Physics → Circuit Elements → Circuit Laws

Maxwell's Equation ⇒

Resistor: $i - v$

Capacitor: $\underline{q - v}$ + $\underline{i = \frac{dq}{dt}}$

Inductor: $\underline{\psi - i}$ + $\underline{v = \frac{d\psi}{dt}}$

(Mut ind): dot convention

↳ ideal transformer $\begin{cases} \frac{v_1}{n_1} = \frac{v_2}{n_2} \\ i_1 n_1 + i_2 n_2 = 0 \end{cases}$

definition of power:

$P = i \cdot v$

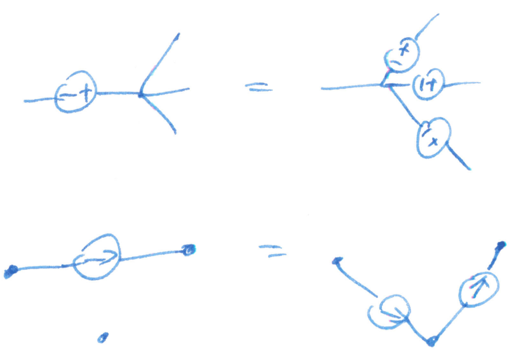
$W = \int P\,dt$

Energy conservation ⇒ KVL ⟹ loop method

Charge conservation ⇒ KCL ⟹ node method.

→ Circuit solving techniques → Special-case circuit:

- Source Shifting:



- Superposition: Keep ONE source at a time ~~Remove ONE~~

- Thevenin / Norton: V_{oc} I_{sc} R_{eq} (R_{th})

↳ source transformation:

- 1^{st} order circuit: (1^{st} order diff eqn)
  1. Write diff eqn based on Law + Technique
  2. Solve for ~~homogeneous~~ general
  3. Solve for particular solution
  4. Plug in initial condition
- 2^{nd} order circuit: (2^{nd} order diff eqn)

↳ SSS: phasor:

$\begin{cases} L \to j\omega L \\ C \to \frac{1}{j\omega C} \\ R \to R \\ M \to j\omega M \end{cases}$

EE10 Final

Department of Electrical Engineering, UCLA

Winter 2014

Instructor: Prof. Gupta

1. Exam is closed book. Calculator and one double sided cheat-sheet is allowed.
2. Cross out *everything* that you don't want me to see. Points will be deducted for everything wrong!
3. Do NOT use Laplace Transforms to solve any problems.
4. No points will be given without proper explanations

Name:

Student ID:

$\boxed{\text{Solution}}$

Student on Left:

Student on Right:

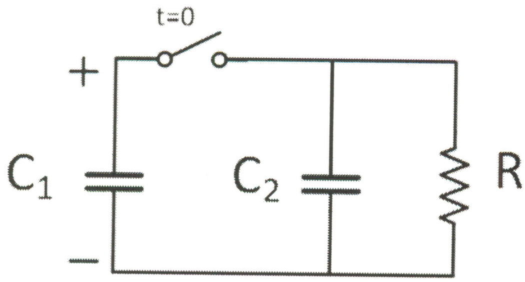
Student in Front:

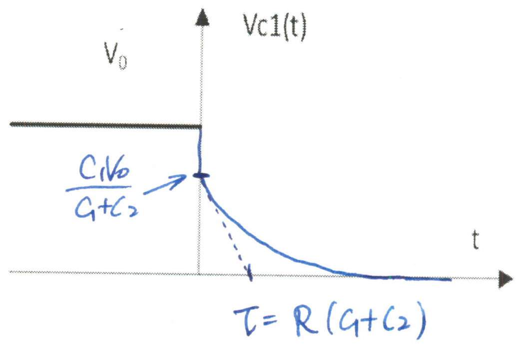
Time: 135 minutes

Problem	Maximum Score	Your Score
1	6	
2	4	
3	10	
4	10	
5	10	
Total	40	

Q1. (6 points) We connect charged capacitor C_1 to a discharged capacitor C_2 and a resistor R with a switch. The capacitor C_1 is charged to voltage V_0.

We close the switch at t=0, derive vC_1 in terms of time and plot it.





At $t=0^+$, $V_{C_1} = V_{C_2} = V_R$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

There is no charge going through R from $t=0^-$ to $t=0^+$,

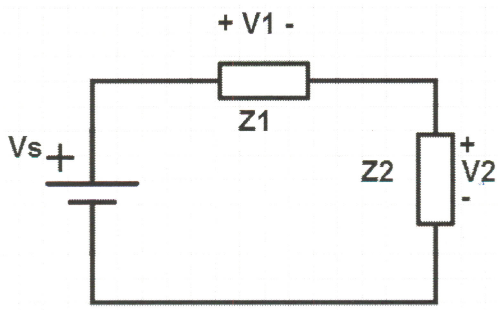
otherwise: $i_R = \frac{dQ_R}{dt} = \infty \implies V_R = \infty \implies$ contradiction

$$\begin{cases} Q_1 + Q_2 = C_1 \times V_0 \\ \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \end{cases} \implies \begin{cases} Q_1 = C_1V_0 \times \frac{C_1}{C_1+C_2} \\ Q_2 = C_1V_0 \times \frac{C_2}{C_1+C_2} \end{cases} \implies V_{C_1} = V_{C_2} = \frac{C_1V_0}{C_1+C_2}$$

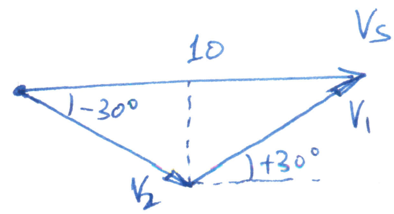
At $t>0^+$, regular 1^{st}-order circuit.

Q2. (4 points) In this circuit: $|V1| = |V2|$, Vs leads V_2 by 30°

For a $V_s(t) = 10 \cos 3t$ find $V_2(t)$ in steady state.



Or graphically:



$$V_s = V_1 + V_2$$

$$10 = V_1 + V_2$$

$$= |V_1| e^{j\phi_1} + |V_2| e^{j\phi_2}$$

$$= |V_1| e^{j\phi_1} + |V_1| e^{j(-30°)}$$

$$\phi_1 = 30° \qquad \phi_2 = -30°$$

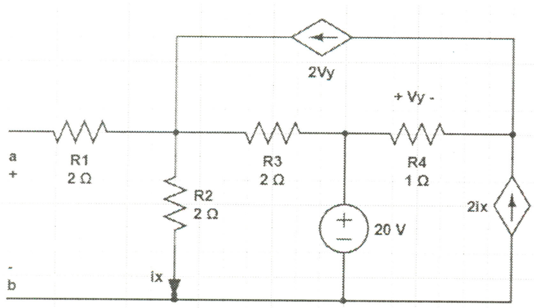
$$|V_1| = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$$

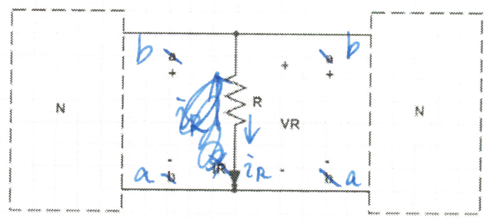
$$V_2(t) = \frac{10\sqrt{3}}{3} \cos(3t - 30°)$$

Q3. (6 + 4 = 10 points)

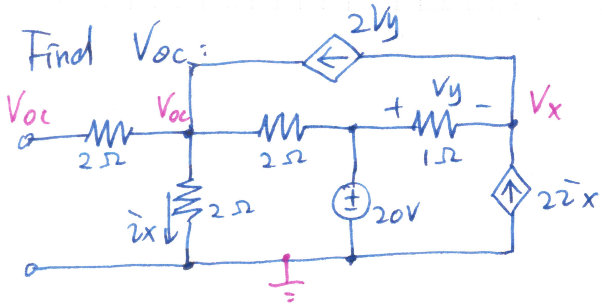
a) Find Thevenin equivalent of the circuit on the left (network N) looking from a & b

b) If we use network N to implement the circuit on the right what is i_R if $V_R = \frac{5}{2} i_R + i_R^2$ (for $i_R > 0$)

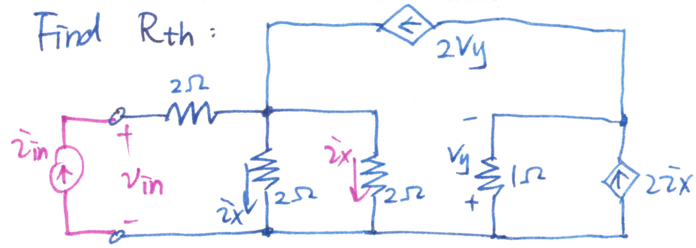




(a) Find V_{oc}:



Find R_{th}:



$$\begin{cases} \frac{20 - V_{oc}}{2} + 2\times(20 - V_x) = \frac{V_{oc}}{2} \\ 2\times\frac{V_{oc}}{2} + \frac{20 - V_x}{1} = 2\times(20 - V_x) \end{cases}$$

$$\Rightarrow V_{oc} = -10V$$

$$\frac{V_y}{1} + 2i_x = 2V_y \Rightarrow V_y = 2i_x$$

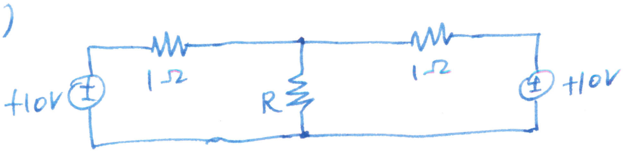
$$i_{in} + 2(2i_x) = i_x + i_x \Rightarrow i_{in} = -2i_x$$

$$V_{in} = i_{in}\times 2 + i_x \times 2$$

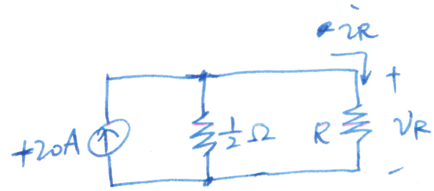
$$= (-2i_x)\times 2 + i_x \times 2 = -2i_x$$

$$\Rightarrow R_{th} = \frac{V_{in}}{i_{in}} = 1$$

(b)



$\Rightarrow$



$$\frac{V_R}{0.5} + i_R = -20$$

$$5i_R + 2i_R^2 + i_R + 20 = 0$$

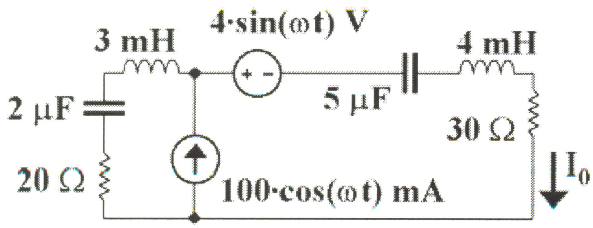
$$i_R = \boxed{2A} \text{ or } \cancel{-5A} \Rightarrow i_R = 2A$$

Q4. (5 + 3 + 2 = 10 points)

Assume $\omega = 10^4$ rad/sec.

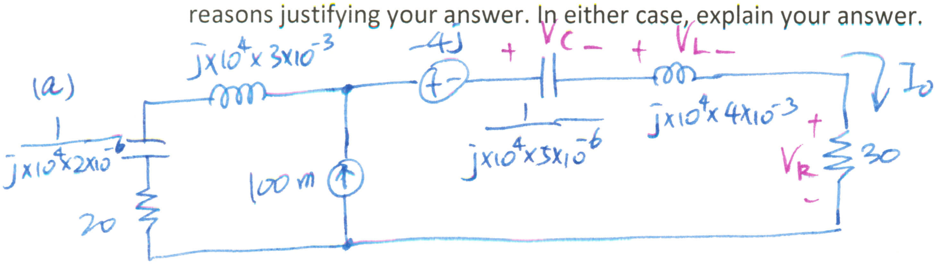
(a) Use source transformation to obtain the current I0. Express your answer in milliamps, both in the phasor notation (rectangular form) and in the time domain.

(b) Draw a phasor diagram showing the voltage across 30ohm resistor, 4mH inductor, 5uF capacitor.

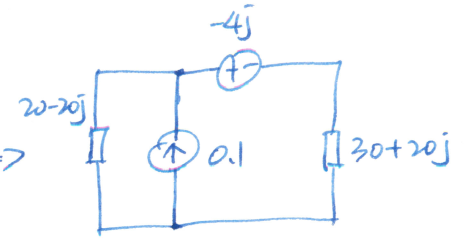


(c) A complicated RLC circuit has a steady state current response of 10cos(100 t + 0.2) amperes when a voltage source of 10cos(100t) volts is applied to it. Now, instead of the original source, a voltage source of 20cos(100t + 0.1) volts is applied to the circuit. Can you find the new current response? Is this information enough to find it? If yes, find the new current response. If not, give reasons justifying your answer. In either case, explain your answer.

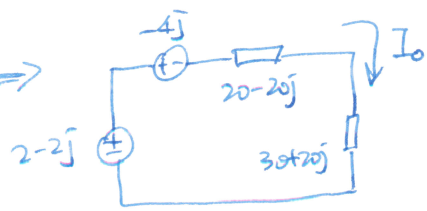
(a)



$\Rightarrow$



$\Rightarrow$



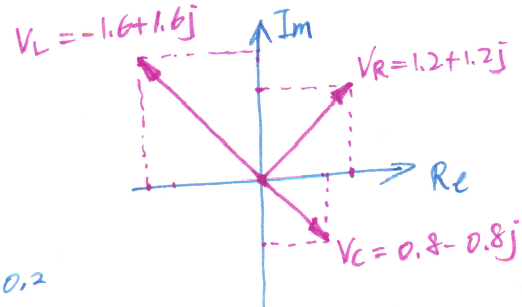
$$I_o = \frac{2+2j}{50} = 0.04 + 0.04j = (40+40j)\ mA$$

$$i_o(t) = 40\sqrt{2} \times \cos(10^4 \times t + 45^\circ)\ mA$$

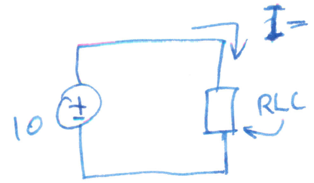
(b) $V_R = 1.2 + 1.2j$

$V_L = -1.6 + 1.6j$

$V_C = 0.8 - 0.8j$



(c)



$\Rightarrow$

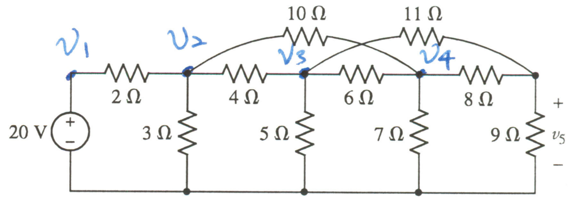
$i'(t) = 20\cos(100t + 0.3)$

$Z_{RLC} = \frac{10}{e^{j.0.2}}$

$I' = \frac{20 \cdot e^{j.0.1}}{Z_{RLC}} = 20 \times e^{j.0.3}$

Q5. (4 + 6 = 10 points)

(a) Label the nodes in this circuit and write down the matrix equation to solve this circuit by node method. You don't need to solve the actual KCL equations.



$$\begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{10} & -\frac{1}{4} & -\frac{1}{10} & 0 \\ 0 & -\frac{1}{4} & \frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{11} & -\frac{1}{6} & -\frac{1}{11} \\ 0 & -\frac{1}{10} & -\frac{1}{6} & \frac{1}{6}+\frac{1}{8}+\frac{1}{7}+\frac{1}{10} & -\frac{1}{8} \\ 0 & 0 & -\frac{1}{11} & -\frac{1}{8} & \frac{1}{8}+\frac{1}{11}+\frac{1}{9} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

(b)

Consider the circuit shown in Figure 1(a). The current i(t), flowing through the inductor was found to obey the straight-line plot shown in Figure 1(b) for $0 < t < 4ms$. Find an expression for v(t) for $0 < t < 4ms$ which satisfies the observation and draw a neat plot for it.

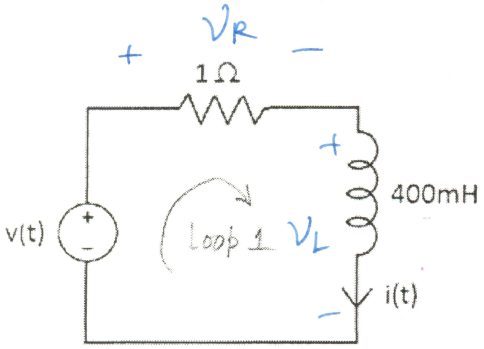


FIGURE 1(a)

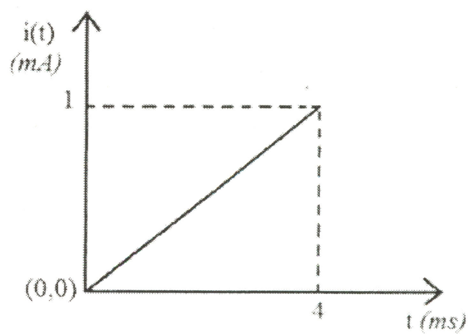


FIGURE 1(b)

$$V_L = L\frac{di}{dt} = 0.4 \times \frac{1}{4} = 0.1\ V$$

$$V_R = R \cdot i = 1 \times i(t) = \frac{1}{4} t\ V$$

$$v(t) = \cancel{(0.1 + 0.25t)\ V \quad t \text{ in s}}$$

$$= (100 + 0.25t)\,mV$$

t in ms