UCLA DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 102: SYSTEMS & SIGNALS

Midterm Examination II
February 23, 2021
Duration: 1 hr 50 min. (+15 min. for Gradescope submission)

INSTRUCTIONS:

- The exam has 5 problems and 17 pages.
- The exam is open-book and open-notes.
- Calculator/MATLAB allowed.
- Show all of your work! No credit given for answers without math steps shown and/or an explanation.
- NO LATE SUBMISSIONS ALLOWED ON GRADESCOPE.

Your name:______________________________

Student ID:______________________________

Table 1: Score Table

Problem	a	b	c	d	e	Score
1	10					10
2	10	3	2			15
3	4	4	4	4	4	20
4	1	3	2	10	4	20
5	6	4	6	4	5	25
Total						90

Table 3.1 One-Sided Laplace Transforms

	Function of Time	Function of s, ROC
1.	$\delta(t)$	1, whole s-plane
2.	$u(t)$	$\frac{1}{s}$, $\mathcal{R}e[s] > 0$
3.	$r(t)$	$\frac{1}{s^2}$, $\mathcal{R}e[s] > 0$
4.	$e^{-at}u(t),\ a > 0$	$\frac{1}{s+a}$, $\mathcal{R}e[s] > -a$
5.	$\cos(\Omega_0 t)u(t)$	$\frac{s}{s^2+\Omega_0^2}$, $\mathcal{R}e[s] > 0$
6.	$\sin(\Omega_0 t)u(t)$	$\frac{\Omega_0}{s^2+\Omega_0^2}$, $\mathcal{R}e[s] > 0$
7.	$e^{-at}\cos(\Omega_0 t)u(t),\ a > 0$	$\frac{s+a}{(s+a)^2+\Omega_0^2}$, $\mathcal{R}e[s] > -a$
8.	$e^{-at}\sin(\Omega_0 t)u(t),\ a > 0$	$\frac{\Omega_0}{(s+a)^2+\Omega_0^2}$, $\mathcal{R}e[s] > -a$
9.	$2A\, e^{-at}\cos(\Omega_0 t + \theta)u(t),\ a > 0$	$\frac{A\angle\theta}{s+a-j\Omega_0} + \frac{A\angle-\theta}{s+a+j\Omega_0}$, $\mathcal{R}e[s] > -a$
10.	$\frac{1}{(N-1)!}\, t^{N-1}u(t)$	$\frac{1}{s^N}$ N an integer, $\mathcal{R}e[s] > 0$
11.	$\frac{1}{(N-1)!}\, t^{N-1}e^{-at}u(t)$	$\frac{1}{(s+a)^N}$ N an integer, $\mathcal{R}e[s] > -a$
12.	$\frac{2A}{(N-1)!}\, t^{N-1}e^{-at}\cos(\Omega_0 t + \theta)u(t)$	$\frac{A\angle\theta}{(s+a-j\Omega_0)^N} + \frac{A\angle-\theta}{(s+a+j\Omega_0)^N}$, $\mathcal{R}e[s] > -a$

Table 3.2 Basic Properties of One-Sided Laplace Transforms

Causal functions and constants	$\alpha f(t),\ \beta g(t)$	$\alpha F(s),\ \beta G(s)$
Linearity	$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
Time shifting	$f(t-\alpha)$	$e^{-\alpha s}F(s)$
Frequency shifting	$e^{\alpha t}f(t)$	$F(s-\alpha)$
Multiplication by t	$t\,f(t)$	$-\frac{dF(s)}{ds}$
Derivative	$\frac{df(t)}{dt}$	$sF(s) - f(0-)$
Second derivative	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0-) - f^{(1)}(0)$
Integral	$\int_{0-}^{t} f(t')dt$	$\frac{F(s)}{s}$
Expansion/contraction	$f(\alpha t)\ \alpha \neq 0$	$\frac{1}{\lvert\alpha\rvert}F\left(\frac{s}{\alpha}\right)$
Initial value	$f(0+) = \lim_{s\to\infty} sF(s)$	
Final value	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$	

Simple Real Poles

If $X(s)$ is a proper rational function

$$X(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{\prod_k (s - p_k)} \tag{3.21}$$

Problem 1 (10 pts)

Solve the following linear differential equation using the Laplace transform. Show your work and label any properties or identities you use.

$$u(t) + r(t) = x(t) + 3\frac{dx(t)}{dt} + 2\frac{d^2x(t)}{dt^2}, \quad 0 < t < \infty$$

$$x(0^-) = 0, \quad x'(0^-) = 1$$

Note: $u(t)$ is the unit step function and $r(t) = tu(t)$ (i.e. the unit ramp function).
Hint: You may find it easier to avoid fully combining fractions when finding $X(s)$.

Problem 2 (15 pts)

A causal LTI system S has the following input-output relationship:

$$y(t) = \int_0^t e^{-3\tau} sin\left(2(\tau - 1)\right) \left(\frac{dx(t-\tau)}{d(t-\tau)}\right) u(\tau - 1) d\tau, \quad x(0) = 0, \quad 0 < t < \infty$$

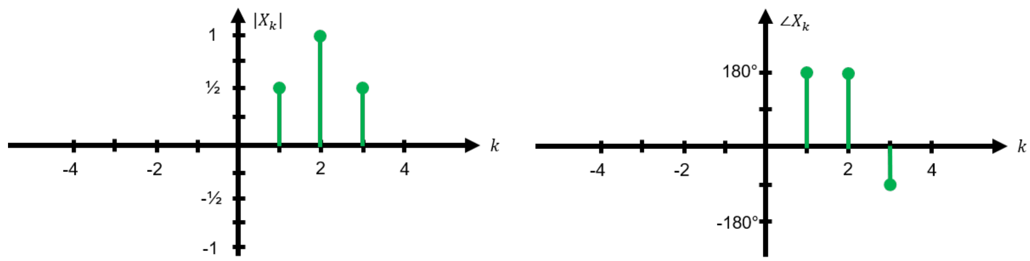
(a) (10 pts) Find the transfer function $H(s)$ for the system S. Show your work and label any properties or identities you use.

(b) (3 pts) Plot the pole-zero plot for the transfer function $H(s)$. Label your plot clearly.

(c) (2 pts) Is the system BIBO stable? Justify your answer.

Problem 3 (20 pts)

Suppose Gene was in a lab measuring a real, periodic signal $x(t)$. He created phase and magnitude spectra plots for the signal, but, as shown below, the spectra for only $k \geq 1$ were saved!



Note that a separate instrument saved 0.5 as the average value of $x(t)$ and $T_0 = 1$ as the period. Also, measurements for $k > 3$ are assumed to be 0.

(a) (4 pts) Let's help Gene out. Fill in the missing spectra and label the values clearly. Justify your answers.

(b) (4 pts) Find the trigonometric Fourier series coefficients, i.e. find coefficients a_k and b_k such that:

$$x(t) = X_0 + 2\sum_{k=1}^{\infty} a_k \cos(k\Omega_0 t) - 2\sum_{k=1}^{\infty} b_k \sin(k\Omega_0 t).$$

Simplify your results and show your work.

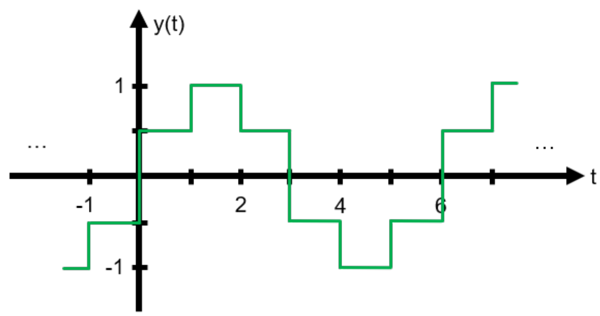
(c) (4 pts) Write an expression for x(t). Your expression should not include any complex exponential terms. Show your work and/or justify your answer.

(d) (4 pts) Was the signal x(t) even, odd, or neither? Justify your answer.

(e) (4 pts) Find the power of the signal $x(t)$. Show your work.

Problem 4 (20 pts)

Suppose we want to build a sine wave generator, but our device is only able to give 4 total output amplitudes. A capture of the periodic output $y(t)$ of our generator, with period $T_0 = 6$ seconds, is shown below:



(a) (1 pt) What is the fundamental frequency of the output $y(t)$?

(b) (3 pts) Our target sine wave $\widehat{y}(t)$ has the same frequency as $y(t)$. Find the complex (exponential) Fourier series coefficients for $\widehat{y}(t)$? Show your work.

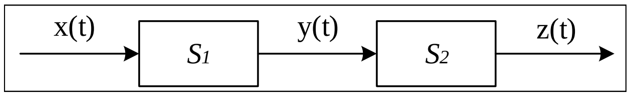
(c) (2 pts) Plot the phase and magnitude spectra of $\widehat{y}(t)$.

(d) (10 pts) Find the complex (exponential) Fourier series coefficients for $y(t)$. You may solve using the Fourier series definition and/or the Laplace transform method. Simplify your coefficients so they do NOT include any complex exponential terms. Show your work.

(e) (4 pts) Plot the phase and magnitude spectra of $y(t)$ for **only** X_1, X_0, X_{-1}. How do they compare to the pure sine wave in (c)?

Problem 5 (25 pts)

Consider a cascade of two systems $S_{12} = S_1 S_2$.



The first system S_1 is described by:

$$y(t) = \int_0^t x(\sigma)d\sigma$$

where $x(t)$ and $y(t)$ are the input and the output, respectively. The second system is described by:

$$z(t) = 2y(t) + 10\int_0^t y(\sigma)d\sigma$$

where $y(t)$ and $z(t)$ and the input and the output, respectively.
The input signal to the system $x(t)$ is periodic with $T_0 = 1$. Each period of $x(t)$ is represented by the following equation:

$$x(t) = e^{-4t}, \quad 0 \leq t < 1$$

(a) (6 pts) Find the complex (exponential) Fourier series of the input signal $x(t)$. Show your work. Simplify any complex exponentials. Your final answer should be of the form:

$$\frac{A}{B + jC}$$

Note A, B, and C should be entirely real and can be functions of k.

(b) (4 pts) Find the phase and magnitude of X_1 and X_{-1}. Show your work. You do not need to simplify any inverse trigonometric functions. Note that the form in part (a) can be rewritten as:

$$\left(\frac{AB}{B^2 - C^2}\right) - j\left(\frac{AC}{B^2 - C^2}\right)$$

(c) (6 pts) Find the transfer functions $H_1(s)$ and $H_2(s)$ of S_1 and S_2 respectively. Show your work.

(d) (4 pts) Find the transfer function $H_{12}(s)$ of the cascaded system S_{12}. Show your work.

(e) (5 pts) Find the complex (exponential) Fourier series coefficients of the output Z_k.