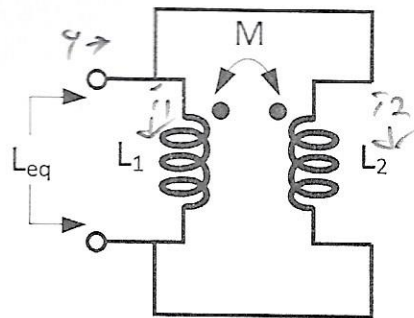


1. Find the equivalent inductance of the circuit below. $L_1 = 5H$, $L_2 = 2H$, and $M = 3H$.



$$i = i_1 + i_2 \quad i' = i_1 + i_2'$$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad \frac{di_1}{dt} = \frac{V_1 - M \frac{di_2}{dt}}{L_1}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\frac{di_2}{dt} = \frac{V_2 - M \frac{di_1}{dt}}{L_2}$$

$$V_1 = V_2 = V$$

$$\frac{di_2}{dt} = \frac{V - M \left(\frac{V - M \frac{di_2}{dt}}{L_1} \right)}{L_2}$$

$$L_1 L_2 \frac{di_2}{dt} = L_1 V - M V + M^2 \frac{di_2}{dt}$$

$$\frac{di_2}{dt} (L_1 L_2 - M^2) = V (L_1 - M)$$

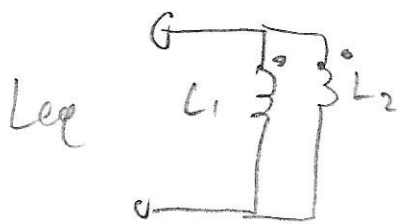
$$\frac{di_2}{dt} = \frac{V (L_1 - M)}{L_1 L_2 - M^2}$$

$$\frac{di_1}{dt} = \frac{V (L_2 - M)}{L_1 L_2 - M^2} + 30$$

$$\frac{di_1}{dt} + \frac{di_2}{dt} = \frac{V (L_1 + L_2 - 2M)}{L_1 L_2 - M^2}$$

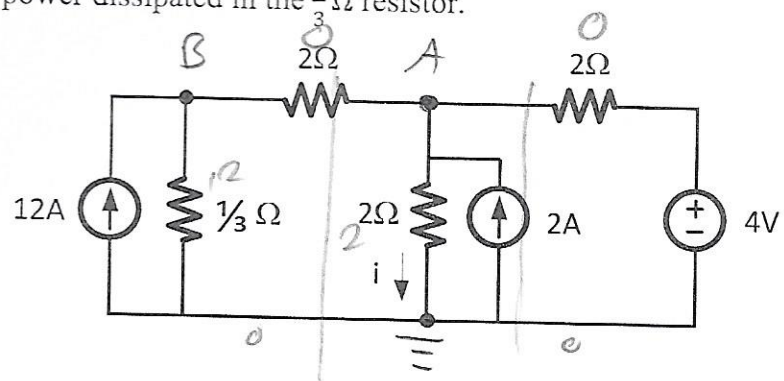
$$i_1 + i_2 = i' = \frac{V}{L_{eq}}$$

$$\therefore L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{5 \cdot 2 - 3^2}{5 + 2 - 6} = \frac{10 - 9}{7 - 6} = 1H$$



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

2. In the circuit below, using node analysis,
- Find the current i .
 - Find the power dissipated in the $\frac{1}{3} \Omega$ resistor.



$$a) \frac{e_A}{2} + \frac{e_A - e_B}{2} + \frac{e_A - 4}{2} - 2 = 0$$

$$3e_B + e_B - e_A - 12 = 0$$

$$6e_B + e_B - e_A = 12$$

$$e_A = 7e_B - 12$$

$$e_A + e_A - e_B + e_A - 4 = 4$$

$$3e_A - e_B = 8$$

$$-e_A + 7e_B = 24$$

$$21e_A - 7e_B = 56$$

$$20e_A = 56 + 24 = 80$$

$$e_A = 4 \text{ V}$$

$$12 - e_B = 8$$

$$e_B = 4 \text{ V}$$

$$-3e_A + 21e_B = 72$$

$$20e_B = 80$$

$$e_B = 4$$

$$i = \frac{e_A}{2} = 2 \text{ A}$$

$$b) 3 \cdot 4^2 = 48 \text{ W}$$

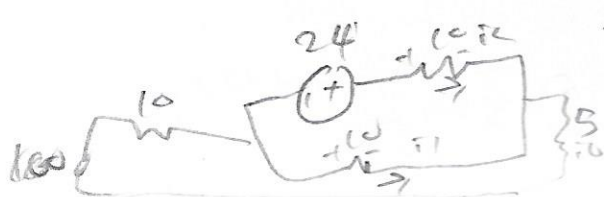
+30

$$3e_B = 12$$

$$i^2 R = \frac{144}{3} = 48$$

$$V = 4 \quad i = 12$$

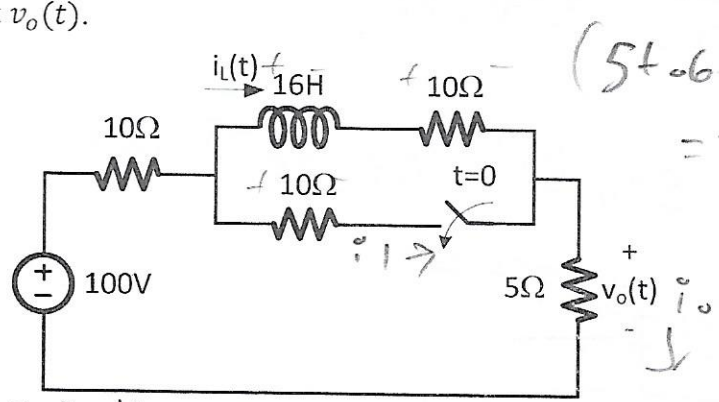
48



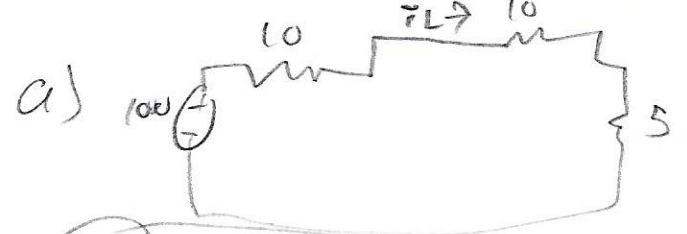
$$\begin{aligned}
 -24 + 10i_L - 10 &= 1 \\
 24 &= 10i_L + 10 \implies 10 = 10i_L \\
 24 &= 10(2) + 10 \implies 10 = 20 \\
 100 &= 15i_o + 10i \\
 100 &= 15i_L + 25i
 \end{aligned}$$

3. The circuit below has been idle for a long time (switch is open). At $t = 0$, the switch is closed.
- Find the inductor current right before and after the switch is closed ($i_L(0^-)$, and $i_L(0^+)$).
 - Find v_o right before and after the switch is closed ($v_o(0^-)$, and $v_o(0^+)$).
 - Find and plot $v_o(t)$.

$$\begin{aligned}
 70 &= 3i_L + 20 \\
 70 &= 30 + 20 \\
 72 &= 30 + 20 \\
 e6 &= \frac{3}{5} i
 \end{aligned}$$



$$\begin{aligned}
 (5 + 0.6e^{-t})15 \\
 = 75 + 9e^{-t} \\
 + 25 - 9e^{-t} = 100
 \end{aligned}$$



$$i_L = \frac{100}{25} = 4A = i_L(0^-)$$

$i_L(0^-) = i_L(0^+) = 4A$

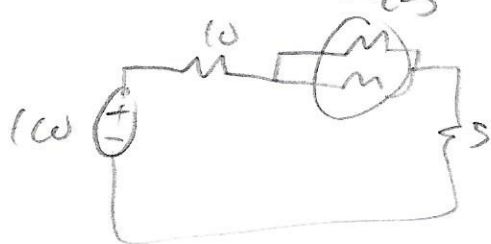
$i_L(0^+) = i_L(0^-) = 4A$
 since infinite voltage is not allowed (finite source)

$v_o(0^-) = 5 \cdot 4 = 20V$ $v_o(0^+) = 5 \cdot (5 + 0.6) = 28V$

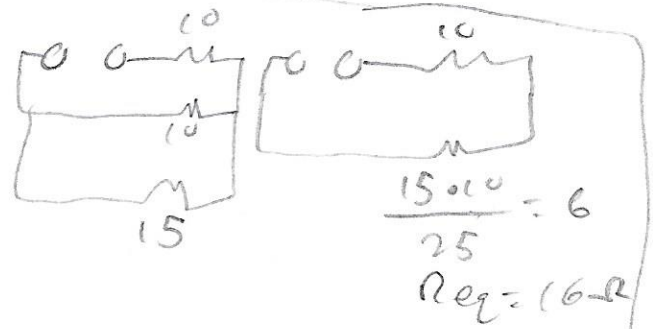
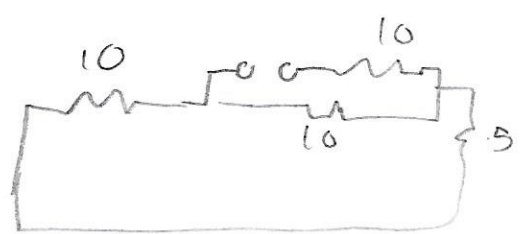
$$\begin{aligned}
 i_L(0^+) &= 4A \\
 i_L(\infty) &= \frac{5}{2}A \\
 R_{eq} &= 5
 \end{aligned}$$

$$i_L(t) = \frac{5}{2} + (4 - \frac{5}{2})e^{-\frac{16}{16}t} = \frac{5}{2} + \frac{3}{2}e^{-t}$$

$$v_L(t) = L \frac{di_L}{dt} = 16 \left(-\frac{3}{2}e^{-t} \right) = -24e^{-t}$$



$$\begin{aligned}
 \frac{100}{20} &= 5A \\
 \frac{5 \cdot 10}{20} &= \frac{5}{2}A
 \end{aligned}$$



$$\begin{aligned}
 \frac{15 \cdot 10}{25} &= 6 \\
 R_{eq} &= 16\Omega
 \end{aligned}$$