UCLA — Electrical Engineering Dept. EE102: Systems and Signals — Midterm Exam Wednesday, October 28, 2015

This exam has 4 questions, for a total of 33 points.

Closed book. One two-sided cheat-sheet allowed. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Please, write your name and ID on the top of each loose sheet!

Name and ID:

Name of person on your left:

Name of person on your right:

Question	Points	Score
1	9	8
2	10	10.
3	8	7.5
4	6	3,5

Total:	33	29
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Multiple-choice questions – Check all the answers that apply.

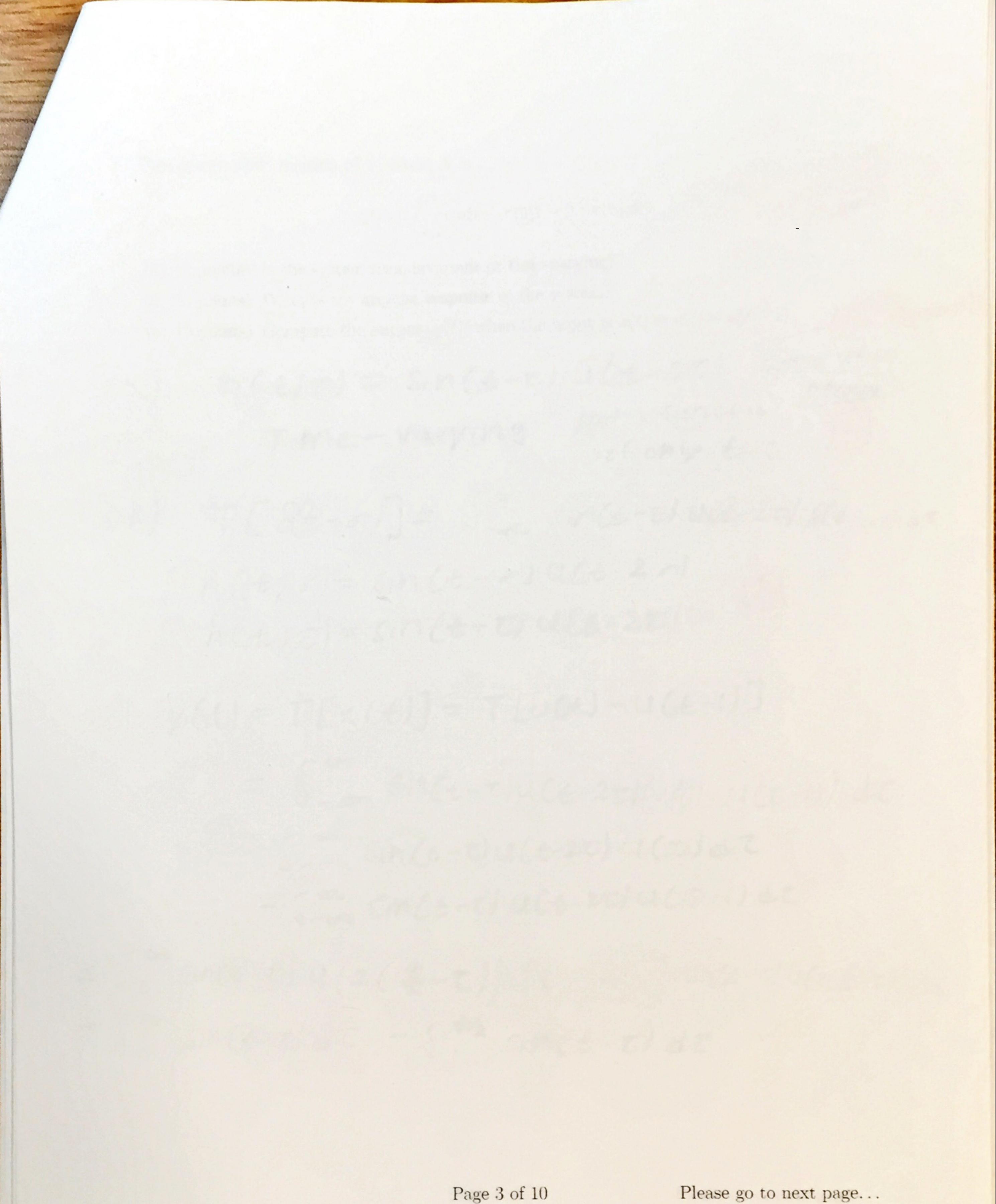
tput relation $K_1 \times (e) + k_2 \times (e)$ $sin (k_1 \times (e) + k_2 \times (e))$ Non (hear) $Z(E) = \times (E - E)$ TEZ(E) = sin (Z(E+1))(a) (3 points) Consider the system described by the input-output relation $y(t) = \sin(x(t+1)).$ What are its properties? □ Linear Time-invariant = $Sin(x(t+1-t_0))$ Causal (b) (3 points) Consider the system described by the input-output relation [S(t-0)] $y(t) = \int_{-\infty}^{\infty} e^{t} e^{-\tau} x(\tau) u(t-\tau) d\tau.$ $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{t} e^{-\tau} x(\tau) u(t-\tau) d\tau.$ $= 5^{\infty} e^{\pm} e^{\pm} S(\Xi - \sigma)$ What are its properties? YCEI= 500 e Linear Time-invariant $h(t; \tau) = e^{t}e^{-\tau} u(t - \tau)$ (c) (3 points) Consider the system described by the input-output relation $\ell - \tau < 0$ $y(t) = x(at), \quad a > 0.$ What is the condition on a for the system to be causal?

X(-0,5) X(-0.25)

XCE)

TESCE-old= Se etet SCE-oluce-tdt Z(E)=(E-to) u(t-to-z)dt $= \int_{-\infty}^{\infty} e^{\pm} e^{-\tau} Z(\tau) u(t-\tau) d\tau$ TEZCEJJ = Stap ete- z x (t-to) u(t-z) dz

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2. The input-output relation of a system S is

of a system S is

$$y(t) = \int_{-\infty}^{\infty} \sin(t-\tau) u(t-2\tau) x(\tau) d\tau. \longrightarrow \text{This is not only outpend}.$$

Not the mon!!!

(a) (2 points) Is the system time-invariant or time-varying? (b) (3 points) What is the impulse response of the system? (c) (5 points) Compute the output, y(t), when the input is x(t) = u(t) - u(t-1)

2a) h(t/z) = sin(t-z) u(t-2z) convolution Time-varying Not a function integral of only t-z [2] $2b) T [\delta(t-\sigma)] = \int_{-\infty}^{\infty} sin(t-\tau) u(t-2\tau) \delta(t-\sigma) d\tau$ $h(t; \sigma) = sin(t - \sigma)u(t - 2\sigma)$ $h(t; \tau) = sin(t - \tau)u(t - 2\tau)$ [3] 2c| y(t) = T[x(t)] = T[u(t) - u(t-1)]

=
$$\int_{-\infty}^{\infty} \sin(t-\tau) u(t-2\tau) (u(t)-u(t-1)) dt$$

=
$$\int_{-\infty}^{\infty} \sin(t-\tau) u(t-2\tau) u(\tau) d\tau$$

- $\int_{-\infty}^{\infty} \sin(t-\tau) u(t-2\tau) u(\tau-1) d\tau$

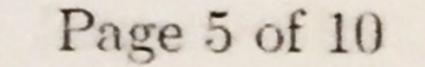
$$= \int_{0}^{\infty} sin(t-\tau) \, u\left(2\left(\frac{t}{2}-\tau\right)\right) d\tau - \int_{1}^{\infty} sin(t-\tau) \, u(2(\frac{t}{2}-\tau)) d\tau \\ = \int_{0}^{t_{2}} sin(t-\tau) \, d\tau - \int_{1}^{t_{2}} sin(t-\tau) \, d\tau$$

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When EKO YCE) = 0 when O<t<2 YCEJ= Sta SinCE-ZIdT $= COS(E-T)/D^{E/2}$ $= \cos(t - t_2) - \cos(t)$ = COS (E/2) - COS (E/ V when E>2 YCEJ = Sta sin(t-tldt - Sta sin(t-t) dt $= \cos(\frac{4}{2}) - \cos(\frac{4}{2}) - \left(\cos(\frac{4}{2}) - \left(\cos(\frac{4}{2}) - \cos(\frac{4}{2}) - \cos(\frac{4}{2})$ 5 = Cos(t-1) - Cos(t) V

 $Y(E) = \left[\cos(\frac{\epsilon}{2}) - \cos(E) \right] U(E) U(2-E)$

+ [cosce-1)-coscel]u(E-2)

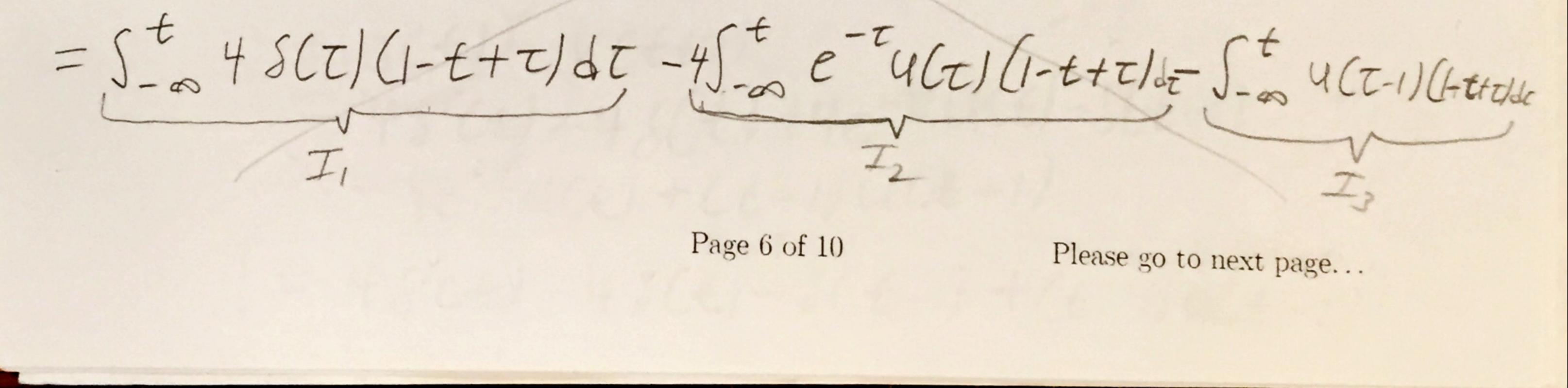


3. Let S be a linear, time-invariant, and causal system. We know that the output corresponding to x(t) = (t-3)u(t-3) is

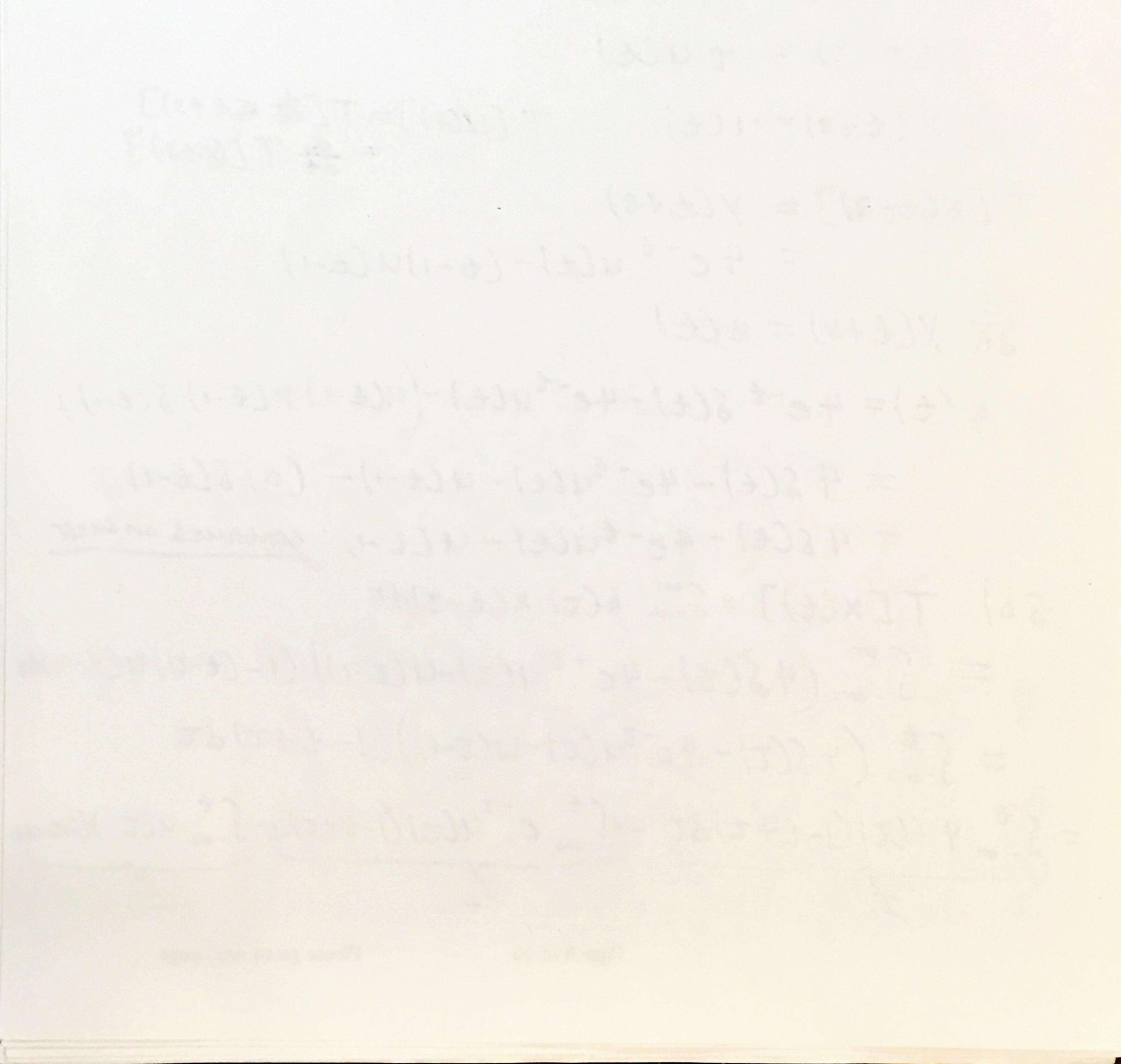
$$y(t) = 4e^{-(t-3)}u(t-3) - (t-4)u(t-4).$$

(a) (3 points) What is the impulse response of S? (Hint: consider that $\frac{d}{dt}tu(t) = u(t)$.) (b) (5 points) Compute the output of S when x(t) = (1 - t)u(t). (c) $X(-t) \rightarrow Y(-t) h(t) = ?$

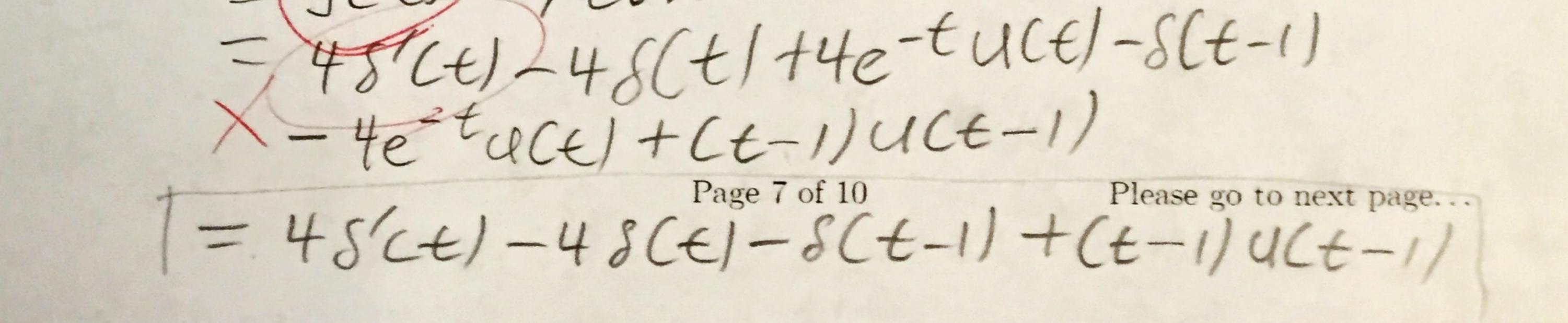
 $X(\pm\pm3) = \pm u(\pm)$ $\frac{d}{dE} \times (E+3) = U(E)$ $T[u(\epsilon)] = T[\frac{1}{\epsilon}x(\epsilon+3)]$ $= \frac{d}{d\epsilon} T \Gamma(\kappa + 3) T$ T[X(t+3)] = Y(t+3)= $4e^{-t}u(t) - (t-1)u(t-1)$ $\frac{d}{dE}$ Y(E+3) = g(E) $g(t) = 4e^{-t}S(t) - 4e^{-t}u(t) - (u(t-1) + (t-1)S(t-1))$ $= 4 S(t) - 4e^{-t}u(t) - u(t-1) - (0) S(t-1)$ = 48(t) - 4e-tucti - 4(t-1) jontinued on back 3b) $T[X(t)] = S_{-\infty}^{\infty} h(t) X(t-t) dt$ $= \int_{-\infty}^{\infty} (4S(\tau) - 4e^{-\tau}u(\tau) - u(\tau)) (1 - (t - \tau))u(t - \tau)br$ $= \int_{-\infty}^{+} (4S(\tau) - 4e^{-\tau}u(\tau) - u(\tau-1))(1 - t + \tau)d\tau$



3al hCEI = de gcel = 4 s'(e) - 4(e^{-t} s(t) - e^{-t} u(t)) - s(t)) $] = 45'CEI - 45CEI + 4e^{-EuCEI - 5CE-I)$



 $T_1 = S_{-\infty} + S(\tau)(1 - t + \tau) + (t - \tau) d\tau$ = 4(1-t)u(t) $T_2 = 45t e^{-\tau}uct/cl - t + t/dt$ $= 4 \int_{0}^{t} e^{-\tau} (1 - t + \tau) d\tau \qquad y = (1 - t + \tau) dv = e^{-\tau} d\tau$ $du = d\tau \qquad v = -e^{-\tau} d\tau$ $= 4 (-e^{-\tau} (1 - t + \tau) |_{0}^{t} + S_{0}^{t} e^{-\tau} d\tau)$ $= 4(-e^{-t}(1)+(1-t)+[-e^{-t}]^{t})$ $= 4(-e^{-t}+1-t-e^{-t}+1)$ $= 4(-2e^{-t}+2-t) = -8e^{-t}+4-4t$ = SF 1- t+ t dt $= \begin{bmatrix} T + t + t + \frac{t^2}{2} \end{bmatrix}_{1}^{t} = t + t^2 + \frac{t^2}{2} - (1 + t + \frac{t}{2})$ $= t^{2} + \frac{t^{2}}{2} - \frac{3}{2} = \frac{3}{2}t^{2} - \frac{3}{2}$ $Y(\mathcal{E}) = I_1 - I_2 - I_3$ = 4(1-t)u(t)+8e-t-4+4+6-3+2+3 = 44(E) - 4Euce + 8e-E + 4E - 3E^2 - 2 361 XCEI = UCEI - EUCEI TEXCEJ= TEURJ- TETUCEJ = get - TEX(tt3)J .0.5 = (gCEJ YCE+3)



4. A system S_1 is described by

$$\frac{\mathrm{d}y}{\mathrm{d}t} + 2y(t) = \frac{\mathrm{d}x}{\mathrm{d}t} - 2x(t), \quad t > 0, x(0) = 0, y(0) = 0.$$

(a) (3 points) Write down the impulse response function $h(t;\tau)$ of S_1 .

(b) (3 points) System S_1 is now cascaded with a second linear, time-invariant system, S_2 whose unit step response, $g_2(t)$ is given by LTI

 $g_2(t) = t e^{-t} u(t).$

Compute the impulse response, $h_{1,2}(t;\tau)$, of the cascaded combination. ta) $M = e^{2dt} = p^{2t}$

I [e^{2t} ycel] = c^{2t} (I = -2x(e)) Stat [e²T YLJ] = St e²T (# - 2xCI) dt e^{2t}y(t) = Stert (dx - 2x(t))dt YCEJ= Sterce -2xCTI)dt 1 $T \left[S(t-\sigma) \right] = \frac{S_{0}^{*} e^{2\tau} \left(\frac{2}{2} - 2S(\tau-\sigma) \right) d\tau}{2}$ p2t = Sterr dx dz - 2 Sterr S(z-o)dz $5 = e^{2\tau} \frac{dx}{d\tau} d\tau - 2 \int_{-\infty}^{\infty} e^{2\tau} u(\tau) u(t-\tau) \delta(t-\sigma) d\tau$ pze $= \frac{S_{0}^{t}e^{2\tau} \frac{dx}{d\tau} d\tau - 2e^{2\sigma} 4(\sigma) 4(t-\sigma)}{e^{2t}}$ $h(t;\tau) = \frac{S_{0}^{t}e^{2\sigma} \frac{dx}{d\sigma} d\sigma - 2e^{2\tau} 4(\tau) 4(t-\tau)}{e^{2t}} (T)$ CT1?

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Szi TLUCEIJ= te-tuce) = 800 de gCEI = h2CEI h2(t) = t'etuce) + t (etuce) $= e^{-t}u(t) + t(e^{-t}s(t) - e^{-t}u(t))$ $= e^{-t}u(t) + t \left(s(t) - e^{-t}u(t)\right)$ $= e^{-t}u(t) + ts(t) - te^{-t}u(t)$ = $e^{-t}u(t) - te^{-t}u(t)$ $\Rightarrow = e^{-t}u(t)(1-t)$ $h_{i,2}(t;\tau) = \int_{-\infty}^{\infty} h_2(t;\sigma) h_i(\sigma;\tau) d\sigma$ = S-00 h2 (t-o) h, (o/t) do = $\int_{-\infty}^{\infty} e^{-(t-\sigma)} u(t-\sigma)(1-t+\sigma) = \int_{0}^{\infty} e^{2\alpha} \frac{dx}{d\alpha} d\alpha - 2e^{2t} u \left(\frac{1}{2} u(t-\tau)\right) \frac{1}{d\sigma} \frac{1}{d\sigma}$ $= e^{-t} \int_{-\infty}^{t} e^{-\sigma} \left(1 + t + \sigma \right) \left(\int_{0}^{\infty} e^{2\alpha} \frac{dx}{d\alpha} d\alpha - 2e^{2t} \mu(t) \mu(\sigma - t) \right) d\sigma$ $= e^{-t} \int_{-\infty}^{t} e^{-\sigma} \left(\frac{1+t+\sigma}{5\sigma} e^{2\sigma} \frac{dx}{d\sigma} d\sigma \right) d\sigma$

 $-\int_{-\infty}^{\infty} e^{-\sigma} (1+t+\sigma) 2e^{2t} u(\tau) u(\sigma-\tau) d\sigma$

So 2000 e2t (1+t++) uco-z/do

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Product to Sum	Sum to Product	
$2\sin(\alpha)\sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta)$	$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$	
$2\cos(\alpha)\cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta)$	$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$	
$2\sin(\alpha)\cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta)$	$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$	
$2\cos(\alpha)\sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta)$	$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$	
Sum/Difference	Pythagorean Identity	
$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$	$\sin^2(\alpha) + \cos^2(\alpha) = 1$	
$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$		
$\tan(\alpha \pm \beta) = \tan(\alpha) \pm \tan(\beta)$		
$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha) \tan(\beta)}$		
Even/Odd	Periodic Identities	
$\sin(-\alpha) = -\sin(\alpha)$	$\sin(\alpha + 2\pi n) = \sin(\alpha)$	
$\cos(-\alpha) = \cos(\alpha)$	$\cos(\alpha + 2\pi n) = \cos(\alpha)$	
$\tan(-\alpha) = -\tan(\alpha)$	$\tan(\alpha + \pi n) = \tan(\alpha)$	
Double-Angle Identities	Half-Angle Identities	
$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$	$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos(\alpha)}{2}}$	
$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$	$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$	
$\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$	$\tan\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos(\alpha)}{1 + \cos(\alpha)}}$	
Laws of Sines, Cosines, and Tangents	Mollweide's Formula	
$\sin(\alpha) \sin(\beta) \sin(\gamma)$	$a+b = \cos\left(\frac{\alpha-\beta}{2}\right)$	
$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\cos(\beta)}{c}$	$c = \sin\left(\frac{\gamma}{2}\right)$	
$a^2 - b^2 + c^2 - 2bc\cos(\alpha)$		
$a - b = \frac{\tan\left(\frac{\alpha - \beta}{2}\right)}{$		
$\frac{\alpha}{a+b} = \frac{\alpha}{\tan\left(\frac{\alpha+\beta}{2}\right)}$		

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