UCLA DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 102: SYSTEMS & SIGNALS

Midterm Examination II
February 23, 2021
Duration: 1 hr 50 min. (+15 min. for Gradescope submission)

INSTRUCTIONS:

- The exam has 5 problems and 17 pages.
- The exam is open-book and open-notes.
- Calculator/MATLAB allowed.
- Show all of your work! No credit given for answers without math steps shown and/or an explanation.
- NO LATE SUBMISSIONS ALLOWED ON GRADESCOPE.

Your name:______________________________

Student ID:______________________________

Table 1: Score Table

Problem	a	b	c	d	e	Score
1	10					10
2	10	3	2			15
3	4	4	4	4	4	20
4	1	3	2	10	4	20
5	6	4	6	4	5	25
Total						90

Table 3.1 One-Sided Laplace Transforms

	Function of Time	Function of s, ROC
1.	$\delta(t)$	1, whole s-plane
2.	$u(t)$	$\frac{1}{s}$, $\mathcal{R}e[s] > 0$
3.	$r(t)$	$\frac{1}{s^2}$, $\mathcal{R}e[s] > 0$
4.	$e^{-at}u(t),\ a > 0$	$\frac{1}{s+a}$, $\mathcal{R}e[s] > -a$
5.	$\cos(\Omega_0 t)u(t)$	$\frac{s}{s^2+\Omega_0^2}$, $\mathcal{R}e[s] > 0$
6.	$\sin(\Omega_0 t)u(t)$	$\frac{\Omega_0}{s^2+\Omega_0^2}$, $\mathcal{R}e[s] > 0$
7.	$e^{-at}\cos(\Omega_0 t)u(t),\ a > 0$	$\frac{s+a}{(s+a)^2+\Omega_0^2}$, $\mathcal{R}e[s] > -a$
8.	$e^{-at}\sin(\Omega_0 t)u(t),\ a > 0$	$\frac{\Omega_0}{(s+a)^2+\Omega_0^2}$, $\mathcal{R}e[s] > -a$
9.	$2A\, e^{-at}\cos(\Omega_0 t + \theta)u(t),\ a > 0$	$\frac{A\angle\theta}{s+a-j\Omega_0} + \frac{A\angle-\theta}{s+a+j\Omega_0}$, $\mathcal{R}e[s] > -a$
10.	$\frac{1}{(N-1)!}\, t^{N-1}u(t)$	$\frac{1}{s^N}$ N an integer, $\mathcal{R}e[s] > 0$
11.	$\frac{1}{(N-1)!}\, t^{N-1}e^{-at}u(t)$	$\frac{1}{(s+a)^N}$ N an integer, $\mathcal{R}e[s] > -a$
12.	$\frac{2A}{(N-1)!}\, t^{N-1}e^{-at}\cos(\Omega_0 t + \theta)u(t)$	$\frac{A\angle\theta}{(s+a-j\Omega_0)^N} + \frac{A\angle-\theta}{(s+a+j\Omega_0)^N}$, $\mathcal{R}e[s] > -a$

Table 3.2 Basic Properties of One-Sided Laplace Transforms

Causal functions and constants	$\alpha f(t),\ \beta g(t)$	$\alpha F(s),\ \beta G(s)$
Linearity	$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
Time shifting	$f(t-\alpha)$	$e^{-\alpha s}F(s)$
Frequency shifting	$e^{\alpha t}f(t)$	$F(s-\alpha)$
Multiplication by t	$t\, f(t)$	$-\frac{dF(s)}{ds}$
Derivative	$\frac{df(t)}{dt}$	$sF(s) - f(0-)$
Second derivative	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0-) - f^{(1)}(0)$
Integral	$\int_{0-}^{t} f(t')dt$	$\frac{F(s)}{s}$
Expansion/contraction	$f(\alpha t)\ \alpha \neq 0$	$\frac{1}{\lvert\alpha\rvert}F\left(\frac{s}{\alpha}\right)$
Initial value	$f(0+) = \lim_{s\to\infty} sF(s)$	
Final value	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$	

Simple Real Poles

If $X(s)$ is a proper rational function

$$X(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{\prod_k (s-p_k)} \tag{3.21}$$

Problem 1 (10 pts)

Solve the following linear differential equation using the Laplace transform. Show your work and label any properties or identities you use.

$$u(t) + r(t) = x(t) + 3\frac{dx(t)}{dt} + 2\frac{d^2x(t)}{dt^2}, \quad 0 < t < \infty$$

$$x(0^-) = 0, \quad x'(0^-) = 1$$

Note: $u(t)$ is the unit step function and $r(t) = tu(t)$ (i.e. the unit ramp function).
Hint: You may find it easier to avoid fully combining fractions when finding $X(s)$.

Solution: First, find the Laplace transform of both sides and reorganize the terms to isolate $X(s) = \mathcal{L}\{x(t)\}$:

$$\begin{aligned}
\mathcal{L}\left\{u(t) + r(t)\right\} &= \mathcal{L}\left\{x(t) + 3\frac{dx(t)}{dt} + 2\frac{d^2x(t)}{dt^2}\right\} \\
\frac{1}{s} + \frac{1}{s^2} &= X(s) + 3(sX(s) - x(0^-)) + 2(s^2X(s) - sx(0^-) - x'(0^-)) \\
\frac{s+1}{s^2} &= X(s) + 3sX(s) + 2s^2X(s) - 2 \\
\frac{s+1}{s^2} + 2 &= X(s)(1 + 3s + 2s^2) = X(s)(s+1)(2s+1) \\
X(s) &= \frac{s+1}{s^2(s+1)(2s+1)} + \frac{2}{(s+1)(2s+1)} \\
X(s) &= \frac{1}{s^2(2s+1)} + \frac{2}{(s+1)(2s+1)} = X_1(s) + X_2(s)
\end{aligned}$$

Then, we can use partial fraction decomposition on each of the two relatively simple fractions:

$$X_1(s) = \frac{1}{s^2(2s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{2s+1}$$
$$A = -2$$
$$B = 1$$
$$C = 4$$
$$\rightarrow X_1(s) = \frac{-2}{s} + \frac{1}{s^2} + \frac{2}{s+\frac{1}{2}}$$
$$x_1(t) = \mathcal{L}^{-1}\left\{X_1(s)\right\} = -2u(t) + r(t) + 2e^{-\frac{1}{2}t}u(t)$$

$$X_2(s) = \frac{2}{(s+1)(2s+1)} = \frac{D}{s+1} + \frac{E}{2s+1}$$
$$D = -2$$
$$E = 4$$
$$\rightarrow X_2(s) = \frac{-2}{s+1} + \frac{2}{s+\frac{1}{2}}$$
$$x_2(t) = \mathcal{L}^{-1}\left\{X_2(s)\right\} = -2e^{-t}u(t) + 2e^{-\frac{1}{2}t}u(t)$$

Thus, the total time domain function for $x(t)$ is:

$$x(t) = x_1(t) + x_2(t) = -2u(t) + r(t) - 2e^{-t}u(t) + 4e^{-\frac{1}{2}t}u(t)$$

Problem 2 (15 pts)

A causal LTI system S has the following input-output relationship:

$$y(t) = \int_0^t e^{-3\tau} sin\left(2(\tau-1)\right)\left(\frac{dx(t-\tau)}{d(t-\tau)}\right)u(\tau-1)d\tau, \quad x(0)=0, \quad 0<t<\infty$$

(a) (10 pts) Find the transfer function $H(s)$ for the system S. Show your work and label any properties or identities you use.

(b) (3 pts) Plot the pole-zero plot for the transfer function $H(s)$. Label your plot clearly.

(c) (2 pts) Is the system BIBO stable? Justify your answer.

Solution:

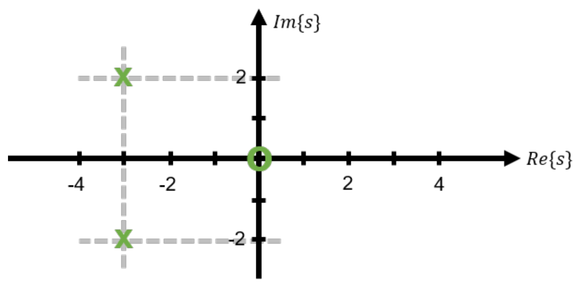
(a) We first notice that the equation can be reorganized as a convolution to simplify Laplace operations:

$$y(t) = e^{-3}\left(e^{-3(t-1)} sin\left(2(t-1)\right)u(t-1)\right) * \left(\frac{dx(t)}{dt}\right)$$

To find the transfer function, we will take the Laplace transform of the equation:

$$\begin{aligned}
\mathcal{L}\{y(t)\} &= e^{-3}\mathcal{L}\left\{e^{-3(t-1)} sin\left(2(t-1)\right)u(t-1)\right\}\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} \\
Y(s) &= e^{-3}\left(e^{-s}\mathcal{L}\left\{e^{-3t} sin\left(2t\right)u(t)\right\}\right)\left(sX(s)-x(0^-)\right) \\
Y(s) &= e^{-3}\left(e^{-s}\frac{2}{(s+3)^2+4}\right)(sX(s)) \\
H(s) &= \frac{Y(s)}{X(s)} = 2se^{-(s+3)}\left(\frac{1}{(s+3)^2+4}\right)
\end{aligned}$$

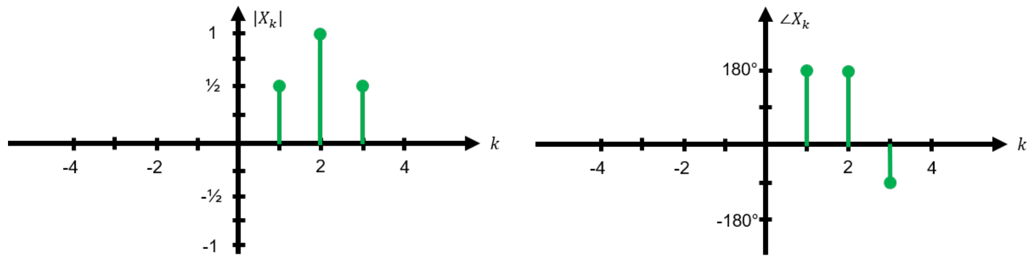
(b) This transfer function has a zero at $s = 0$ and two complex poles at $s = -3 \pm 2j$, translating to the following pole-zero plot:



(c) The system is BIBO stable, since the poles are all to the left of the imaginary axis.

Problem 3 (20 pts)

Suppose Gene was in a lab measuring a real, periodic signal $x(t)$. He created phase and magnitude spectra plots for the signal, but, as shown below, the spectra for only $k \geq 1$ were saved!



Note that a separate instrument saved 0.5 as the average value of $x(t)$ and $T_0 = 1$ as the period. Also, measurements for $k > 3$ are assumed to be 0.

(a) (4 pts) Let's help Gene out. Fill in the missing spectra and label the values clearly. Justify your answers.

(b) (4 pts) Find the trigonometric Fourier series coefficients, i.e. find coefficients a_k and b_k such that:

$$x(t) = X_0 + 2\sum_{k=1}^{\infty} a_k \cos(k\Omega_0 t) - 2\sum_{k=1}^{\infty} b_k \sin(k\Omega_0 t).$$

Simplify your results and show your work.

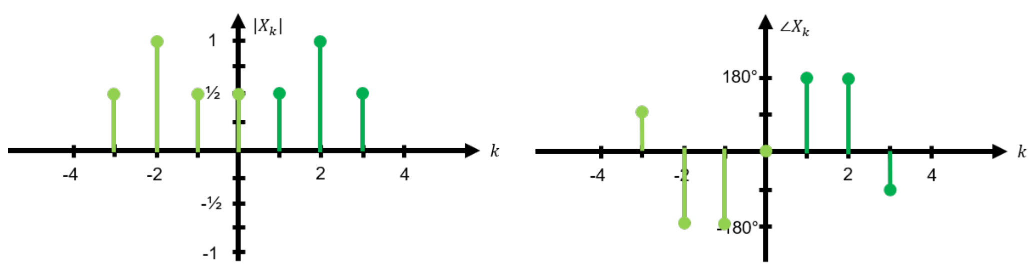
(c) (4 pts) Write an expression for x(t). Your expression should not include any complex exponential terms. Show your work and/or justify your answer.

(d) (4 pts) Was the signal x(t) even, odd, or neither? Justify your answer.

(e) (4 pts) Find the power of the signal $x(t)$. Show your work.

Solution:

(a) The filled in spectra are shown below. Since $x(t)$ is a real signal, the magnitude spectra should be even with respect to k while the phase spectra should be odd with respect to k. One common mistake was forgetting to fill in the DC component, which should be 0.5 given the average value measurement.



(b) To find the trigonometric Fourier series coefficients, we will use the following relationship between this formulation of the trig. FS coefficients and the complex (exponential) FS coefficients X_k:

$$a_k = Re\{X_k\}$$
$$b_k = Im\{X_k\}$$

From the magnitude and phase plots, we can simply find the complex

FS coefficients using the complex polar representation $X_k = |X_k|e^{j\angle X_k}$:

$$
\begin{aligned}
X_1 &= \frac{1}{2}e^{j\pi} = \frac{-1}{2} \\
&\rightarrow a_1 = Re\{\frac{-1}{2}\} = \frac{-1}{2} \\
&\rightarrow b_1 = Im\{\frac{-1}{2}\} = 0 \\
X_2 &= 1e^{j\pi} = -1 \\
&\rightarrow a_2 = Re\{1\} = -1 \\
&\rightarrow b_2 = Im\{-1\} = 0 \\
X_3 &= \frac{1}{2}e^{j\frac{-\pi}{2}} = \frac{-1}{2}j \\
&\rightarrow a_3 = Re\{\frac{-1}{2}j\} = 0 \\
&\rightarrow b_3 = Im\{\frac{-1}{2}j\} = \frac{-1}{2} \\
X_k &= 0, k > 3 \\
&\rightarrow a_k = 0, b_k = 0, k > 3
\end{aligned}
$$

Note that these trig. FS coefficients are ONLY defined for $k \geq 1$. The conversion to this trig. FS formulation already accounts for the negative and DC coefficients.

(c) To write an expression for $x(t)$ without complex exponentials, we can simply use the trig. From the period $T_0 = 1$, we can find the fundamental frequency to be $\Omega_0 = \frac{2\pi}{T_0} = 2\pi$. FS representation we just computed:

$$
\begin{aligned}
x(t) &= X_0 + 2\sum_{k=1}^{\infty} a_k \cos(k\Omega_0 t) - 2\sum_{k=1}^{\infty} b_k \sin(k\Omega_0 t) \\
&= \frac{1}{2} + 2\left(\frac{-1}{2}cos(1 \times 2\pi t)\right) + 2\left(-1cos(2 \times 2\pi t)\right) - 2\left(\frac{-1}{2}sin(3 \times 2\pi t)\right) \\
&= \frac{1}{2} - cos(2\pi t) - 2cos(4\pi t) + sin(6\pi t)
\end{aligned}
$$

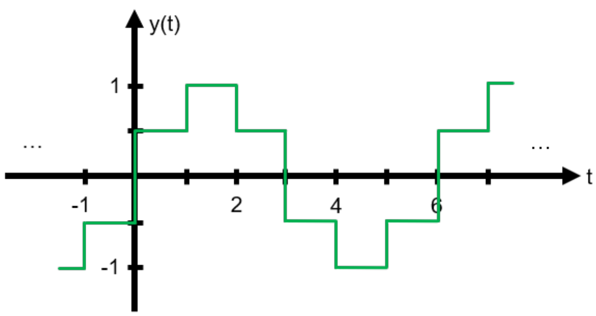
(d) The signal was neither odd nor even, since the signal is a sum of both (non-shifted) sine (purely odd) and cosine (purely even) functions. Alternatively, we can also observe that $a_k \neq 0$ and $b_k \neq 0$ for all k, meaning that $x(t)$ cannot be fully even nor odd.

(e) To find the power of the signal, we will use Parseval's power relation:

$$\begin{aligned} P_x &= \sum_{k=-\infty}^{\infty} |X_k|^2 \\ &= |X_{-3}|^2 + |X_{-2}|^2 + |X_{-1}|^2 + |X_0|^2 + |X_1|^2 + |X_2|^2 + |X_3|^2 \\ &= |X_0|^2 + 2|X_1|^2 + 2|X_2|^2 + 2|X_3|^2 \\ &= \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^2 + 2\,(1)^2 + 2\left(\frac{1}{2}\right)^2 \\ &= \frac{13}{4} \end{aligned}$$

Problem 4 (20 pts)

Suppose we want to build a sine wave generator, but our device is only able to give 4 total output amplitudes. A capture of the periodic output $y(t)$ of our generator, with period $T_0 = 6$ seconds, is shown below:



(a) (1 pt) What is the fundamental frequency of the output $y(t)$?

(b) (3 pts) Our target sine wave $\widehat{y}(t)$ has the same frequency as $y(t)$. Find the complex (exponential) Fourier series coefficients for $\widehat{y}(t)$? Show your work.

(c) (2 pts) Plot the phase and magnitude spectra of $\widehat{y}(t)$.

(d) (10 pts) Find the complex (exponential) Fourier series coefficients for $y(t)$. You may solve using the Fourier series definition and/or the Laplace transform method. Simplify your coefficients so they do NOT include any complex exponential terms. Show your work.

(e) (4 pts) Plot the phase and magnitude spectra of $y(t)$ for **only** X_1, X_0, X_{-1}. How do they compare to the pure sine wave in (c)?

Solution:

(a) The fundamental frequency of $y(t)$ is just based on the period T_0:

$$\Omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{3}$$

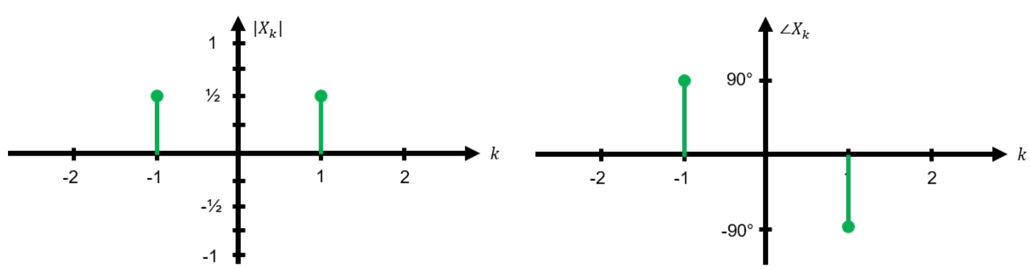
(b) The target sine wave is thus $\widehat{y}(t) = sin\left(\frac{\pi}{3}t\right)$. Then, to find the complex FS coefficients, we will simply decompose the sine function using Euler's identity:

$$\widehat{y}(t) = \frac{1}{2j}\left(e^{j\frac{\pi}{3}t} - e^{-j\frac{\pi}{3}t}\right)$$

Fitting this equation to the definition of the complex FS representation, we find:

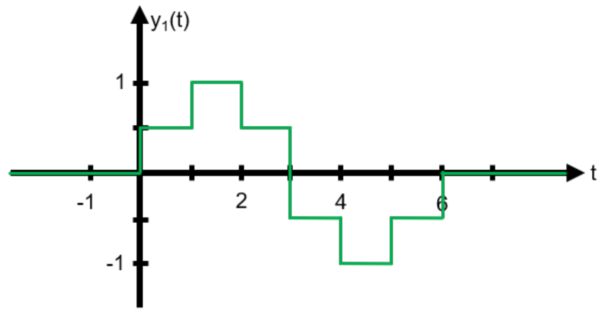
$$X_k = \begin{cases} \frac{-1}{2j}, & k = -1 \\ \frac{1}{2j}, & k = 1 \\ 0 & otherwise \end{cases}$$
$$= \begin{cases} \frac{j}{2}, & k = -1 \\ \frac{-j}{2}, & k = 1 \\ 0 & otherwise \end{cases}$$

(c) The spectra are shown below:



(d) To find the complex FS coefficients for the full $y(t)$ shown, we will use the Laplace transform of a single period of $y(t)$ and the translation to the FS coefficients:

$$X_k = \frac{1}{T_0}\mathcal{L}\left\{y_1(t)\right\}|_{s=jk\Omega_0}$$



$$
\begin{aligned}
y_1(t) &= \frac{1}{2}\left(u(t)+u(t-1)-u(t-2)-2u(t-3)-u(t-4)+u(t-5)+u(t-6)\right) \\
\mathcal{L}\{y_1(t)\} = Y_1(s) &= \frac{1}{2s}\left(1+e^{-s}-e^{-2s}-2e^{-3s}-e^{-4s}+e^{-5s}+e^{-6s}\right) \\
X_k &= \frac{1}{6\times 2\left(j\frac{\pi}{3}k\right)}\left(1+e^{-j\frac{\pi}{3}k}-e^{-j2\frac{\pi}{3}k}-2e^{-j3\frac{\pi}{3}k}-e^{-j4\frac{\pi}{3}k}+e^{-j5\frac{\pi}{3}k}+e^{-j6\frac{\pi}{3}k}\right) \\
&= \frac{-j}{4\pi k}\left(1+e^{-j\frac{\pi}{3}k}-e^{-\frac{2\pi}{3}k}-2e^{-j\pi k}-e^{j\frac{2\pi}{3}k}+e^{j\frac{\pi}{3}k}+e^{-j2\pi k}\right) \\
&= \frac{-j}{4\pi k}\left(1+\left(e^{j\frac{\pi}{3}k}+e^{-j\frac{\pi}{3}k}\right)-\left(e^{-j\frac{2\pi}{3}k}+e^{j\frac{2\pi}{3}k}\right)-2(-1)^k+1\right) \\
&= \frac{-j}{4\pi k}\left(2+2cos\left(\frac{\pi}{3}k\right)-2cos\left(\frac{2\pi}{3}k\right)-2(-1)^k\right) \\
&= \frac{-j}{2\pi k}\left(1+cos\left(\frac{\pi}{3}k\right)-cos\left(\frac{2\pi}{3}k\right)-(-1)^k\right)
\end{aligned}
$$

The DC coefficient $X_0 = 0$, since the average value of this function is 0.

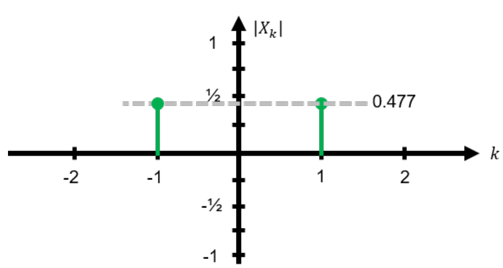
(e) We first find the FS coefficients just at $k \in \{-1, 1, 0\}$:

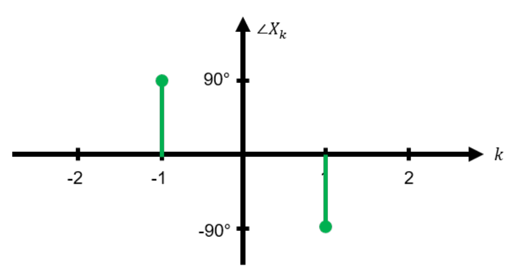
$$
\begin{aligned}
X_0 &= 0 \\
X_{-1} &= \frac{j}{2\pi}\left(1 + cos\left(\frac{-\pi}{3}\right) - cos\left(\frac{-2\pi}{3}\right) - (-1)\right) \\
&= \frac{j}{2\pi}\left(2 + 0.5 - (-0.5))\right) \\
&= \frac{3}{2\pi}j \\
X_1 &= \frac{-j}{2\pi}\left(1 + cos\left(\frac{\pi}{3}\right) - cos\left(\frac{2\pi}{3}\right) - (-1)\right) \\
&= \frac{-j}{2\pi}\left(2 + 0.5 - (-0.5))\right) \\
&= \frac{-3}{2\pi}j
\end{aligned}
$$

Thus, the magnitude and phase spectra are shown below for these coefficients. The phase is exactly the same as the pure sine from part (b), but the magnitude is slightly smaller (0.477 instead of 0.5).

You do not need to write this to get credit, but here is some additional insight into what we should expect:

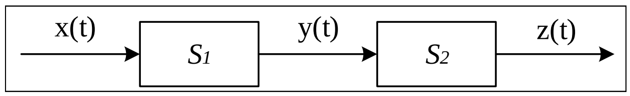
We do not expect the magnitudes to be the same, since the sine approximation will distribute its power over more harmonics than just the fundamental. Although we do not know if the power of these signals are the same, it is unlikely the signals are scaled perfectly to match their fundamental frequency components.





Problem 5 (25 pts)

Consider a cascade of two systems $S_{12} = S_1 S_2$.



The first system S_1 is described by:

$$y(t) = \int_0^t x(\sigma) d\sigma$$

where $x(t)$ and $y(t)$ are the input and the output, respectively. The second system is described by:

$$z(t) = 2y(t) + 10 \int_0^t y(\sigma) d\sigma$$

where $y(t)$ and $z(t)$ and the input and the output, respectively.
The input signal to the system $x(t)$ is periodic with $T_0 = 1$. Each period of $x(t)$ is represented by the following equation:

$$x(t) = e^{-4t}, \quad 0 \leq t < 1$$

(a) (6 pts) Find the complex (exponential) Fourier series of the input signal $x(t)$. Show your work. Simplify any complex exponentials. Your final answer should be of the form:

$$\frac{A}{B + jC}$$

Note A, B, and C should be entirely real and can be functions of k.

(b) (4 pts) Find the phase and magnitude of X_1 and X_{-1}. Show your work. You do not need to simplify any inverse trigonometric functions. Note that the form in part (a) can be rewritten as:

$$\left(\frac{AB}{B^2 - C^2}\right) - j\left(\frac{AC}{B^2 - C^2}\right)$$

(c) (6 pts) Find the transfer functions $H_1(s)$ and $H_2(s)$ of S_1 and S_2 respectively. Show your work.

(d) (4 pts) Find the transfer function $H_{12}(s)$ of the cascaded system S_{12}. Show your work.

(e) (5 pts) Find the complex (exponential) Fourier series coefficients of the output Z_k.

Solution:

(a) First, we will find the DC coefficient by taking the average value of a single period:

$$\begin{aligned}
X_0 &= \int_0^1 e^{-4t} dt \\
&= \frac{e^{-4}}{-4} - \frac{e^0}{-4} \\
&= \frac{1 - e^{-4}}{4}
\end{aligned}$$

For the other coefficients, we then use the complex FS definition with the fundamental frequency $\Omega_0 = \frac{2\pi}{T_0} = 2\pi$:

$$\begin{aligned}
X_k &= \int_0^1 e^{-4t} e^{-jk\Omega_0 t} dt \\
&= \int_0^1 e^{-(4+j2\pi k)t} dt \\
&= \frac{1}{-(4 + j2\pi k)} e^{-(4+j2\pi k)t} |_0^1 \\
&= \frac{e^{-4} e^{-j2\pi k} - 1}{-(4 + j2\pi k)} \\
&= \frac{1 - e^{-4}}{4 + j2\pi k}
\end{aligned}$$

(b) To find the magnitude and phase of these coefficients, we will use the hint to find an expression for the magnitude and phase using the values for A, B, and C:

$$\begin{aligned}
X_k &= \left(\frac{AB}{B^2 - C^2}\right) - j\left(\frac{AC}{B^2 - C^2}\right) \\
|X_k| &= \sqrt{\left(\frac{AB}{B^2 - C^2}\right)^2 + \left(\frac{AC}{B^2 - C^2}\right)^2} \\
&= \left(\frac{A}{B^2 - C^2}\right)\sqrt{B^2 + C^2} \\
\angle X_k &= \tan^{-1}\left(\frac{\left(\frac{-AC}{B^2-C^2}\right)}{\left(\frac{AB}{B^2-C^2}\right)}\right) \\
&= \tan^{-1}\left(\frac{-C}{B}\right)
\end{aligned}$$

Then, using the values found in part (a) (i.e. $A = 1 - e^{-4}$, $B = 4$, and $C = 2\pi k$), we compute the magnitude and phase for X_1:

$$\begin{aligned}
|X_1| &= \left(\frac{1 - e^{-4}}{4^2 - (2\pi k)^2}\right)\sqrt{4^2 + (2\pi k)^2} \\
&= \left(\frac{1 - e^{-4}}{16 - 4\pi^2}\right)\sqrt{16 + 4\pi^2} \\
\angle X_1 &= \tan^{-1}\left(\frac{-2\pi k}{4}\right) \\
&= \tan^{-1}\left(\frac{-\pi}{2}\right)
\end{aligned}$$

Then, we can easily find the magnitude and phase for X_{-1} using the properties of FS coefficients for real signals:

$$
\begin{aligned}
|X_{-1}| &= |X_1| \\
&= \left(\frac{1-e^{-4}}{16-4\pi^2}\right)\sqrt{16+4\pi^2} \\
\angle X_{-1} &= -\angle X_1 \\
&= -\tan^{-1}\left(\frac{-\pi}{2}\right)
\end{aligned}
$$

(c) For H_1, we notice that this integral is just a convolution for a causal system:

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\sigma)u(\sigma)u(t-\sigma)d\sigma \\
&= x(t) * u(t)
\end{aligned}
$$

Thus, the impulse response of H_1 is just $h_1(t) = u(t)$ and the transfer function is $H_1(s) = \frac{1}{s}$ with ROC: $Re\{s\} > 0$.

For H_2, we can convert the IPOP into a convolution integral:

$$
\begin{aligned}
z(t) &= 2\int_{-\infty}^{\infty} y(\sigma)\delta(t-\sigma)d\sigma + 10\int_{-\infty}^{\infty} y(\sigma)u(\sigma)u(t-\sigma)d\sigma \\
&= \int_{-\infty}^{\infty} y(\sigma)\left(2\delta(t-\sigma) + 10u(t-\sigma)\right)d\sigma
\end{aligned}
$$

Thus, the impulse response of H_2 is $h_2(t) = 2\delta(t) + 10u(t)$. The transfer function is then $H_2(s) = 2 + \frac{10}{s} = \frac{2(s+5)}{s}$ with ROC: $Re\{s\} > 0$.

(d) The transfer function of the cascaded system is:

$$
\begin{aligned}
H_{12}(s) &= H_1(s)H_2(s) \\
&= \left(\frac{1}{s}\right)\left(\frac{2(s+5)}{s}\right) \\
&= \frac{2(s+5)}{s^2}
\end{aligned}
$$

(e) To find the complex Fourier series coefficients of the output Z_k, we will just evaluate the cascaded transfer function at $s = j\Omega_0 k$ and multiply by the input coefficients:

$$\begin{aligned} Z_k &= H_{12}(j\Omega_0 k)X_k \\ &= \left(\frac{2(j2\pi k + 5)}{(j2\pi k)^2}\right)\left(\frac{1 - e^{-4}}{j2\pi k + 4}\right) \end{aligned}$$