EE 10 : Circuit Analysis I

Winter 2012

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Solutions to Practice Midterm 2

(1)

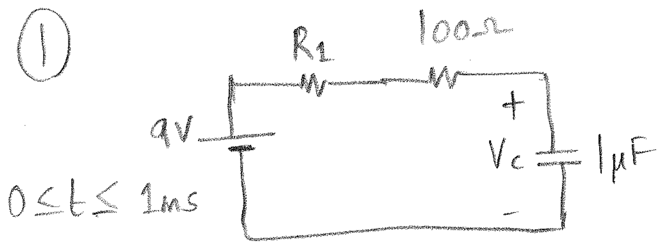


Fig. 1

Steady state voltage if the circuit connections remained as in Fig. 1 for ever.

$$V_c(t) = V_c(\infty) - [V_c(\infty) - V_c(0)]\, e^{-t/\tau_1}.$$

$$V_c(\infty) = 9V$$

$$V_c(0) = 0V$$

$$\tau_1 = (R_1 + 100)C.$$

$$\therefore V_c(t) = 9\left(1 - e^{-t/\tau_1}\right) \quad \text{for } 0 \le t \le 1ms.$$

Given $V_c(1ms) = 8 \Rightarrow 9\left(1 - e^{-1m/\tau_1}\right) = 8.$

$$\Rightarrow \tau_1 = \frac{1ms}{\ln 9} = (R_1 + 100) \times 10^{-6}$$

$$\therefore R_1 = \frac{10^3}{\ln 9} - 100 \simeq 355.12\,\Omega$$

For $1ms < t < \infty$

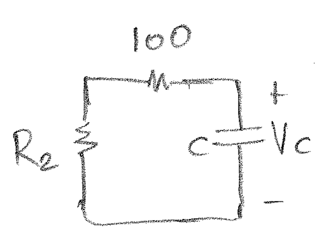


Fig. 2.

$$V_c(t) = V_c(\infty) - [V_c(\infty) - V_c(1ms)]\, e^{-(t-1ms)/\tau_2}$$

Time shift!

Steady state voltage if the circuit connections remained as in Fig. 2. for ever.

$$V_c(\infty) = 0V$$

$$V_c(1ms) = 8V \text{ (Given).}$$

$$\tau_2 = (R_2 + 100)C.$$

$$\therefore V_c(t) = 8e^{-(t-1ms)/\tau_2} \quad \text{for } 1ms < t < \infty.$$

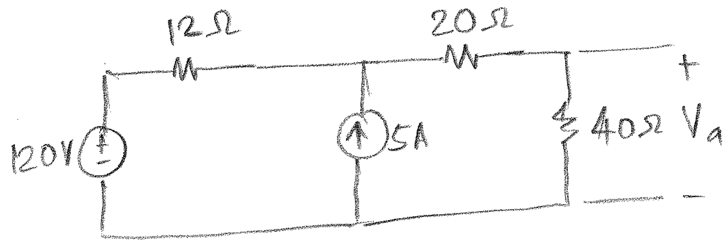
Given $V_c(2ms) = 1V$

$$\Rightarrow 1 = 8e^{-(2-1)ms/\tau_2} \Rightarrow \tau_2 = \frac{1ms}{\ln 8} = (R_2+100)\times 10^{-6}$$

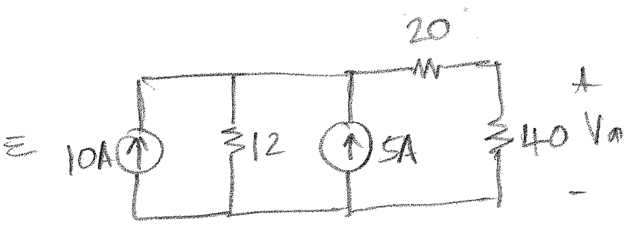
$$\therefore R_2 = \frac{10^3}{\ln 8} - 100 \simeq 380.89\Omega.$$

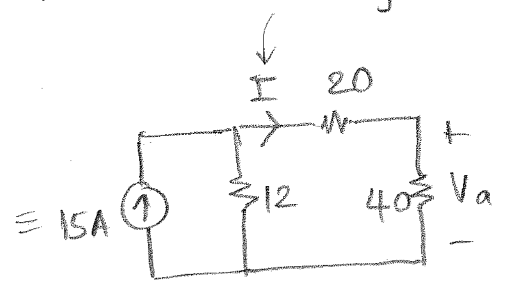
$$\therefore \boxed{\begin{aligned} R_1 &= 355.12\Omega \\ R_2 &= 380.89\Omega \end{aligned}}$$

(2) In steady state, an inductor is equivalent to a short circuit, a capacitor is equivalent to an open circuit. Using this, the equivalent circuit to the given circuit becomes



Find by current divider.

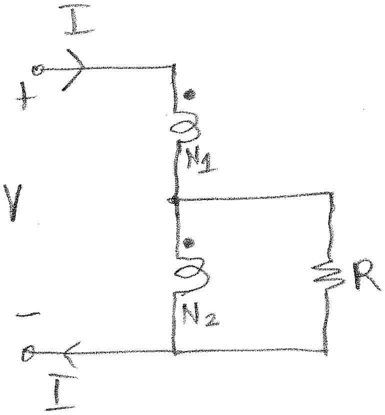




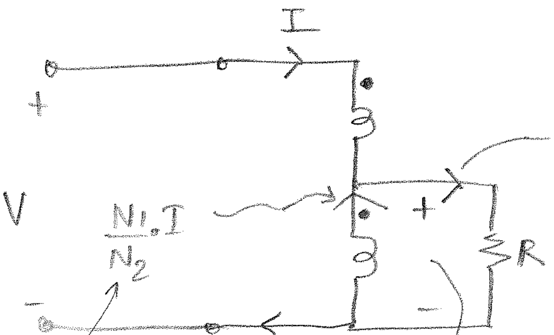
$$\therefore V_a = \underbrace{\frac{12}{12+(20+40)} \times 15}_{I} \times 40 = \frac{12}{72} \times 15 \times 40 = 100V$$

$$\therefore \boxed{V_a = 100V}$$

(3)



Assuming V & I as the only variables, we will find all other voltages & currents in terms of these.



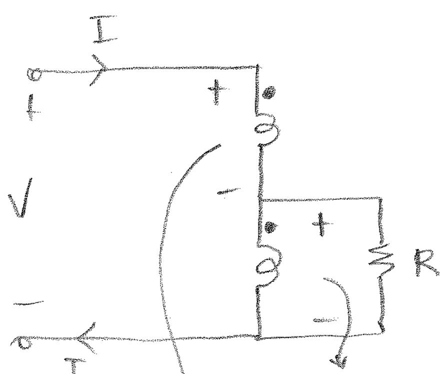
KCL ⇒

$$\frac{N_1}{N_2} I + I = \left(\frac{N_1}{N_2} + 1\right) I$$

$$\left(\frac{N_1}{N_2} + 1\right) I \times R.$$

Using transformer current equation.

⇓



$$\left(\frac{N_1}{N_2} + 1\right) IR$$

$$\frac{N_1}{N_2}\left(\frac{N_1}{N_2} + 1\right) IR$$

Using transformer voltage equation.

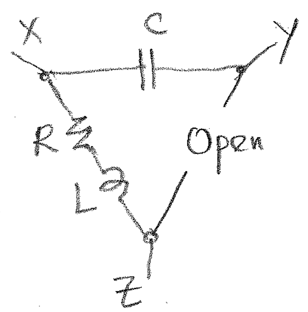
$$V = \frac{N_1}{N_2}\left(\frac{N_1}{N_2} + 1\right) IR + \left(\frac{N_1}{N_2} + 1\right) IR$$

$$\Rightarrow \frac{V}{I} = \left(\frac{N_1}{N_2} + 1\right)\left[\frac{N_1}{N_2} + 1\right] R$$

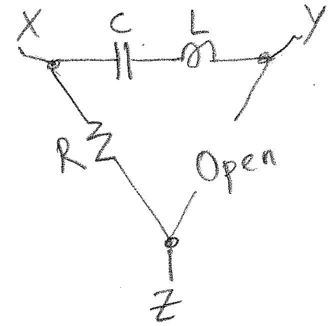
$$\therefore \boxed{R_{eq} = \left(\frac{N_1}{N_2} + 1\right)^2 R.}$$

(4) Possible network connections:

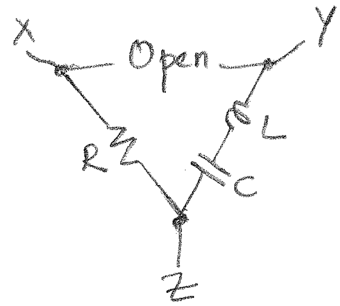
(a)



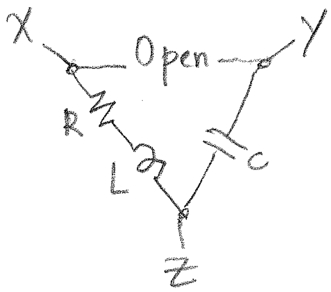
(b)



(c)

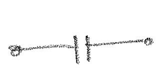


(d)

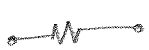


$R = 10\Omega$ in all the connections. L & C can be of any arbitrary values

Since the equivalent resistance between XY & YZ must be ∞, there should be a DC open circuit between (X & Y) and (Y & Z) both. We know that elements that qualify for such a condition are a capacitor, a series LC or an actual open!

i.e.  or  or 

Since the Req between X & Z has to be 10Ω DC wise, elements that qualify for this are an actual resistor of 10Ω or a series L-R with $R = 10\Omega$.

i.e.  or 

$R = 10\Omega$ $R = 10\Omega$