ECE102, Fall 2020 | **Midterm Exam**

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UCLA True Bruin academic integrity principles apply.
Open: Notes, Book, Calculator.
Closed: Internet, except to download files from CCLE and check Piazza.
2:00-3:50pm.
Monday, 9 Nov 2020.

State your assumptions and reasoning. No credit without reasoning. Show all work.
You may do this exam on these pages or on separate sheets of paper.
Upload your work to Gradescope by 4pm PST.

Do not send a copy of this exam to any classmates or discuss the exam on Piazza until after Wednesday, 11 Nov 2020. There are students who will take the exam at different times.

Name: ____________________

Signature: ____________________

ID#: ____________________

Problem 1	_____	/ 30
Problem 2	_____	/ 40
Problem 3	_____	/ 30
BONUS	_____	/ 6 bonus points
Total	_____	/ 100 points + 6 bonus points

1. **Signal and System Properties** (30 points).

   (a) (6 points) What are the phase and amplitude of the following number?

$$x = (1 + j)e^{3j} \tag{1}$$

   (b) (6 points) If $x(t)$ is an even function, and $x(t-1)$ is also even, is $x(t)$ periodic? Explain your reasoning.

(c) (6 points) Evaluate the expression

$$\int_0^{\infty} \delta(t-1)(t-2)^2 dt.$$

(d) (12 points) Consider the system with input $x(t)$ and output $y(t)$:

$$y(t) = \int_{-\infty}^{t} x(\lambda)u(t+1)d\lambda.$$

Determine if the system is:

i. Linear
ii. Time-invariant
iii. Causal
iv. Stable

You must justify your answer to receive full credit. (The next page is intentionally left blank as additional space to show work for this question.)

Additional space for question 1d.

2. **Impulse response, LTI systems and Convolution** (40 points).

Note: Parts (a) and (b) of this problem are independent. In part (a) of the problem, you might find the following convolution result useful:

$$e^{at}u(t) * e^{bt}u(t) = \frac{1}{a-b}e^{at}u(t) + \frac{1}{b-a}e^{bt}u(t)$$

(a) Consider the LTI system characterized by the Input/Output relationship:

$$\text{System 1 :} \qquad y(t) = \int_{-\infty}^{t} e^{-2(t-\tau)}x(\tau)d\tau$$

i. (5 points) Write the impulse response of the system, $h_1(t)$.

ii. (10 points) Determine a closed-form expression for the output ($y(t)$) of System 1 for the input

$$x(t) = 3e^{-4t}u(t-2)$$

iii. (10 points) Determine a closed-form expression for the output ($y(t)$) of System 1 for the input

$$x(t) = u(t) - u(t-3)$$

iv. (5 points) Determine the output of System 1, $y(t)$, for the input

$$x(t) = \delta(t) - \delta(t-3)$$

(b) If $y(t) = x(t) * h(t)$ is the output of an LTI system with input $x(t)$ and impulse response $h(t)$, then show the following properties. Note, you will not receive credit if you simply state that differentiation and convolution are LTI systems and can be interchanged. Although this is true, we want you to show your work with the convolution integral.

Hint: You may interchange the order of integration and differentiation.

i. (5 points) $\frac{d}{dt}y(t) = x(t) * (\frac{d}{dt}h(t))$

ii. (5 points) $\frac{d}{dt}y(t) = (\frac{d}{dt}x(t)) * h(t)$

3. **Fourier Series** (30 points).

(a) (15 points) Find the Fourier series of the function: $f(x) = \begin{cases} -1 & \text{if } -\pi \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq \pi \end{cases}$. To receive full credit, you must simplify your answer, including complex exponentials if possible. For this part of the problem, you may assume that $f(x)$ has a fundamental period of 2π.

(b) (15 points) Let $x(t)$ be a periodic signal whose Fourier series coefficients are:

$$c_k = \begin{cases} 2 & k = 0 \\ j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases}$$

Use Fourier series properties to answer the following questions:

i. Is $x(t)$ real?

ii. Is $x(t)$ even?

iii. Is $\frac{dx(t)}{dt}$ even?

Bonus Question (6 points)

Consider two signals, $x(t) = \text{rect}(t - 2.5)$ and $h(t) = \text{rect}(t)$. We convolve $x(2t)$ and $h(2t)$, i.e.,

$$y(t) = x(2t) * h(2t)$$

Using the flip and drag technique, compute the values of:

- $y(0.5)$
- $y(1)$

To receive full credit, you must draw a sketch of the flip and drag convolution computation you use to compute $y(0.5)$ and $y(1)$. You will not receive any partial credit for solving this analytically, although you may of course use an analytical answer to check your work. (This is because the purpose of this question is to test your understanding of signal operations and computing convolution with flip and drag.)