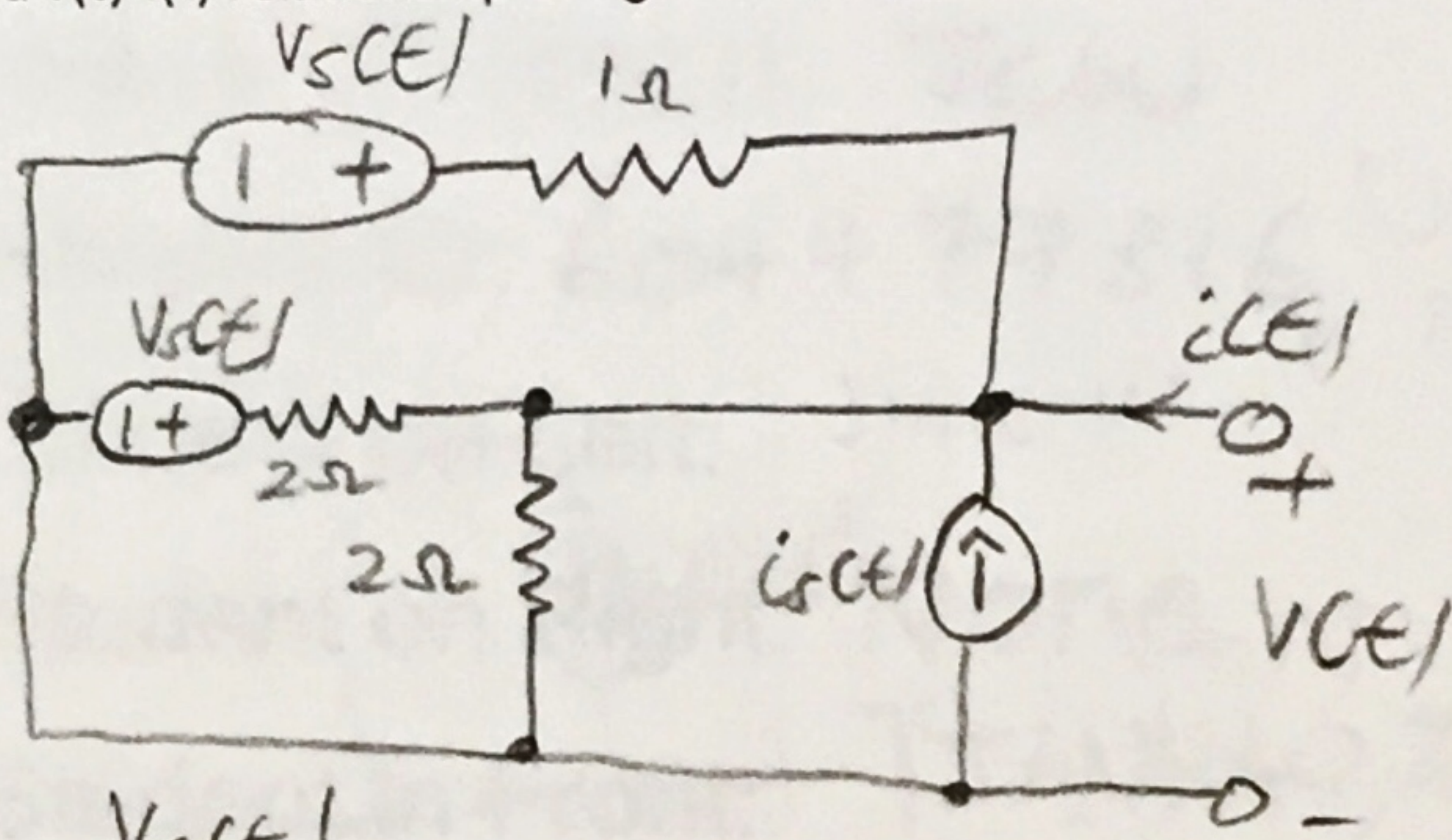
$$P_{15\Omega} = I^2 R = (i_1 - i_2)^2 15\Omega$$

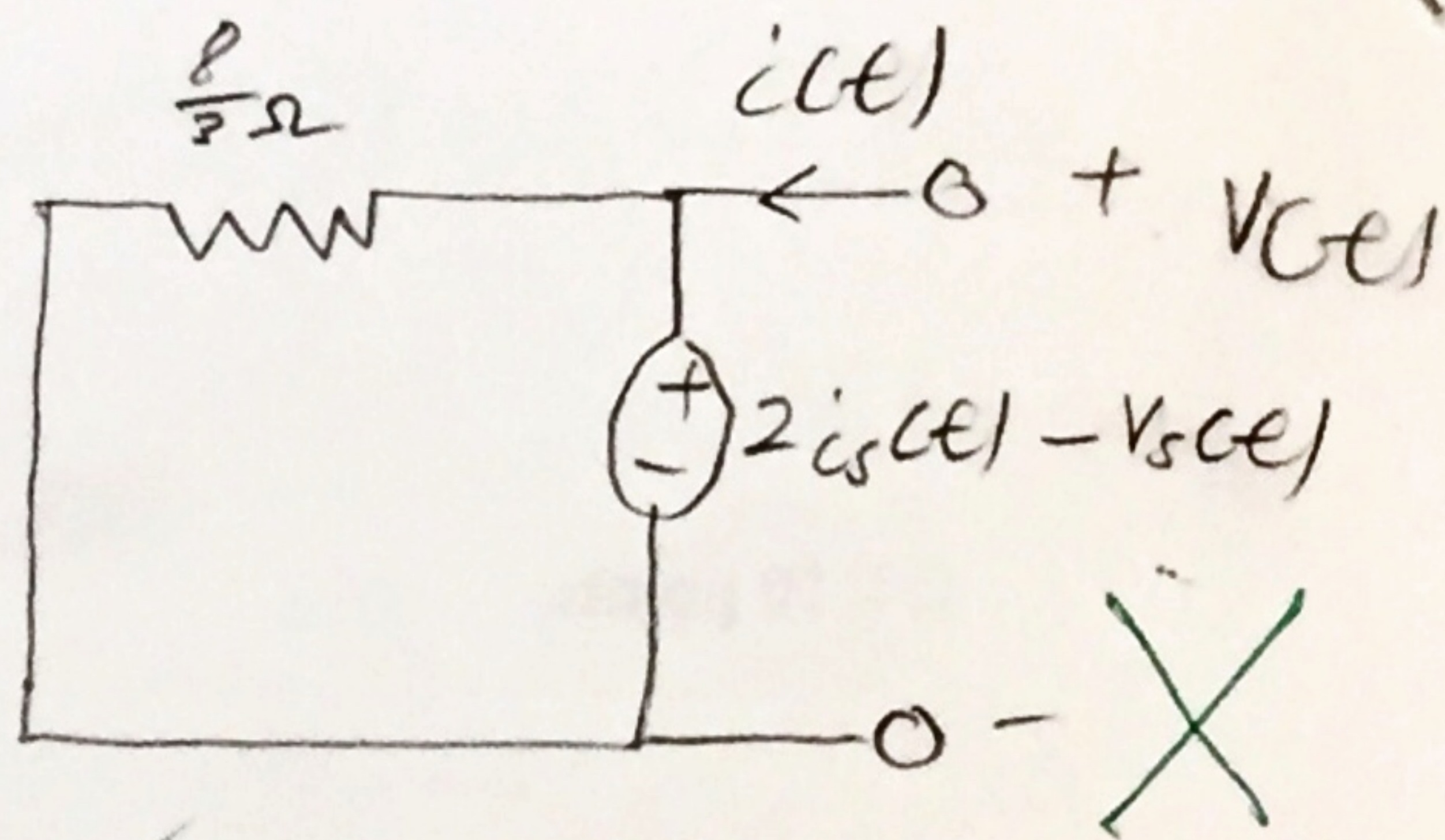
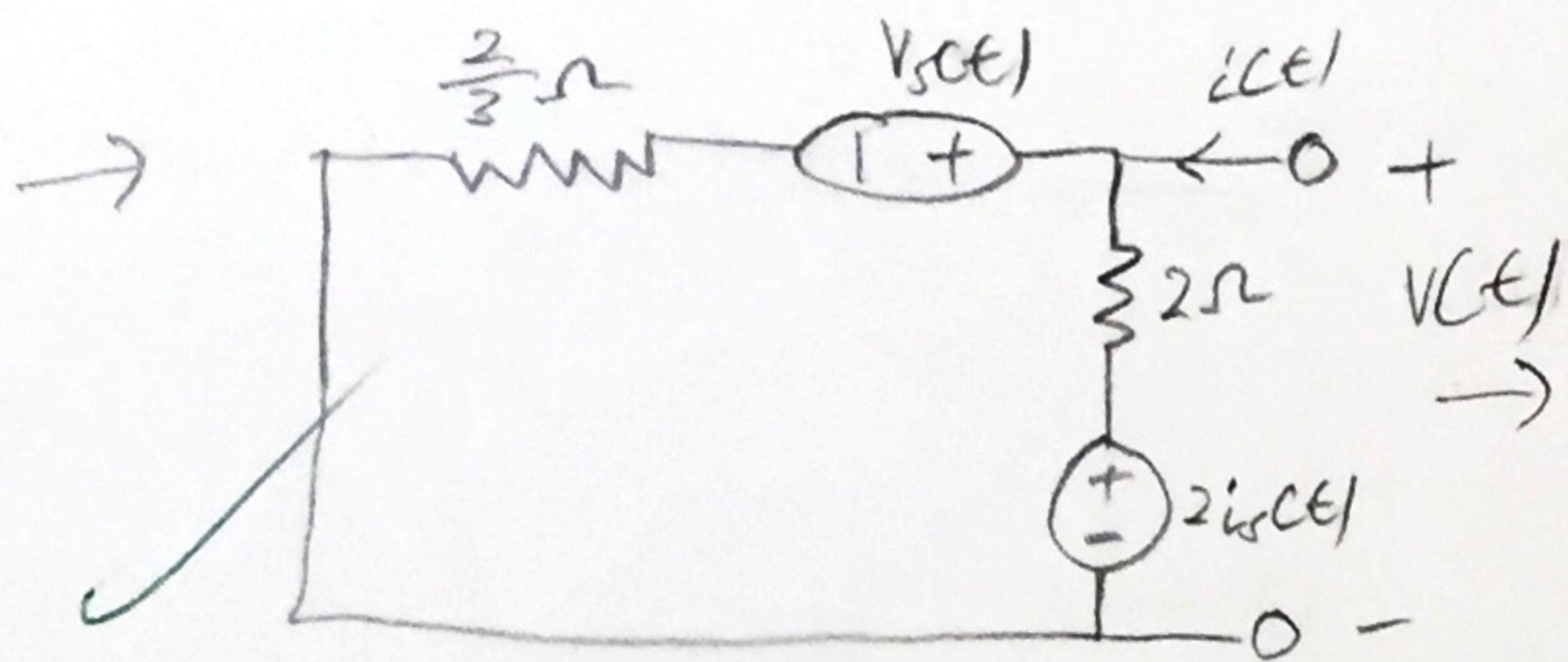
$$= \left(\frac{2}{5}A\right)^2 15\Omega = \boxed{\frac{12}{5}W}$$

Q4. 10 points



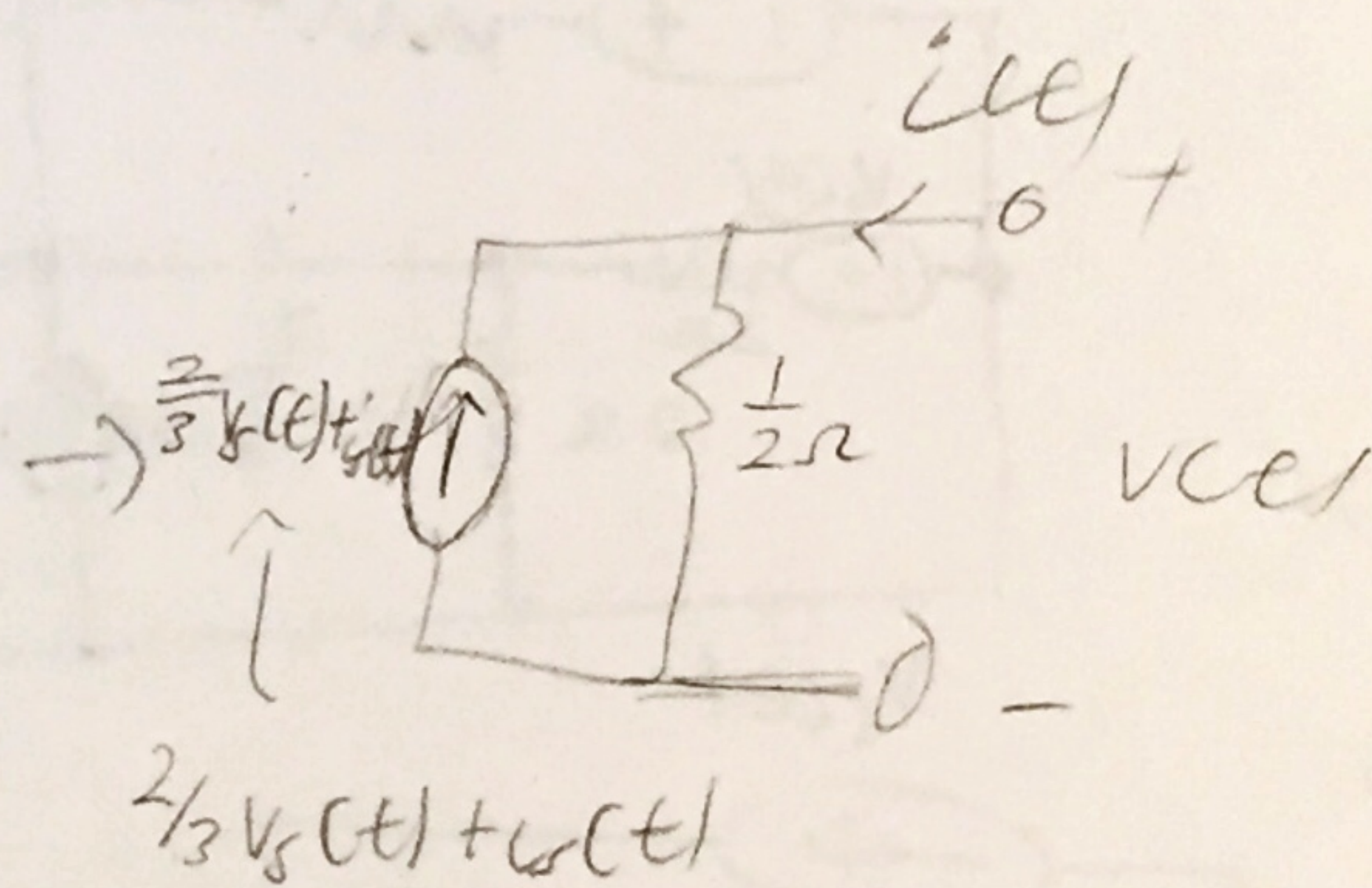
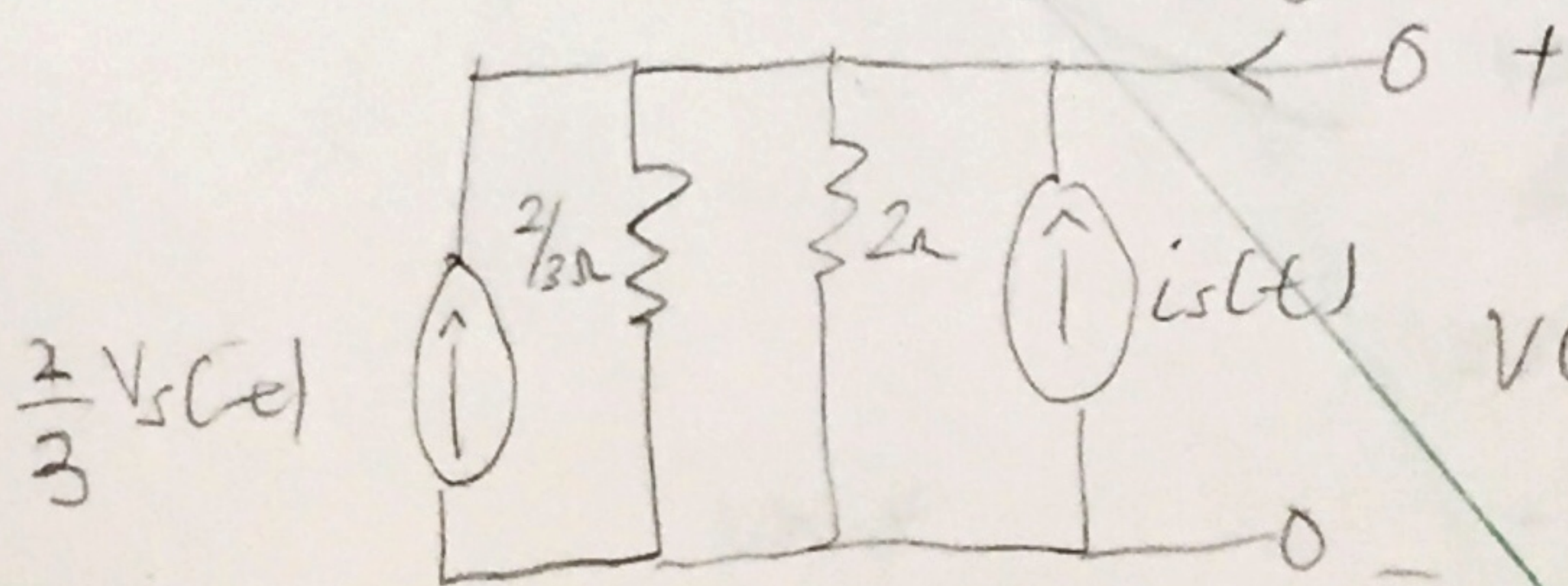
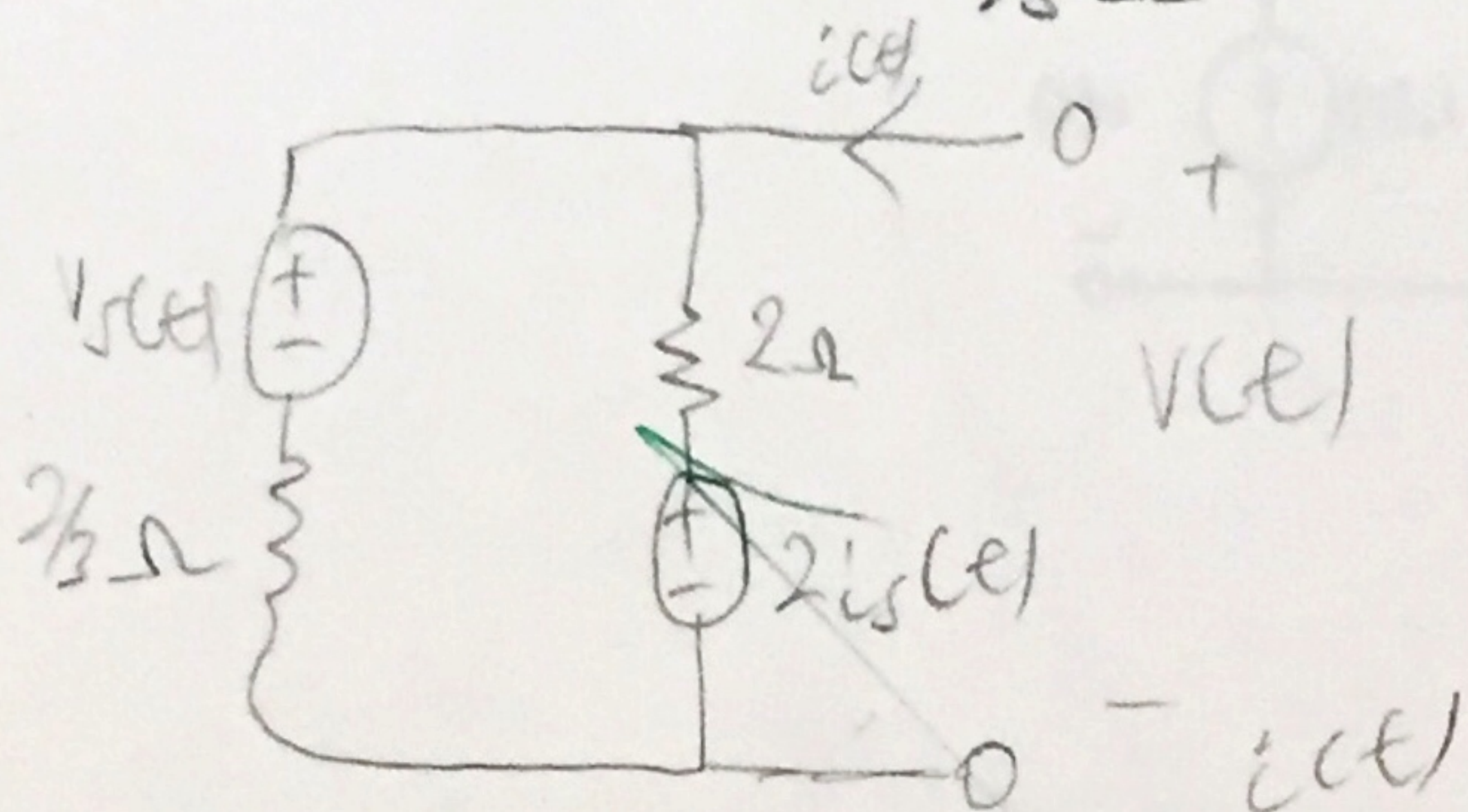
Find $v(t)$ - $i(t)$ relationship using source transformations.





$$V(t) = 2i_s(t) - V_s(t) \quad \times$$

$$i(t) = \frac{2i_s(t) - V_s(t)}{8/3 \Omega} = \frac{3}{4}i_s(t) - \frac{3}{8}V_s(t) \quad \times$$



$$\frac{2}{3}V_s(t) + i_s(t)$$

See Notes