Question 1

(i) (10 pt) The IPOP relation of a L system S is:

$$y(t) = \int_{-\infty}^{\infty} t(t-\tau)u(t-\tau)x(\tau)d\tau, \ t \in (-\infty, \infty),$$

where

$$x(t) \rightarrow [S] \rightarrow y(t).$$

Write down the IRF $h(t,\tau)$ of S. Then compute its output $y(t)$ given that its input $x(t)$ is

$$x(t) = u(t)u(3-t), \ t \in (-\infty, \infty).$$

(ii) (5 pt) S is: TV? TI? C? NC?

Question 2

Compute

$$y(t) = \int_{-\infty}^{\infty} h(t-\sigma)u(t-\sigma)x(\sigma)d\sigma,$$

for

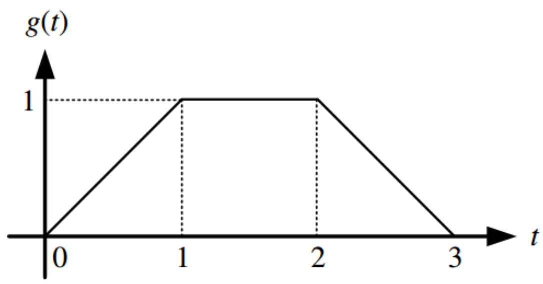
(i) (5 pt) $h(t) = t^2 u(t)$, $x(t) = e^{-t}u(t-5)$.

Question 3

(i) (2pts) Find the even and odd parts of this function

$$g(t) = t(2 - t^2)(1 + 4t^2)$$

(ii) Given the signal:



Graph these signals:

(a) (2pts) $g(2t + 3)$

(b) (2pts) $g(-2t + 3)$

(c) (2pts) $g\left(\frac{t}{2} + 1\right)$

Question 4

The IPOP of system S is:

$$y(t) = \int_{-\infty}^{t} e^{-\tau} x(t-\tau) d\tau, \quad t \in (-\infty, \infty)$$

(i) (3pts) Find IRF $h(t, \tau)$
(ii) (3pts) State properties of S: TV/TI? C/NC?
(iii) (4pts) Find output due to $\delta(t-2) + u(t-3)$.

Question 5

Let system S_1 be described by the IPOP relation:

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau) d\tau, \; t \in (-\infty, \infty)$$

$$x(t) \rightarrow [S_1] \rightarrow y(t)$$

and let system S_2 be described by the IPOP relation:

$$v(t) \rightarrow [S_2] \rightarrow w(t)$$

$$w(t) = v(t) - \int_{-\infty}^{\infty} u(t-\tau) v(\tau) d\tau, \; t \in (-\infty, \infty)$$

Compute the IRF $h_{21}(t)$ of the cascaded system $S_{21} := S_2 S_1$.

Question 1

Q.1

(i) IPOP: $y(t) = \int_{-\infty}^{\infty} t(t-\tau)\, u(t-\tau)\, x(\tau)\, d\tau, \quad t \in (-\infty, \infty)$

IRF $h(t,\tau) = t(t-\tau)\, u(t-\tau)$

using $x(\tau) = u(\tau)\, u(3-\tau)$

$$y(t) = \int_{-\infty}^{\infty} t(t-\tau)\, u(t-\tau)\, u(\tau)\, u(3-\tau)\, d\tau$$

$y(t) = 0$ for $t < 0$

$$y(t) = \int_{0}^{t} t(t-\tau)\, d\tau, \quad 0 \leq t < 3$$

$$= t \cdot \left[t^2 - \frac{t^2}{2}\right]$$

$$= \frac{t^3}{2}.$$

$$y(t) = \int_{0}^{3} t(t-\tau)\, d\tau, \quad t \geq 3$$

$$= t\left[t \cdot \tau - \frac{\tau^2}{2}\right]_0^3 = t\left[3t - \frac{9}{2}\right]$$

$$\therefore\ y(t) = \begin{cases} 0 & , \quad t < 0 \\ \dfrac{t^3}{2} & , \quad 0 \le t < 3 \\ t\left[3t - \dfrac{9}{2}\right] & ; \quad t \geq 3 \end{cases}$$

(ii) From IRF : $h(t,\tau) = t\,(t-\tau)u(t-\tau)$

S is TV and Causal.

Question 2

Q.2

$$y(t) = \int_{-\infty}^{\infty} h(t-\sigma)\, u(t-\sigma)\, x(\sigma)\, d\sigma$$

~~xt~~ substitute $h(t) = t^2 u(t)$, $x(t) = e^{-t} u(t-5)$

$$y(t) = \int_{-\infty}^{\infty} (t-\sigma)^2 u(t-\sigma) u(t-\sigma)\, e^{-\sigma} u(\sigma-5)\, d\sigma$$

$$= \begin{cases} \int_5^t (t-\sigma)^2 e^{-\sigma}\, d\sigma & , t \geq 5 . \\ 0 & \text{o.w.} \end{cases}$$

(2)

in $\int_5^t (t-5)^2 e^{-\sigma} d\sigma$ use $dv = e^{-\sigma}$, $u = (t-5)^2$

using ~~[crossed out]~~ $\int u\,dv = uv - \int v\,du$

$$\int_5^t (t-5)^2 e^{-\sigma} d\sigma = \frac{(t-\sigma)^2 e^{-\sigma}}{-1}\Bigg|_5^t - 2\left[\frac{(t-\sigma)e^{-\sigma}}{-1}\Bigg|_5^t - \int_5^t e^{-\sigma} d\sigma\right]$$

$$= (t-5)^2 e^{-5} - 2\left[(t-5)e^{-5} + e^{-t} - e^{-5}\right]$$

$$\Rightarrow \boxed{y(t) = e^{-5}\left[(t-5)^2 - 2(t-5) - 2e^{-(t-5)} + 2\right] u(t-5)}$$

Question 3

Q.3

(i) $g(t) = t(2-t^2)(1+4t^2)$

Even part $g_e(t) = \frac{1}{2}[g(t) + g(-t)]$

$= \frac{1}{2}[t(2-t^2)(1+4t^2) - t(2-t^2)(1+4t^2)]$

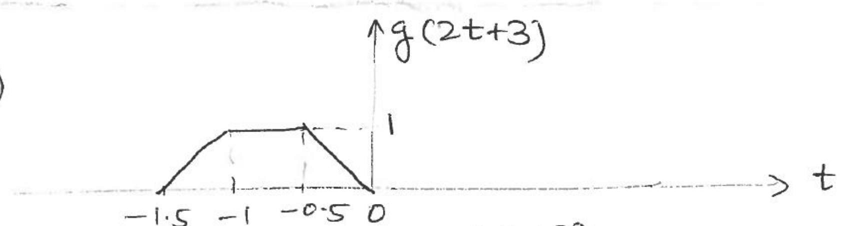
$= 0$

Odd part: $g_o(t) = \frac{1}{2}[g(t) - g(-t)]$

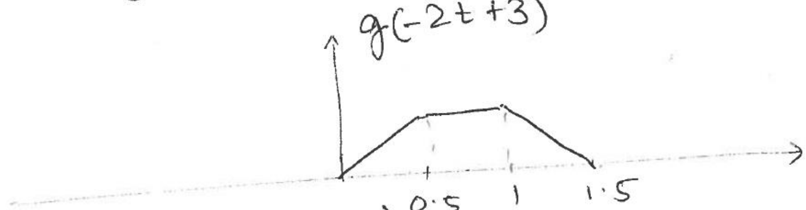
$= \frac{1}{2}[t(2-t^2)(1+4t^2) - (-t)(2-t^2)(1+4t^2)]$

$= t(2-t^2)(1+4t^2)$

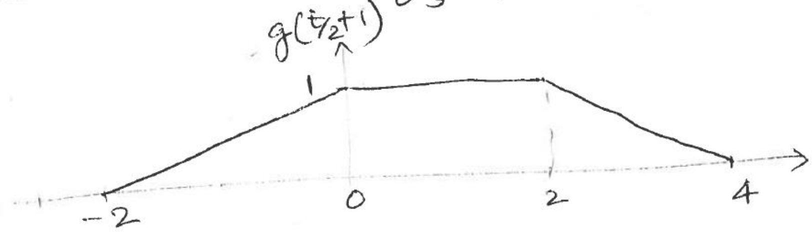
(ii) (a)



(b)



(c)



Question 4

(i) Rewrite IPOP output as

$$y(t) = \int_{-\infty}^{t} e^{-\sigma} x(t-\sigma) d\sigma$$

change the variable of integration to σ in order to reserve τ for impulse $\delta(t-\tau)$.

Using $x(t) = \delta(t-\tau)$ as input, the output is IRF.

$$\begin{aligned} h(t,\tau) &= \int_{-\infty}^{t} e^{-\sigma} \delta(t-\sigma-\tau) d\sigma \\ &= \int_{-\infty}^{+\infty} e^{-\sigma} \delta(t-\sigma-\tau) u(t-\sigma) d\sigma \\ &= e^{\tau-t} u(\tau) \end{aligned}$$

(ii) System is TV since IRF is not function of $t-\tau$.

System is NC since IRF can be nonzero when $t < \tau$.

(iii) Using the given input signal

$$\begin{aligned} y(t) &= \int_{-\infty}^{t} e^{-\tau} \delta(t-2-\tau) d\tau + \int_{-\infty}^{t} e^{-\tau} u(t-3-\tau) d\tau \\ &= e^{2-t} + \int_{-\infty}^{t-3} e^{-\tau} d\tau \\ &= \infty \end{aligned}$$

Note: the second integral is not a finite value.

Question 5

Using $v(t) = \delta(t)$, the output of the second system is

$$\begin{aligned} w(t) =& \delta(t) - \int_{-\infty}^{+\infty} u(t-\tau)\delta(\tau)d\tau \\ =& \delta(t) - u(t). \end{aligned}$$

Use such $w(t)$ as input of the first system, namely $x(t) = \delta(t) - u(t)$, the output is

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}\delta(\tau)d\tau - \int_{-\infty}^{t} e^{-(t-\tau)}u(\tau)d\tau$$

The first integral is $e^{-t}u(t)$, while the second is

$$\int_{-\infty}^{t} e^{-(t-\tau)}u(\tau)d\tau = \begin{cases} \int_0^t e^{-(t-\tau)}d\tau, & \text{if } t > 0 \\ 0, & \text{if } t < 0 \end{cases} = \left(1 - e^{-t}\right) u(t)$$

Therefore $h_{21}(t) = \left(2e^{-t} - 1\right) u(t)$.