ECE 102 Final

TOTAL POINTS

177.5 / 215

QUESTION 1

Signal and System 40 pts

1.1 (a)i 6.5 / 8

- **- 0 pts** Correct with sufficient justification.
- **- 1 pts** F(jw)G(jw) is correct but conclusion is wrong. Forget to transfer it back to time domain.
- ✓ **- 1.5 pts Properties of F(jw) and G(jw) are correct. But F(jw)G(jw) is wrong.**
- **- 2 pts** Properties of F(jw) or G(jw) is incorrect.
- **- 2 pts** Proof by definition of convolution. Sign is incorrect in intermedia steps.
- **- 3 pts** correct conclusion but insufficient justification.
- **- 6 pts** incorrect justification.
- **- 8 pts** No justification

1.2 (a)ii 7 / 8

- **- 0 pts** Correct counterexample or sufficient justification
- ✓ **- 1 pts insufficient justification**
- **- 3 pts** The counterexample is not LTI or not a system.
- **- 3 pts** incorrect justification but show correct understanding on the concepts
- **- 4 pts** justification does not concern stability.
- **- 6 pts** no justification but correct conclusion.
- **- 8 pts** no justification

1.3 (b) 12 / 12

- ✓ **+ 12 pts Correct**
- **+ 9 pts** Correct answer, compute the correct H(jw), but fell short to provide an sufficient argument
- **+ 6 pts** Correct answer, correct Y(jw) and X(jw), but irrelevant argument
- **+ 4 pts** Correct answer but with incorrect justification
- **+ 4 pts** Incorrect answer with correct Y(jw) and X(jw)
- **+ 2 pts** Wrong answer, with irrelevant argument or correct answer with no argument
- **+ 0 pts** No answer

1.4 (c) 8 / 12

- **+ 12 pts** Correct answer and correct justification
- ✓ **+ 8 pts Get one property correctly; get the other correctly with incorrect justification, or incorrectly but state the correct property definition**
- **+ 6 pts** Get one property correctly; get the other incorrectly
- **+ 4 pts** For both properties, either correct answer with incorrect justification, or incorrect but state the correct property definition
- **+ 2 pts** Get one property correctly with incorrect justification, or incorrectly but state the correct property definition
- **+ 0 pts** Evaluate both properties Incorrectly

QUESTION 2

Frequency Response 45 pts

2.1 (a) 8 / 8

- ✓ **+ 8 pts Correct**
- **+ 5 pts** Identify one property correctly
- **+ 0 pts** Incorrect

2.2 (b) 4 / 8

- **+ 8 pts** Correct answer with sufficient justification
- ✓ **+ 4 pts Correct answer, make an argument but justify insufficiently or with an incorrect example**
- **+ 2 pts** Correct, with no argument
- **+ 2 pts** Incorrect but with some argument
- **+ 0 pts** Incorrect, no work

2.3 (c) **15 / 15**

✓ **+ 15 pts** Correct

+ 4 pts Correct H1

+ 4 pts Correct H2

+ 4 pts Corect H3

+ 1.5 pts Made an algebra error or did not simplify when computing Heq

+ 1.5 pts Modeledz Heq = H1 + H2*H3, and compute correctly

+ 2 pts Correct Heq in terms of H1, H2 and H3, if any subsystem is evaluated incorrect

2.4 (d) **8 / 14**

+ 14 pts Correct answer with computation of frequency response or eigenfunction

+ 11 pts Clearly express A and theta in terms of H(j \pi/3)

+ 8 pts Clearly state the implication of eigenfunction property or equivalent statement

✓ **+ 8 pts** **correct X(jw), and clear and explicit expression of W(jw) or w(t) in terms of H(jw) and X(jw), or equivalent in time domain**

+ 5 pts Clearly write W(jw) in terms of frequency response, and with computation steps

+ 3 pts Any partial credit, such as correct X(jw)

+ 13 pts Correct answer when using the frequency response H(jw) from part (c) (****Please request a regrade if you could confirm this applied to your answer)

+ 0 pts No answer

QUESTION 3

Sampling and Modulation 40 pts

3.1 (a) **5 / 5**

✓ **- 0 pts** Correct

- 0.5 pts minor mistakes

- 2.5 pts incorrect due to false understanding of Bandwidth or Nyquist rate or etc.

- 5 pts Not attempted

3.2 (b) **5 / 5**

✓ **- 0 pts** correct

- 0.5 pts minor mistake

- 2 pts Nyquist rate incorrect/not answered

- 3 pts Sketch incorrect

- 5 pts not attempted

3.3 (c) **8 / 10**

- 0 pts Correct

- 1.5 pts T is incorrect but the answer shows some correct intermediate steps.

- 3 pts T is incorrect.

- 1 pts Magnitude is incorrect but applied multiplication property of FT correctly.

✓ **- 2 pts** **magnitude is incorrect.**

- 2 pts The shape of frequency spectrum is wrong. But shows the correct calculation of convolution or correct understanding of sampling theorem in frequency domain.

- 4 pts shape of frequency spectrum is wrong.

- 0.5 pts minor mistakes

- 10 pts incorrect, no justification, no reasoning

3.4 (d) **17 / 20**

- 0 pts Correct

- 1 pts minor mistakes.

- 3 pts part of the design or specification is unclear. e.g. claim to shift the positive baseband without using any filter ahead, no math expression of the modulation signal and etc.

- 3 pts fail to recover magnitude from Xp(jw).

✓ **- 3 pts** **fail to recover the shape of X(jw). But the output looks very similar.**

- 6 pts fail to recover the spectrum shape.

- 6 pts use bandpass/highpass filter to recover the signal.

- 14 pts Explain in words but no flow diagram, missing many details like cutoff frequency and necessary LPF.

- 16 pts very limited design and detail information. Cannot recover signal

- 20 pts not attempted, irrelevant writing.

QUESTION 4

Laplace Transform 30 pts

4.1 (a) 9 / 10

+ **10 pts** Correct.

✓ + **9 pts** **Very minor algebra error.**

+ **8 pts** Found 1/H(s).

+ **7 pts** Factored numerator incorrectly.

+ **6 pts** Found 1/H(s) and factored incorrectly.

+ **6 pts** Missed an s or s^2 term in the derivative; or mixed up s and s^2.

+ **5 pts** Did not take Laplace Transform of y(t) or x(t) correctly.

+ **2 pts** Did not take Laplace Transform of both y(t) and x(t) correctly.

4.2 (b) 20 / 20

✓ - **0 pts** **Correct, given ans to part (a).**

Example ans:

Correct: 1/2*exp(t) + 9/2 * exp(-t) - 5*exp(-2t)
c/(s+2): c*exp(-t) - c*exp(-2t)
Solved it considering x(0) = 1: 3 * exp(t) + 9*exp(-t) - 6*exp(-2t)

Or other correct work from an incorrect starting point.

- **1 pts** Minor algebraic mistake

- **3 pts** Moderate algebraic mistake (e.g., one coefficient off due to algebra, or took the Laplace Xfm of the input incorrectly)

- **3 pts** Made a numerator factorization error or wrote 6s+3 or did not factor numerator

- **5 pts** In part (a), had 1/H(s), but did not make the fraction a constant + strictly proper term before doing partial fraction. (This leads to -3/4 * exp(t/2) + 2/3 * exp(-t))

- **5 pts** Claimed eigenfunction property, however, we never showed H(s) is LTI (it isn't), but otherwise did partial fractions correctly.

- **5 pts** Had a quadratic factor, but did not make the numerator r_1 * s + r_2; or had a repeated pole but did not have all partial fraction powers.

- **10 pts** Did partial fractions for an expression correct, but initial expression is incorrect (e.g., did not consider the input X(s) = 1/(s+1)) or did not follow logically as far as I could tell.

- **10 pts** Correct set up but did not do partial fractions (may have shown other work, like completion of squares).

QUESTION 5

Feedback System 45 pts

5.1 (a) 10 / 10

✓ - **0 pts** **Correct**

- **1 pts** Minor error

- **2 pts** Partially Correct

- **6 pts** Incorrect

- **10 pts** No answer

- **10 pts** See comment

5.2 (b) 10 / 10

✓ - **0 pts** **Correct**

- **3 pts** H(s) incorrect

- **2 pts** H(jw) incorrect

- **0.75 pts** Partially correct explanation for low-pass

- **2 pts** Incorrect explanation for low-pass characteristic

5.3 (c) 10 / 10

✓ - **0 pts** **Correct**

- **1 pts** Small mistake

- **2.5 pts** Partially correct

- **5 pts** Incorrect

- **10 pts** No answer

5.4 (d) 5 / 15

- **0 pts** Correct

- **2 pts** Partially correct Heq(s)

- **4 pts** incorrect F(s), Heq(s) unsimplified (no heq(t))

- **3 pts** Correct F(s), incorrect Heq(s)

- **0 pts** Correct Heq(s) using an incorrect F(s)

✓ **- 10 pts** **Incorrect**

- 1.5 pts Incorrect filter type for answered Heq(s)

- 1 pts incorrect heq(t) using answered Heq(s)

- 15 pts No answer or work for heq

QUESTION 6

Bonus 15 pts

6.1 (a) 8 / 8

✓ **- 0 pts** **Correct**

- 1 pts Almost fully correct

- 4 pts Partially correct

- 6 pts Incorrect

- 8 pts No answer or too little work

6.2 (b) 2 / 7

- 0 pts Correct

✓ **- 5 pts** **Incorrect**

- 7 pts No substantial answer

gradescope

ECE102, Fall 2019 **Final**
Department of Electrical and Computer Engineering Prof. J.C.Kao
University of California, Los Angeles TAs: W. Feng, J. Lee & S. Wu

UCLA True Bruin academic integrity principles apply.
Open: Four cheat sheets allowed.
Closed: Book, computer, internet.
8:00-11:00am.
Wednesday, 11 Dec 2019.

State your assumptions and reasoning.
No credit without reasoning.
Show all work on these pages.

Name: .

Signatu

ID#: _

Problem 1	____	/ 40
Problem 2	____	/ 45
Problem 3	____	/ 40
Problem 4	____	/ 30
Problem 5	____	/ 45
BONUS	____	/ 15 bonus points
Total	____	/ 200 points + 15 bonus points

1. **Signal and System Basics** (40 points)

   (a) (16 points) For each statement below, determine whether it is true or false. You must justify your answer to receive full credit.

   i. (8 points) If $f(t)$ is a real and even signal, and $g(t)$ is a real and odd signal, the convolution of $f(t)$ and $g(t)$ is real and odd.

$$r(t) = f(t) * g(t)$$

$$R(j\omega) = \underset{\uparrow \text{ real, even}}{F(j\omega)} \underset{\uparrow \text{ img; odd}}{G(j\omega)}$$

$\therefore$ $R(j\omega)$ = Imaginary and even

$\therefore$ $r(t) = f(t) * g(t)$ = Imaginary and even

False.

   ii. (8 points) All LTI systems are stable.

False The system can go to infinity

$$\int_{-\infty}^{\infty} 1\,dt = t \Big|_{-\infty}^{\infty} \Leftarrow \text{unstable}$$

(b) (12 Points) Suppose we have an unknown system (black box). We input

$$x(t) = \text{sinc}(t)$$

into the system, and measure that its output is

$$y(t) = e^{-t}u(t).$$

Can this system be LTI? You must justify your answer to receive full credit.

NO. sinc has frequencies between $-\pi$ and π, while $e^{-t}u(t)$ has frequencies beyond that. it expands in the Fourier domain which is a characteristic of a non LTI system.

(c) (12 Points) Determine whether the following system is (1) causal, and (2) stable.

$$y(t) = \int_{-\infty}^{t} (x(\tau) + e^{-\tau})u(\tau + 1)d\tau$$

Yes it is Causal. it does not rely on values of $x(\cdot t)$ for the future.

$$y(t) = \int_{-1}^{t} x(\tau) + e^{-\tau} d\tau$$

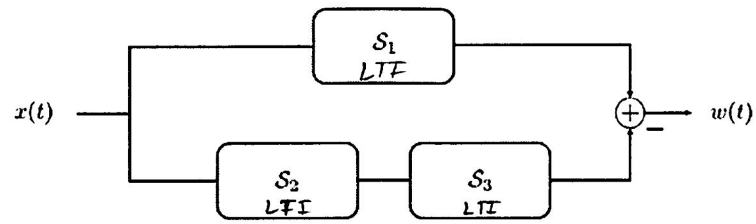
$$y(t) = \bar{X}(t) - \bar{X}(-1) - e^{-t} + e^{1}$$

Assume $|x(t)| \leq M_x$

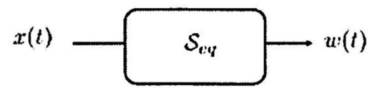
$$y(t) = \underbrace{\int_{-1}^{t} x(\tau)\, d\tau}_{\leq M} + \underbrace{\int_{-1}^{t} e^{-\tau} d\tau}_{\leq \text{some const (decays)}}$$

$y(t)$ is stable.

2. **Frequency Response and LTI system** (45 points)
Suppose the three systems are interconnected as shown below.



And we denote the equivalent system as below.



(a) (8 points) Suppose S_1, S_2 and S_3 are all LTI systems. Is the equivalent system S_{eq} an LTI system? Please justify your answer to receive full credit.

$$w(t) = S_1[x(t)] - S_3[S_2[x(t)]]$$

$$w(t) = h_1(t) * x(t) - h_3(t) * h_2(t) * x(t)$$

$\rightarrow$ let $x(t) = [a\,x(t) + b\,y(t)]$

$$w(t) = h_1(t) * (a\,x(t) + b\,y(t)) - (h_3(t) * h_2(t)) * (a\,x(t) + b\,y(t)))$$

$$w(t) = a\,h_1(t) * x(t) + b\,h_1(t) * y(t) - a\cdot(h_3(t) * h_2(t)) * x(t)$$
$$- b\,(h_3(t) * h_2(t)) * y(t)$$

$$w(t) = a\cdot h_1(t) * x(t) - a\cdot h_3(t) * h_2(t) * x(t)$$
$$+ b\,h_1(t) * y(t) - b\cdot h_3(t) * h_2(t) * y(t)$$
$$= a\Big[S_1[x(t)] - S_3[S_2[x(t)]]\Big] + b\Big[S_1[y(t)] - S_3[S_2[y(t)]]\Big]$$

True. Seq is LTI

(b) (8 points) Suppose the equivalent system $\mathcal{S}_{eq}$ is an LTI system. Are $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$ all necessarily LTI systems? Please justify your answer to receive full credit.

Not Necessarily

Imagine S_1 was $h_1(t) = t u(t)$, anything $\times u(t)$ is not LTI

a non-LTI system, we would still be able to hold LTI-ness in the h_{eq} system.

Nothing about the previous proof changes

(c) (15 points) Suppose $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$ are each characterized by an LTI system,

- The first system $\mathcal{S}_1$, with frequency response $H_1(j\omega)$, is given by its input-output relationship: $y(t) = x(t-3)$;
- The second system $\mathcal{S}_2$, with frequency response $H_2(j\omega)$, is given by its impulse response: $h_2(t) = u(t-3)$;
- The third system $\mathcal{S}_3$, with frequency response $H_3(j\omega)$, is given by its input-output relationship: $y(t) = \frac{d}{dt}x(t) + \frac{d^2}{dt^2}x(t)$.

Determine the frequency responses $H_1(j\omega)$, $H_2(j\omega)$ and $H_3(j\omega)$ of each system as well as $H_{eq}(j\omega)$ of the equivalent system.

H_3 $\quad y(t) = x''(t) + x'(t)$

$$Y(j\omega) = (j\omega)^2 X(j\omega) + (j\omega) X(j\omega)$$

$$H_3(j\omega) = (j\omega)^2 + (j\omega)$$

H_2 $\quad h_2(t) = u(t-3)$

$$H_2(j\omega) = e^{-3j\omega}\pi\delta(\omega) + e^{-3j\omega}\frac{1}{j\omega}$$

H_1 $\quad y(t) = x(t-3)$

$$Y(j\omega) = e^{-3j\omega}X(j\omega)$$

$$H_1(j\omega) = e^{-3j\omega}$$

$$W(j\omega) = e^{-3j\omega}X(j\omega) - [(j\omega)^2 + (j\omega)][e^{-3j\omega}\pi\delta(\omega) + e^{-3j\omega}\frac{1}{j\omega}]X(j\omega)$$

$$H_{eq}(j\omega) = e^{-3j\omega} - [(j\omega)^2 + (j\omega)]e^{-3j\omega}[\pi\delta(\omega) + \frac{1}{j\omega}]$$

$$= e^{-3j\omega}\left[1 - [(j\omega)^2 \cdot \pi\delta(\omega) + (j\omega) + (j\omega)\pi\delta(\omega) + 1]\right]$$

$$= e^{-3j\omega}\left[1 - [\cancel{\pi \cdot (0)}\,\delta(\omega) + (j\omega) + \cancel{0} + 1]\right]$$

$$= e^{-3j\omega}[-(j\omega)]$$

$$H_{eq}(j\omega) = -(j\omega)e^{-3j\omega}$$

(d) (14 points) For the system in part(c), the output $w(t)$ to an input $x(t) = \overline{e^{j\pi t/3}}$ can be written as:

Shift $\pi/3$

$$w(t) = Ae^{j\theta}x(t).$$

Determine A and θ.

$H(j\omega) = -(j\omega)\, e^{-3j\omega}$

$X(j\omega) = 2\pi\, \delta(\omega - \frac{\pi}{3})$

$W(j\omega) = -(j\omega)\, e^{-3j\omega} \cdot 2\pi\, \delta(\omega - \frac{\pi}{3})$

$W(j\omega) = \cancel{-}(\frac{j\pi}{3})\, \cancel{e^{-j\pi}} \cdot 2\pi \cdot \delta(\omega - \frac{\pi}{3})$

$W(j\omega) = \frac{2\pi^2}{3} j\, \delta(\omega - \frac{\pi}{3})$

$w(t) = \frac{2\pi^2}{3} j\, e^{j\frac{\pi}{3}t}$

$w(t) = \frac{2\pi^2}{3} j\, x(t)$

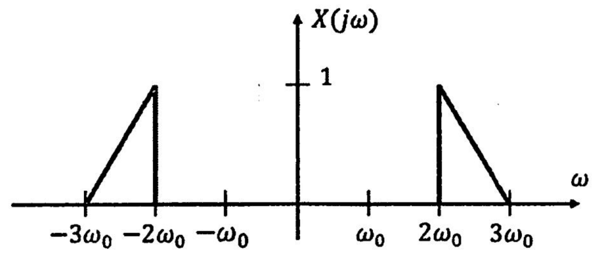
$w(t) = \frac{2\pi^2}{3} e^{j\frac{\pi}{2}}\, x(t)$

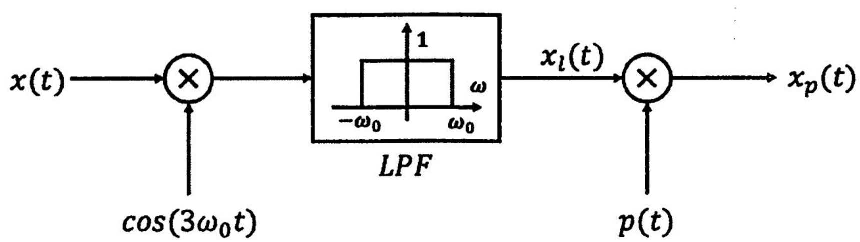
$e^{\theta i} = \cancel{\cos\theta}^{\,0} + i\cancel{\sin\theta}^{\,1} \qquad \theta = \frac{\pi}{2}$

$A = \frac{2\pi^2}{3}$

$\theta = \frac{\pi}{2}$

3. **Sampling and Modulation** (40 points)
Assume we have a continuous bandpass signal $x(t)$ with frequency spectrum as shown below. We also assume that $x(t)$ is real. The sampling theorem states that, to recover a signal without distortion, a signal must be sampled at a rate greater than twice its bandwidth. However, since $x(t)$ has most of its energy concentrated in a narrow band, it would seem reasonable to expect that a sampling rate lower than the Nyquist rate could be used. Now consider the system shown below where $p(t)$ is the sampling function.





(a) (5 points) What is the Nyquist rate of $x(t)$?

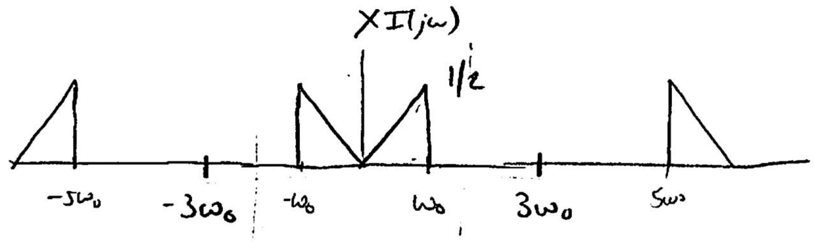
$2\pi B = 3\omega_0$

Nyquist rate $= 2B = \frac{3}{\pi}\omega_0 = \frac{6}{T}$

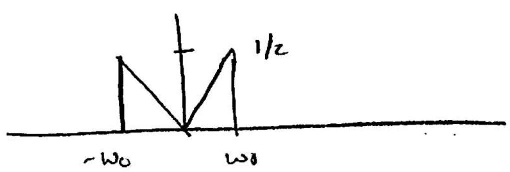
(b) (5 points) What is the Nyquist rate of $x_l(t)$? Sketch the frequency spectrum after the low pass filter, i.e. $X_l(j\omega)$.

$x_i(t) = x(t)\cos(3\omega t)$

$X_i(j\omega) = \frac{1}{2}\left[X(j(\omega - 3\omega_0)) + X(j(\omega + 3\omega_0))\right]$



$X_l(j\omega)$



Nyquist rate $2\pi B = \omega_0$

Nyquist $= \frac{\omega_0}{\pi}$

(c) (10 points) If the sampling function is an impulse train

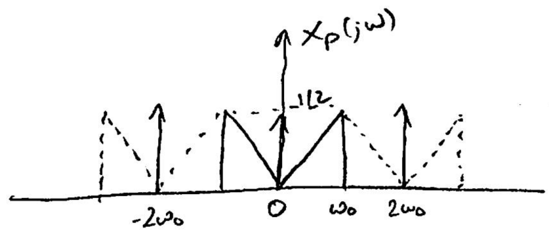
$$p(t) = \sum_{k=-\infty}^{k=+\infty} \delta(t - kT)$$

find the maximum sampling period T such that $x(t)$ is recoverable from $x_p(t)$. Sketch the output frequency spectrum $X_p(jw)$.

$$\omega = \frac{2\pi}{T}$$

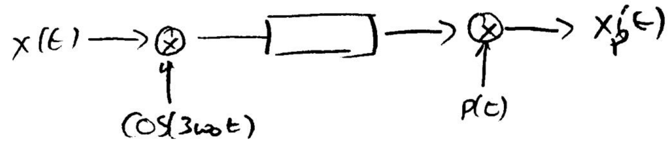
sampling at $2\omega_0$ is maximum ω.

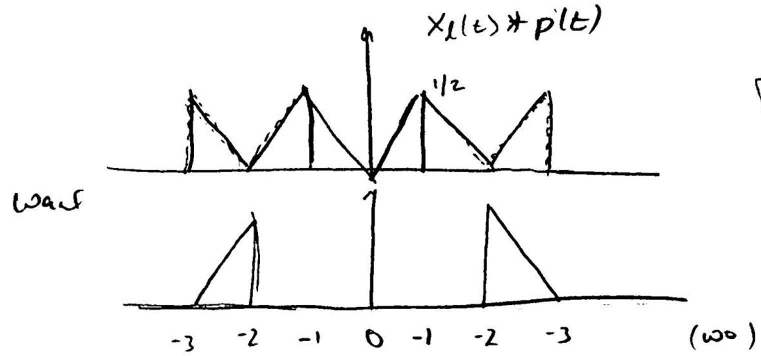
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\omega_0} = \frac{\pi}{\omega_0}$$

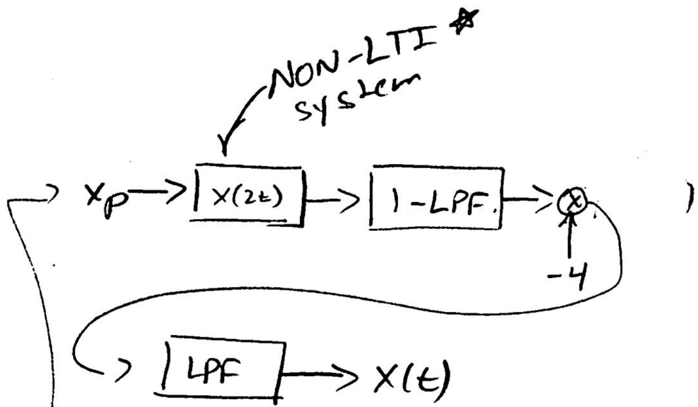


$$p(t) = \sum_{k=-\infty}^{\infty} \delta\left(t - k\frac{\pi}{\omega_0}\right)$$

(d) (20 points) With the $p(t)$ found in part (c), design a system to recover $x(t)$ from $x_p(t)$ without using a bandpass or highpass filter. Note that the recovered signal should have the same amplitude as $x(t)$ in frequency spectrum. Draw a flow diagram of your system and clearly state each component (including cutoff frequencies of any lowpass filter). Write out the explicit mathematical expression of any signal involved.







1) Double eveythiy

-> $x(at) = \frac{1}{a} X(j\frac{\omega}{a})$

for $a = 2$

-> Send through h(1-LPF) (cut $2\omega_0$) $x(at) = \frac{1}{a} X(j\frac{\omega}{a})$ $a = 2$

-> send through LPF (cut $3\omega_0$)

-> mul by -4 to fix amplitude $\text{rect}\left(\frac{\omega}{2\omega}\right)$

$$-4\left[\left(x_p(2t) * \left[1 - 2\omega_0 \,\text{sinc}\left(\frac{2\omega_0}{\pi}\right)\right]\right) * 3\omega_0 \,\text{sinc}\left(\frac{3\omega\omega_0}{2\pi}\right)\right]$$

Its not the most effecient, but it works

4. **Laplace Transform** (30 points)

A system can be described by the following differential equation:

$$y''(t) + y'(t) - 2y(t) = 6x'(t) - 3x(t)$$

where the initial conditions are all zero, i.e. $y''(0) = 0$, $y'(0) = 0$ and $y(0) = 0$.

(a) (10 points) Find the transfer function $H(s) = Y(s)/X(s)$. Assume $x(0) = 0$.

$$s^2 Y(s) + sY(s) - 2Y(s) = 6s X(s) - 3X(s)$$

$$Y(s)[s^2 + s - 2] = X(s)[s - 3]$$

$$\frac{Y}{X} = \frac{s-3}{s^2+s-2} = H(s)$$

(b) (20 points) If the input is

$$x(t) = e^{-t}u(t)$$

then find the output $y(t)$.

$X(s) = \frac{1}{s+1}$ ROC > -1

$Y(s) = \frac{1}{s+1} \cdot \frac{s-3}{s^2+s-2}$

$= \frac{s-3}{(s+1)(s-1)(x+2)}$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{-1 \pm \sqrt{1-4(1)(-2)}}{2}$

$\frac{-1 \pm 3}{2} = -2$ or 1

$\frac{s-3}{(s+1)(s-1)(s+2)} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s+2}$

$A(s-1)(s+2) + B(s+1)(s+2) + C(s+1)(s-1) = s-3$

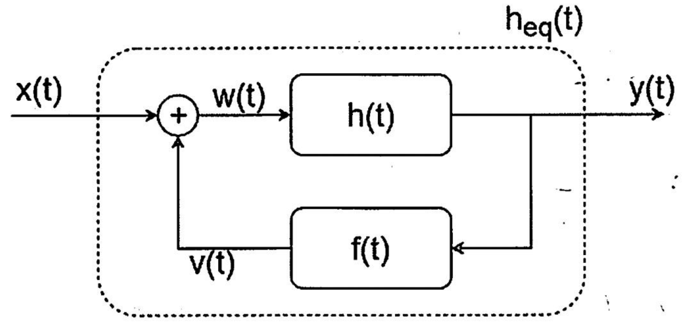
$s=-1$ $A(-2)(1) = -4$ $A = 2$

$s=1$ $B(2)(3) = -2$ $B = \frac{-1}{3}$

$s=-2$ $C(-1)(-3) = -5$ $C = \frac{-5}{3}$

$y(t) = 2e^{-t}u(t) - \frac{1}{3}e^{t}u(t) - \frac{5}{3}e^{-2t}u(t)$

5. **Feedback System** (45 points)
Consider the feedback system shown below (all components are LTI):



where $h(t) = e^{-2t}u(t)$ and $y(0) = 0$.

(a) (10 points) Show that

$$H_{eq}(s) = \frac{H(s)}{1 - H(s)F(s)}$$

$$Y(s) = [Y(s)\cdot F(s) + X(s)]\,H(s)$$

$$Y(s) = Y(s)F(s)H(s) + X(s)H(s)$$

$$Y(s)[1 - F(s)H(s)] = X(s)H(s)$$

$$\frac{Y(s)}{X(s)} = H_{eq}(s) = \frac{H(s)}{1 - F(s)H(s)}$$ ✓

(b) (10 points) Find the Laplace Transform $H(s)$ of $h(t)$. What is the frequency response $H(j\omega)$? Why is this a low-pass filter?

$h(t) = e^{-2t}u(t)$

$H(s) = \frac{1}{s+2}$ ROC: $Re\{s\} > -2$

includs $j\omega$

$H(j\omega) = \frac{1}{j\omega + 2}$

High frequences make $H(j\omega) \to 0$

Its not an ideal LPF

Low frequencys are close to $\frac{1}{2}$

High frequencys tend to 0

(c) (10 points) $v(t)$ and $y(t)$ satisfy the differential equation

$$v(t) = \frac{d}{dt}y(t) + y(t) - 10\int_0^t y(\tau)d\tau$$

What is $F(s)$?

$$V(s) = sY(s) + Y(s) - 10\,\frac{Y(s)}{s}$$

$$\frac{V(s)}{Y(s)} = F(s) = \left[s+1 - \frac{10}{s}\right]$$

(d) (15 points) Using $F(s)$ found in part c, what is $h_{eq}(t)$? Is this a low-pass, band-pass, or high-pass filter?

$$H_{eq}(s) = \frac{H(s)}{1 + H(s)F(s)}$$

$$\frac{\frac{1}{s+2}}{1 + \frac{1}{s+2}\cdot\left[s+1 - \frac{10}{s}\right]}$$

$$\frac{1}{s+2+\left[s+1-\frac{10}{s}\right]}$$

$$\frac{1}{2s+3-\frac{10}{s}}$$

$$H_{eq}(s) = \frac{s}{2s^2+3s-10}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{-3 \pm \sqrt{9-4(2)(-10)}}{4}$$

$$\frac{-3 \pm \sqrt{9+80}}{4}$$

$$\frac{-3 \pm \sqrt{89}}{4}$$

$$2\left(s^2+\tfrac{3}{2}s-5\right)$$

$$2\left(s^2+\tfrac{3}{2}s+\tfrac{9}{16}-\tfrac{9}{16}-5\right)$$

$$2\left(\left(s+\tfrac{3}{4}\right)^2-\tfrac{89}{16}\right)$$

$$\frac{3}{16} \quad \frac{5}{0}$$

$$5+\frac{9}{16}$$

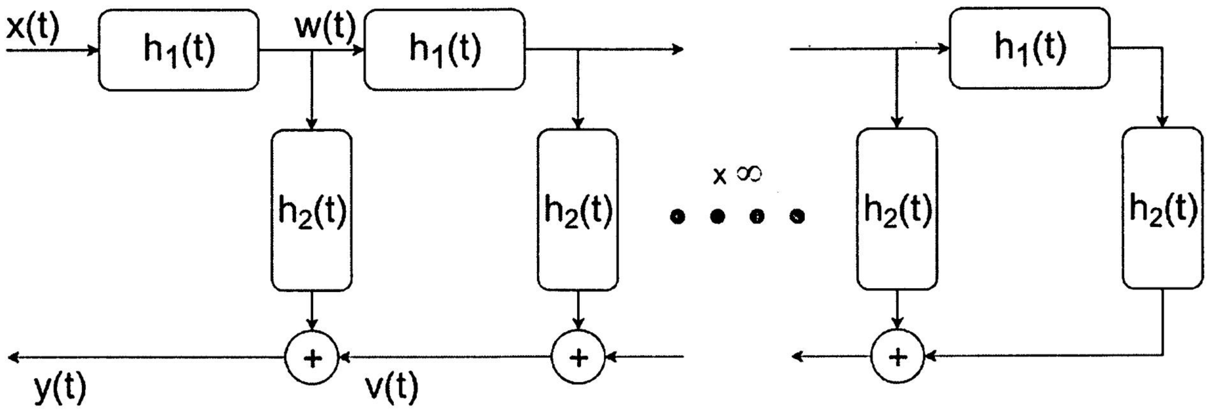
$$\frac{89}{16}$$

$$H_{eq}(s) = \frac{1}{2}\left[\frac{s+\frac{3}{4}}{\left(s+\frac{3}{4}\right)^2-\frac{89}{16}} - \frac{3}{4}\cdot\sqrt{\frac{16}{89}}\,\frac{\sqrt{\frac{89}{16}}}{\left(s+\frac{3}{4}\right)^2-\frac{89}{16}}\right]$$

$$h_{eq}(t) = \frac{1}{2}e^{-3/4t}\cos\left(\frac{\sqrt{89}}{4}t\right)u(t) - \frac{1}{2}\sqrt{\frac{9}{89}}\,e^{-3/4t}\sin\left(\frac{\sqrt{89}}{4}t\right)u(t)$$

$$h_{eq}(t) = \frac{1}{2}e^{-3/4t}\left(\cos\left(\frac{\sqrt{89}}{4}t\right) - \frac{3}{\sqrt{89}}\sin\left(\frac{\sqrt{89}}{4}t\right)\right)u(t)$$

Bonus (15 points) Consider the LTI system S shown below, which is a system ladder with an infinite number of rungs. Let $y(t) = S[x(t)]$.



(a) (8 points) In terms of $H_1(s)$ and $H_2(s)$, what is the equivalent transfer function $H_{eq}(s)$ between $Y(s)$ and $X(s)$? *Hint: how does* $\frac{V(s)}{W(s)}$ *relate to* $\frac{Y(s)}{X(s)}$*?*

$$y(t) = H_1 X(t) H_2 + H_1 \cdot H_1 \cdot X(t) \cdot H_2 + H_1 \cdot H_1$$

$$Y(s) = \sum_{n=1}^{n=\infty} (H_1(s))^n X(s) H_2(s)$$

$$\frac{Y(s)}{X(s)} = \sum_{n=1}^{n=\infty} (H_1(s))^n H_2(s) = H_{eq}(s)$$

(b) (7 points) Suppose $h_1(t) = e^{-a_1 t}u(t)$ and $h_2(t) = e^{-a_2 t}u(t)$, where a_1 and a_2 are real and positive. For what values of a_1 is S BIBO stable?

$$H_{eq} = \sum_{1}^{\infty} (H_1(s))^n H_2(s)$$

BIBO stable if

$$H_1(s) = \frac{1}{s+a_1}$$

$$H_2(s) = \frac{1}{s+a_2}$$

$$H_{eq} = \sum_{1}^{\infty} \left(\frac{1}{s+a_1}\right)^n \frac{1}{s+a_2}$$

$a_1 < |s|$

$$\frac{1}{s+a_1} \leq M$$

$$1 \leq Ms + Ma_1$$

$$\frac{1-Ms}{m} \leq a_1$$

$$\boxed{a_1 = -s + n}$$

$s + a_1$ needs to be as far away from zero as possible.