UCLA

DEPARTMENT OF ELECTRICAL ENGINEERING

Fall 2008

Your Name (LAST, Middle, First):————————————-

(Fill in the following lines — if applicable)

NAME OF FRIEND ON YOUR LEFT:————————————

NAME OF FRIEND ON YOUR RIGHT:————————————

EE102: SYSTEMS & SIGNALS

November 3

MIDTERM EXAMINATION

Instructions:

(i) Closed Book. Calculators, Cell Phones, IPods and IPhones Are NOT Allowed.
A Two-Sided Sheet (8.5 x 11.0) of Notes Is Allowed

(ii) Please Write On ONE Side Only — otherwise, <u>-5 pts/(second-sided) page</u>

(iii) Questions Are Equally Weighted, Unless Otherwise Indicated

(iv) Please Put The First Letter of Your LAST NAME In The Upper Right Hand Corner of This Page

(v) Staple These Examination Papers With Your Papers — using your own stapler

Part I: Time-Domain Analysis

(Do NOT use Laplace Transforms Here)

1. (15%) The IPOP relation of a SISO system S is:

$$\mathbf{x(t)} \longrightarrow [\mathbf{S}] \longrightarrow \mathbf{y(t)}$$

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau)\, d\tau, \quad t \in (-\infty, \infty)$$

Write down the IRF $h(t,\tau)$ of S. Then compute its OP $y(t)$ given that its IP $x(t)$ is

$$x(t) = (t-1)\, U(t-1) + (t+1)\, U(t+1), \quad t \in (-\infty, \infty)$$

SOLS: Rewrite $y(t)$ as:

$$y(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)}\, U(t-\tau)\, x(\tau)\, d\tau, \quad t \in (-\infty, \infty)$$

Then since S is L, it follows from BT that

$$\underline{h(t,\tau) = e^{-(t-\tau)}\, U(t-\tau), \quad t, \tau \in (-\infty, \infty)}$$

Next, set

$$x_1(t) = (t-1)\, U(t-1), \quad \text{and} \quad x_2(t) = (t+1)\, U(t+1)$$

Then consider:

$$t\, U(t) \longrightarrow [S] \longrightarrow \widehat{y}_1(t)$$

where

$$\widehat{y}_1(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \tau U(\tau) d\tau = \int_{0}^{t} e^{-(t-\tau)} \tau\, d\tau, \quad t \geq 0$$

We find

$$\widehat{y}_1(t) = [t-1+e^{-t}]\, U(t) \quad \Rightarrow \quad y_1(t) = \widehat{y}_1(t-1) = [(t-1)-1-e^{-(t-1)}]\, U(t-1)$$

Similarly,

$$y_2(t) = \widehat{y}_1(t+1) = [(t+1) - 1 + e^{-(t+1)}]\, U(t+1)$$

Finally

$$y(t) = y_1(t) + y_2(t)$$

or

$$\underline{y(t) = [t - 2 + e^{-(t-1)}]\, U(t-1) + [t + e^{-(t+1)}]\, U(t+1)}$$

2. A system S_1 is described by the IPOP relation:

$$x(t) \longrightarrow \ [S_1] \ \longrightarrow y(t), \quad y(t) = \int_{-\infty}^{\infty} \sinh(t-\tau)\, U(t-\tau)\, x(\tau)\, d\tau, \quad t \in (-\infty, \infty)$$

and a second system S_2 is described by the IPOP relation:

$$v(t) \longrightarrow \ [S_2] \ \longrightarrow w(t)$$

$$w(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)}\, U(t-\tau)\, v(\tau)\, d\tau, \quad t \in (-\infty, \infty)$$

Compute the IRF $h_{21}(t)$ of the cascaded system $S_{21} := S_2\, S_1$, i.e., the system

$$IP \longrightarrow \ [\ [S_2] \rightarrow \ [S_1]\ ] \ \longrightarrow OP$$

SOLS: You have

$$h_1(t) = \sinh t U(t) \quad \text{and} \quad h_2(t) = e^{-t} U(t)$$

Therefore

$$h_{12}(t) = \int_{-\infty}^{\infty} e^{-(t-\sigma)}\, U(t-\sigma)\, \sinh\sigma\, U(\sigma)\, d\sigma$$

You find:

$$\underline{h_{12}(t) = \frac{1}{2}[\sinh t - te^{-t}]\, U(t)}$$

3. **(15%)** Given the following information regarding a system S:

$$U(t-1) \longrightarrow \ \mathbf{[L,TI,C;\ IRF:\ h(t)U(t)]} \ \longrightarrow \widehat{g}(t)$$

Is it true that

$$\widehat{g}(t+1) = \int_{-\infty}^{\infty} U(t-\tau)\, h(\tau)\, U(\tau)\, d\tau\ ?$$

Please give details of your answer.

Find $h(t)$ given that

$$\widehat{g}(t+1) = \sin t\, U(t).$$

SOLS: We have

$$U(t-1) \longrightarrow [S] \longrightarrow \widehat{g}(t)$$

Therefore, since S is TI:

$$U(t) \longrightarrow [S] \longrightarrow \widehat{g}(t+1)$$

In other words $\widehat{g}(t+1)$ is the Unit Step Response (USR) $g(t)$ of S:

$$\widehat{g}(t+1) = g(t)$$

But you know that

$$g(t) = \int_0^t h(\tau)\, d\tau, \quad t \geq 0.$$

Therefore

$$\widehat{g}(t+1) = \int_0^t h(\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, U(t-\tau)\, U(\tau)\, d\tau.$$

Finally

$$h(t) = \frac{d}{dt}\, h(t) = \frac{d}{dt}\, \widehat{g}(t+1) = \frac{d}{dt}\, \sin t\, U(t)$$

or

$$\underline{h(t) = \cos t\, U(t) + \sin t\, \delta(t) = \cos t\, U(t)}.$$

Q4] $S_1: \sinh(t)u(t) \rightarrow H_1(s) = \frac{1}{s^2-1}$

$S_2: \quad e^{-t}u(t) \rightarrow H_2(s) = \frac{1}{s+1}$

$$S_{21} = H_1(s)H_2(s) = \frac{1}{s^2-1}\,\frac{1}{s+1} = \cancel{A}\,\frac{1}{(s-1)(s+1)^2}$$

$$= \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$A = S_{21} \times (s-1)\Big|_{s=1} = 1/4$$

$$C = S_{21} \times (s+1)^2\Big|_{s=-1} = -1/2$$

To get B: $\frac{1}{(s-1)(s+1)^2} \equiv \frac{1/4}{(s-1)} + \frac{B}{(s+1)} - \frac{1/2}{(s+1)^2}$

$$\therefore 1 = \frac{1}{4}(s+1)^2 + B(s+1)(s-1) - \frac{1}{2}(s-1)$$

comparing coef of s^2:

$$0 = \frac{1}{4} + B \Rightarrow B = -1/4$$

$$\therefore S_{21}: \quad H_1(s)H_2(s) = \frac{1/4}{s-1} - \frac{1/4}{s+1} - \frac{1/2}{(s+1)^2}$$

$$h_{21}(t) = \left[\frac{1}{4}e^{t} - \frac{1}{4}e^{-t} - \frac{1}{2}te^{-t}\right]u(t)$$

Q5] i) $Y(s) = \frac{s}{s^2+5s+4} = \frac{s}{(s+4)(s+1)} = \frac{4/3}{s+4} - \frac{1/3}{s+1}$

$$y(t) = \left[\frac{4}{3}e^{-4t} - \frac{1}{3}e^{-t}\right]u(t)$$

ii) $Y(s) = X(s)\,H(s) = \frac{s}{(s+4)(s+1)}$

There are infinite ways of choosing $X(s)$ and $H(s)$

eg: $X(s) = \frac{s}{s+4}$, $H(s) = \frac{1}{s+1}$

$X(s) = \frac{\cancel{8}\,7}{s+1}$, $H(s) = \cancel{\frac{1}{8}}\ \frac{1}{7}\frac{s}{s+4}$

$\vdots$

choosing $X(s) = \frac{s}{s+4}$, $H(s) = \frac{1}{s+1}$

$X(s) = \frac{s+4-4}{s+4} = 1 - \frac{4}{s+4} \rightarrow x(t) = \delta(t) - 4e^{-4t}u(t)$

$H(s) = \frac{1}{s+1} \rightarrow h(t) = e^{-t}u(t)$

Q6] i) $\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt}$

$$[S^2Y(s) - S\cancel{y(0)} - \cancel{\dot{y}(0)}] + 2[SY(s) - \cancel{y(0)}] + Y(s) = SX(s) - \cancel{x(0)}$$

$$\therefore \; Y(s)\,[s^2 + 2S + 1] = S\,X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{S}{s^2 + 2S + 1} = \frac{S}{(S+1)^2}$$

ii) $x(t) = e^{-2t}u(t) \rightarrow x(0) = 1$

$$X(S) = \frac{1}{S+2}$$

$$\cancel{Y(s) = X(s)H(s) = \frac{S}{(S+1)^2}\,\frac{1}{S+2}}$$

Back to the DE:

$$Y(s)\,(s^2 + 2S + 1) = SX(S) - \cancel{X(s)}\,x(0)$$
$$= SX(s) - 1$$

as $x(0) \neq 0$ anymore !!

$$Y(s) = \frac{S\,X(s)}{(S+1)^2} - \frac{1}{(S+1)^2} = \frac{S}{(S+1)^2(S+2)} - \frac{(S+2)}{(S+1)^2(S+1)}$$

$$= \frac{-2}{(S+1)^2(S+2)} = \frac{-2}{(S+1)^2} + \frac{2}{(S+1)} + \frac{-2}{S+2}$$

$$y(t) = (-2te^{-t} + 2e^{-t} - 2e^{-2t})\,u(t)$$

Note: Almost all of the students didn't put the fact that $x(0) = 1$ in part ii, No marks were deducted for this mistake.

7. $x(t) \longrightarrow \boxed{S_1} \longrightarrow \boxed{S_2} \longrightarrow z(t)$

$x(t) = U(t) \longleftrightarrow X(s) = \frac{1}{s}$

$z(t) = [\cos(t) - \sin(t) - e^{-t}]U(t)$

S_1: $y(t) = \int_{-\infty}^{t} \sin(t-\sigma)x(\sigma)\,d\sigma$, $t > -\infty$ $x(t) \rightarrow \boxed{S_1} \rightarrow y(t)$

$= \int_{-\infty}^{\infty} \sin(t-\sigma)U(t-\sigma)x(\sigma)\,d\sigma$

$h_1(t) = \sin(t)U(t) \longleftrightarrow H_1(s) = \frac{1}{s^2+1}$

$Z(s) = \frac{s}{s^2+1} - \frac{1}{s^2+1} - \frac{1}{s+1} = \frac{-2}{(s+1)(s^2+1)}$

$Z(s) = H_1(s) \cdot H_2(s) \cdot X(s)$

$\frac{-2}{(s+1)(s^2+1)} = \frac{1}{s^2+1} \cdot \frac{1}{s} \cdot H_2(s)$

$H_2(s) = \frac{-2(s)(s^2+1)}{(s+1)(s^2+1)} = -\frac{2s}{s+1} = -2\left[1 - \frac{1}{s+1}\right]$

$h_2(t) = -2[\delta(t) - e^{-t}]U(t)$

IPOP for S_2: $y(t) = \int_{-\infty}^{\infty} -2[\delta(t-\sigma) - e^{-(t-\sigma)}]U(t-\sigma)x(\sigma)\,d\sigma$

$x(t) \rightarrow \boxed{S_2} \rightarrow y(t)$ $= \int_{-\infty}^{t} -2[\delta(t-\sigma) - e^{-(t-\sigma)}]x(\sigma)\,d\sigma$, $t > -\infty$

A Common Alternative

$h_{12}(t) = \frac{d}{dt} z(t)$

$= [-\sin(t) - \cos(t) + e^{-t}]U(t)$

$H_{12}(s) = -\frac{1}{s^2+1} - \frac{s}{s^2+1} + \frac{1}{s} = \frac{-2s}{(s+1)(s^2+1)}$

$H_2(s) = \frac{H_{12}(s)}{H_1(s)} = \frac{-2s}{(s+1)(s^2+1)} \cdot \frac{(s^2+1)}{1} = \frac{-2s}{s+1}$

8. $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau)\, d\tau - x(t)$

i) $y(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)} U(t-\tau) x(\tau)\, d\tau - \int_{-\infty}^{\infty} \delta(t-\tau) x(\tau)\, d\tau$

$= \int_{-\infty}^{\infty} [e^{-(t-\tau)} U(t-\tau) - \delta(t-\tau)] x(\tau)\, d\tau$

$h(t-\tau) = e^{-(t-\tau)} U(t-\tau) - \delta(t-\tau)$

$h(t) = e^{-t} U(t) - \delta(t)$

$H(s) = \frac{1}{s+1} - 1 = \frac{-s}{s+1}$

ii) $x(t) = \sin(t) U(t)$

$X(s) = \frac{1}{s^2+1}$

$Y(s) = H(s) X(s)$

$= \left[\frac{1}{s^2+1}\right]\left[\frac{-s}{s+1}\right]$

$\frac{-s}{(s+1)(s^2+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+1}$

$A = \frac{-s}{s^2+1}\Big|_{s=-1} = \frac{1}{2}$

$-s = \frac{1}{2}(s^2+1) + (Bs+C)(s+1) \Rightarrow C = -\frac{1}{2}$

$\hookrightarrow B - \frac{1}{2} = -1 \Rightarrow B = -\frac{1}{2}$

$Y(s) = \frac{1}{2}\left[\frac{1}{s+1} - \frac{s+1}{s^2+1}\right]$

$y(t) = \frac{1}{2}[e^{-t} - \cos(t) - \sin(t)] U(t)$

iii) $y(t) = \sin t\, U(t) \longleftrightarrow Y(s) = \frac{1}{s^2+1}$

$X(s) = \frac{Y(s)}{H(s)} = \frac{1}{s^2+1} \cdot \frac{s+1}{-s} = -\frac{1}{s} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$

$x(t) = [-1 + \cos(t) - \sin(t)] U(t)$