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SPRING 2012 Put first letter of LAST name in the corner →→↗↗

UCLA

DEPARTMENT OF ELECTRICAL ENGINEERING

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EE 102: SYSTEMS & SIGNALS

May 8, 2012
MIDTERM EXAMINATION

Instructions:

(i) Closed Book, Calculators Are NOT Allowed
(ii) Please Write On ONE Side Only
(iii) Staple These Examination Papers With Your Papers

Problem 1 (15 pts)	15
Problem 2 (15 pts)	15
Problem 3 (15 pts)	10
Problem 4 (15 pts)	15
Problem 5 (10 pts)	10
Problem 6 (15 pts)	15
Problem 7 (15 pts)	15
Total (100 pts)	

95

Part I: Time-Domain Analysis

<u>(Do NOT Use Laplace Transforms here)</u>

1. (15 pts)
(Do NOT use Laplace Transforms here)
(i) The IPOP relation of a L system S is:

$$y(t) = \int_{-\infty}^{\infty} t(t-\tau)U(t-\tau)x(\tau)d\tau, \quad t \in (-\infty, \infty)$$

where

$$x(t) \rightarrow [S] \rightarrow y(t)$$

Write down the IRF $h(t,\tau)$ of S. Then compute its output $y(t)$ given that its input $x(t)$ is

$$x(t) = U(t)U(3-t), \quad t \in (-\infty, \infty)$$

(ii) S is: TV? TI? C? NC?

2. (15 pts)
(i) Let system S_1 be described by the IPOP relation:

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau)d\tau, \quad t \in (-\infty, \infty)$$

$$x(t) \rightarrow [S_1] \rightarrow y(t)$$

and let system S_2 be described by the IPOP relation:

$$v(t) \rightarrow [S_2] \rightarrow w(t)$$

$$w(t) = v(t) - \int_{-\infty}^{\infty} U(t-\tau)v(\tau)d\tau, \quad t \in (-\infty, \infty)$$

Compute the IRF $h_{21}(t)$ of the cascaded system $S_{21} := S_2 S_1$.

(ii) S_1 is TV? TI? C? NC?
S_2 is TV? TI? C? NC?

1.i) linear

$y(t) = \int_{-\infty}^{\infty} t(t-\tau)u(t-\tau)\, x(\tau)\, d\tau \quad t \in (-\infty, \infty)$

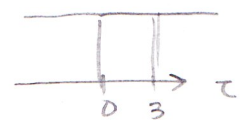
$h(t,\tau) = ?$

convolution integral: $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t,\tau)\, d\tau$

$\therefore \boxed{h(t,\tau) = t(t-\tau)u(t-\tau)}$ and $x(\tau) = x(\tau)$ ✓

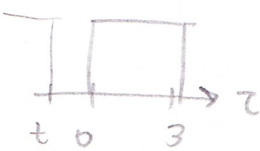
$x(t) = u(t)u(3-t) \quad t \in (-\infty, \infty)$

$y(t) = \int_{-\infty}^{\infty} t(t-\tau)u(t-\tau)\underline{u(\tau)u(3-\tau)}\, d\tau$



$= \int_0^3 t(t-\tau)u(t-\tau)\, d\tau$

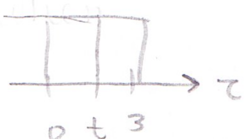
when $t<0$, $y(t) = 0$ ✓



when $0 \leq t < 3$, $y(t) = \int_0^t t(t-\tau)\, d\tau$

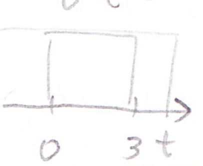
$= t\left[t\tau - \frac{1}{2}\tau^2\right]\Big|_0^t = t\left[t^2 - \frac{1}{2}t^2\right] - t[0]$

$= t\left[\frac{1}{2}t^2\right] = \frac{1}{2}t^3$ ✓



when $t \geq 3$, $y(t) = \int_0^3 t(t-\tau)\, d\tau$

$= t\left[t\tau - \frac{1}{2}\tau^2\right]\Big|_0^3 = t\left[3t - \frac{9}{2}\right] - t[0]$ ✓



$= t\left(3t - \frac{9}{2}\right)$

$\boxed{y(t) = \frac{1}{2}t^3 u(t)u(3-t) + t\left(3t - \frac{9}{2}\right)u(t-3), \quad t \in (-\infty, \infty)}$

1.ii) time variance

$S[x(t-\sigma)] = \int_{-\infty}^{\infty} t(t-\tau)u(t-\tau)\, x(\tau-\sigma)\, d\tau$

$y(t-\sigma) = \int_{-\infty}^{\infty} (t-\sigma)(t-\sigma-\tau)u(t-\sigma-\tau)\, x(\tau)\, d\tau$ ✓

$= \int_{-\infty}^{\infty} t(t-\lambda-\sigma)u(t-\lambda-\sigma)\, x(\lambda)\, d\lambda$

let $\lambda = \tau - \sigma$, $\tau = \lambda + \sigma$

$d\lambda = d\tau$

$\tau = -\infty \rightarrow \lambda = -\infty$

$\tau = \infty \rightarrow \lambda = \infty$

$\boxed{\text{time variant because } y(t-\sigma) \neq S[x(t-\sigma)]}$ ✓

causality: $y(t) = \int_{-\infty}^{\infty} t(t-\tau)u(t-\tau)\, x(\tau)\, d\tau = \int_{-\infty}^{t} t(t-\tau)\, x(\tau)\, d\tau$

causal, because output only depends on values of input at past and present times (no future inputs needed)

2i) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau)\, d\tau \quad , \; t \in (-\infty, \infty)$ $\qquad h_{21}(t) = ?$

$x(t) \longrightarrow \boxed{S_1} \longrightarrow y(t)$

$w(t) = v(t) - \int_{-\infty}^{\infty} u(t-\tau) v(\tau)\, d\tau, \; t \in (-\infty, \infty)$

$v(t) \longrightarrow \boxed{S_2} \longrightarrow w(t) \qquad (y(t) = v(t))$

convolution: $y(t) = \int_{\infty}^{\infty} h(t,\tau)\, x(\tau)\, d\tau$

$h_1(t,\tau) = e^{-(t-\tau)} u(t-\tau)$

$w(t) = v(t) - \int_{-\infty}^{\infty} u(t-\tau) v(\tau)\, d\tau$

decomposition

$= \int_{-\infty}^{\infty} v(\tau) \delta(t-\tau)\, d\tau - \int_{-\infty}^{\infty} u(t-\tau) v(\tau)\, d\tau$

$= \int_{-\infty}^{\infty} v(\tau) [\delta(t-\tau) - u(t-\tau)]\, d\tau$

$h_2(t,\tau) = \delta(t-\tau) - u(t-\tau)$

$h_{21}(t,\tau) = \int_{-\infty}^{\infty} h_2(\sigma,\tau) h_1(t,\sigma)\, d\sigma$

$= \int_{-\infty}^{\infty} [\delta(\sigma-\tau) - u(\sigma-\tau)][e^{-(t-\sigma)} u(t-\sigma)]\, d\sigma$

$= \int_{-\infty}^{\infty} \delta(\sigma-\tau) e^{-(t-\sigma)} u(t-\sigma)\, d\sigma - \int_{-\infty}^{\infty} u(\sigma-\tau) e^{-(t-\sigma)} u(t-\sigma)\, d\sigma$

$= \int_{-\infty}^{t} \delta(\sigma-\tau) e^{-(t-\sigma)}\, d\sigma - \int_{\tau}^{t} e^{-(t-\sigma)}\, d\sigma$

when $\tau > t$, $= 0$

when $\tau \le t$, $= e^{-(t-\tau)} - [e^{-(t-\sigma)}]\Big|_{\tau}^{t} = e^{-(t-\tau)} - [e^{0} - e^{-(t-\tau)}] =$

$2e^{-(t-\tau)} - 1$

$h_{21}(t,\tau) = (2e^{-(t-\tau)} - 1) u(t-\tau)$

$\boxed{h_{21}(t) = (2e^{-t} - 1) u(t)}$

ii) <u>time variance</u>

$S_1: h_1(t,\tau) = e^{-(t-\tau)} = h_1(t-\tau)$ TI

$S_2: h_2(t,\tau) = \delta(t-\tau) - u(t-\tau) = h_2(t-\tau)$ TI

<u>causality</u>

$S_1: h_1(t,\tau) = e^{-(t-\tau)} = h_1(t,\tau) u(t)$ C

$S_2: h_2(t,\tau) = \delta(t-\tau) - u(t-\tau) = [\delta(t-\tau) - 1] u(t-\tau)$ C

$\boxed{\text{Both } S_1 \text{ and } S_2 \text{ are TI and C.}}$

check

$H_1(s) = \frac{1}{s+1}$ $\quad H_2(s) = 1 - \frac{1}{s} = \frac{s-1}{s}$

$H_{12}(s) = \frac{s-1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$ $\quad A = -1$, $B = 2$

$h_{12}(t) = -u(t) + 2e^{-t} u(t)$

Part II: s-Domain Analysis

(Use Laplace Transforms here)

3. (15 pts)

Given the following information regarding a system S:

$$x(t) = e^{-t}tU(t) \rightarrow [\boldsymbol{S}: \mathbf{L}, \mathbf{TI}, \mathbf{C}] \rightarrow e^{-t}t(1-t)U(t) = y(t)$$

Your problem is:

(i) Find the IRF $h(t)$ of S,

and,

(ii) Find an IP $x(t)$ to S so that:

$$x(t)(\text{To Be Found}) \rightarrow [\boldsymbol{S}: \mathbf{L}, \mathbf{TI}, \mathbf{C}] \rightarrow \cos t\, U(t)$$

4. (15 pts)

Compute

$$L_s\{\int_0^t \cos(t-\tau)\, e^{-(t+\tau)} \sin(\tau)\, d\tau\}$$

5. (10 points)

Express the signal $f(t) = (\cos(t) + \sin(t) - 1)U(t)$ as a convolution integral.

3.i) $x(t) = e^{-t}tu(t) \rightarrow X(s) = \frac{1}{(s+1)^2}$

$y(t) = e^{-t}t(1-t)u(t) = e^{-t}tu(t) - e^{-t}t^2u(t) \rightarrow Y(s) = \frac{1}{(s+1)^2} - \frac{d}{ds}\left[\frac{1}{(s+1)^2}\right]$

$Y(s) = \frac{1}{(s+1)^2} - \left[-2\frac{1}{(s+1)^3}\right] = \frac{1}{(s+1)^2} + \frac{2}{(s+1)^3} = \frac{s+1+2}{(s+1)^3} = \frac{s+3}{(s+1)^3}$

due to LTI, C, $Y(s) = X(s)H(s)$

$H(s) = \frac{s+3}{(s+1)^3} \cdot (s+1)^2 = \frac{s+3+1-1}{s+1} = 1 + \frac{2}{s+1}$

$\boxed{h(t) = \delta(t) + 2e^{-t}u(t)}$ X -1

3ii) $x(t) = ?$

$y(t) = \cos t \cdot u(t) \rightarrow Y(s) = \frac{s}{s^2+1}$

$H(s) = \frac{s+3}{s+1}$

$X(s) = \frac{Y(s)}{H(s)} = \frac{s}{s^2+1}\left(\frac{s+1}{s+3}\right) = \frac{s(s+1)}{(s^2+1)(s+3)} = \frac{As+B}{s^2+1} + \frac{C}{s+3}$

$s(s+1) = (As+B)(s+3) + C(s^2+1)$

let $s=-3 \rightarrow -3(-2) = C(10) \rightarrow C = 6/10 = 3/5$

$s^2+s = \cancel{As^2} + 3As + Bs + 3B + \cancel{\frac{3}{5}s^2} + \frac{3}{5}$

$A + \frac{3}{5} = 1 \rightarrow A = \frac{2}{5}$, $3B + \frac{3}{5} = 0 \rightarrow B = -\frac{1}{5}$, $3A + B = 1 \rightarrow \frac{6}{5} - \frac{1}{5} = 1$ ✓

cont'd

3ii) $X(s) = \dfrac{\frac{2}{5}s - \frac{1}{5}}{s^2+1} + \dfrac{\frac{3}{5}}{s+3} = \frac{1}{5}\left[\dfrac{2s-1}{s^2+1} + \dfrac{3}{s+3}\right]$

$= \frac{1}{5}\left[2\dfrac{s}{s^2+1} - \dfrac{1}{s^2+1} + \dfrac{3}{s+3}\right]$

$$\boxed{x(t) = \frac{1}{5}\left[2\cos t\cancel{u(t)} - \sin t\cancel{u}(t) + 3e^{-3t}u(t)\right]}$$

-4

4. $\mathcal{L}_s\left\{\int_0^t \cos(t-\tau)e^{-(t+\tau)}\sin(\tau)d\tau\right\} = \mathcal{L}_s\left[e^{-t}\underbrace{\int_0^t \cos(t-\tau)e^{-\tau}\sin\tau d\tau}_{y(t)\,\rightarrow}\right]$

$\int_0^t \cos(t-\tau)e^{-\tau}\sin\tau d\tau = \int_{-\infty}^{\infty} \underbrace{\cos(t-\tau)}_{h(t-\tau)}\underbrace{e^{-\tau}\sin\tau\, u(\tau)}_{x(\tau)}\underbrace{u(t-\tau)}_{h(t-\tau)}d\tau$

$e^{-t-\tau} = e^{-t}e^{-\tau}$

system is LTI, C

by inspection of $h(t-\tau)$

$\therefore \int_0^t \cos(t-\tau)\sin\tau e^{-\tau}d\tau = y(t)$

$Y(s) = H(s)X(s)$

$H(s) = \mathcal{L}_s[\cos(t)u(t)] = \dfrac{s}{s^2+1}$

$X(s) = \mathcal{L}_s[\sin t\, u(t)e^{-t}] = \dfrac{1}{(s+1)^2+1}$

$Y(s) = \dfrac{s}{(s^2+1)((s+1)^2+1)}$

original problem $= e^{-t}\int_0^t \cos(t-\tau)e^{-\tau}\sin\tau d\tau \rightarrow Y(s+1)$

$\rightarrow \boxed{\dfrac{s+1}{((s+1)^2+1)((s+2)^2+1)}}$

5. $f(t) = (\cos t + \sin t - 1)u(t)$

$F(s) = Y(s) = \dfrac{s}{s^2+1} + \dfrac{1}{s^2+1} - \dfrac{1}{s} = \dfrac{s+1}{s^2+1} - \dfrac{1}{s} = \dfrac{s^2+s-s^2-1}{s(s^2+1)} = \dfrac{s-1}{s(s^2+1)}$

let $H(s) = \dfrac{s-1}{s} = 1 - \dfrac{1}{s} \rightarrow h(t) = \delta(t) - u(t)$

$X(s) = \dfrac{1}{s^2+1} \longrightarrow x(t) = \sin t\, u(t)$

$$\boxed{f(t) = \int_{-\infty}^{\infty}[\delta(t-\tau) - u(t-\tau)]\sin\tau\cancel{u}(\tau)d\tau} = \boxed{\int_0^t[\delta(t-\tau)-1]\sin\tau d\tau}$$

Part III: Time-Domain and / or s-Domain

(Use whatever method(s) which you are most comfortable with)

6. (15 pts)

A system S : $x(t) \to [S] \to y(t)$ is described by the differential equation:

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} + x(t), t > 0$$

$$y(0) = y'(0) = 0 = x(0)$$

Find the IRF $h(t)$ and the USR (Unit Step Response) $g(t)$ of the system S.

7. (15 pts)

Find the system function $H(s)$ of the system S described by the IPOP relation

$$x(t) \to [S] \to y(t)$$

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau)d\tau + \int_{-\infty}^{\infty} U(t-\tau)x(\tau)d\tau.$$

Then, write down the differential equation involving any input $x(t)$ and corresponding output $y(t)$ of S.

6. $s^2Y(s) + sY(s) + Y(s) = sX(s) + X(s)$

$Y(s)(s^2+s+1) = X(s)(s+1)$

$\frac{Y(s)}{X(s)} = \frac{s+1}{s^2+s+1} = H(s) = \frac{s+1+\frac{1}{2}-\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} = \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} + \frac{\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}}$

$= \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} + \frac{\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{3}}\right)}{(s+\frac{1}{2})^2+\frac{3}{4}} \to \boxed{h(t) = e^{-t/2}\cos\frac{\sqrt{3}}{2}t\,u(t) + \frac{1}{\sqrt{3}}e^{-t/2}\sin\frac{\sqrt{3}}{2}t\,u(t)}$

$X(s) = \frac{1}{s}$

$Y(s) = G(s) = \frac{1}{s}\left(\frac{s+1}{s^2+s+1}\right) = \frac{As+B}{s^2+s+1} + \frac{C}{s}$

$s+1 = (As+B)s + C(s^2+s+1) \xrightarrow{s=0} C=1$

$s+1 = As^2 + Bs + \cancel{s^2} + s + 1 \to A+1=0 \to A=-1$

$B+1=0 \to B=-1$

$G(s) = \frac{1}{s} - \frac{s+1}{(s+\frac{1}{2})^2+\frac{3}{4}} \Rightarrow u(t) - h(t)$

$\boxed{g(t) = u(t) - e^{-t/2}\cos\frac{\sqrt{3}}{2}t\,u(t) - \frac{1}{\sqrt{3}}e^{-t/2}\sin\frac{\sqrt{3}}{2}t\,u(t)}$

7. $x(t) \rightarrow \boxed{S} \rightarrow y(t)$

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau)\,d\tau + \int_{-\infty}^{\infty} u(t-\tau)\,x(\tau)\,d\tau \qquad H(s) = ?$$

$$y(t) = \int_{-\infty}^{\infty} \left[e^{-(t-\tau)} x(\tau)\,u(t-\tau) + u(t-\tau)\,x(\tau) \right] d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) \underbrace{\left[\left(e^{-(t-\tau)} + 1\right)\left(u(t-\tau)\right) \right]}_{h(t-\tau)} d\tau$$

$$h(t) = \left(e^{-t} + 1\right) u(t) = e^{-t} u(t) + u(t)$$

$$H(s) = \frac{1}{s+1} + \frac{1}{s} = \frac{s + s + 1}{s(s+1)} = \boxed{\frac{2s+1}{s(s+1)}} = H(s)$$

$$Y(s) = X(s)\,H(s)$$

$$Y(s) = X(s)\left[\frac{2s+1}{s^2+s}\right] \rightarrow Y(s)(s^2+s) = X(s)(2s+1)$$

from inspection of $h(t-\tau)$, system is LTI, C, so all initial values are 0

$$s^2 Y(s) + sY(s) = 2s\,X(s) + X(s)$$

$$\boxed{\frac{d^2 y(t)}{dt^2} + \frac{dy(t)}{dt} = 2\frac{dx(t)}{dt} + x(t)}$$