UCLA DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 102: SYSTEMS & SIGNALS

Midterm Examination SOLUTION
February 2, 2021
Duration: 1 hr 50 min. (+15 min. for Gradescope submission)

INSTRUCTIONS:

- The exam has 6 problems and 18 pages.
- The exam is open-book and open-notes.
- Calculator/MATLAB allowed.
- Show all of your work! No credit given for answers without math steps shown and/or an explanation.
- NO LATE SUBMISSIONS ALLOWED ON GRADESCOPE.

Your name:______________________________

Student ID:______________________________

Table 1: Score Table

Problem	a	b	c	d	e	Score
1	8	7				15
2	3	3	2	4		12
3	2	2	4	3	4	15
4	6	6	4	2		18
5	3	5				8
6	4	8				12
Total						80

Table 3.1 One-Sided Laplace Transforms

	Function of Time	Function of s, ROC
1.	$\delta(t)$	1, whole s-plane
2.	$u(t)$	$\frac{1}{s}$, $\mathcal{R}e[s] > 0$
3.	$r(t)$	$\frac{1}{s^2}$, $\mathcal{R}e[s] > 0$
4.	$e^{-at}u(t),\ a > 0$	$\frac{1}{s+a}$, $\mathcal{R}e[s] > -a$
5.	$\cos(\Omega_0 t)u(t)$	$\frac{s}{s^2+\Omega_0^2}$, $\mathcal{R}e[s] > 0$
6.	$\sin(\Omega_0 t)u(t)$	$\frac{\Omega_0}{s^2+\Omega_0^2}$, $\mathcal{R}e[s] > 0$
7.	$e^{-at}\cos(\Omega_0 t)u(t),\ a > 0$	$\frac{s+a}{(s+a)^2+\Omega_0^2}$, $\mathcal{R}e[s] > -a$
8.	$e^{-at}\sin(\Omega_0 t)u(t),\ a > 0$	$\frac{\Omega_0}{(s+a)^2+\Omega_0^2}$, $\mathcal{R}e[s] > -a$
9.	$2A\, e^{-at}\cos(\Omega_0 t + \theta)u(t),\ a > 0$	$\frac{A\angle\theta}{s+a-j\Omega_0} + \frac{A\angle-\theta}{s+a+j\Omega_0}$, $\mathcal{R}e[s] > -a$
10.	$\frac{1}{(N-1)!}\, t^{N-1}u(t)$	$\frac{1}{s^N}$ N an integer, $\mathcal{R}e[s] > 0$
11.	$\frac{1}{(N-1)!}\, t^{N-1}e^{-at}u(t)$	$\frac{1}{(s+a)^N}$ N an integer, $\mathcal{R}e[s] > -a$
12.	$\frac{2A}{(N-1)!}\, t^{N-1}e^{-at}\cos(\Omega_0 t + \theta)u(t)$	$\frac{A\angle\theta}{(s+a-j\Omega_0)^N} + \frac{A\angle-\theta}{(s+a+j\Omega_0)^N}$, $\mathcal{R}e[s] > -a$

Table 3.2 Basic Properties of One-Sided Laplace Transforms

Causal functions and constants	$\alpha f(t),\ \beta g(t)$	$\alpha F(s),\ \beta G(s)$
Linearity	$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
Time shifting	$f(t-\alpha)$	$e^{-\alpha s}F(s)$
Frequency shifting	$e^{\alpha t}f(t)$	$F(s-\alpha)$
Multiplication by t	$t\,f(t)$	$-\frac{dF(s)}{ds}$
Derivative	$\frac{df(t)}{dt}$	$sF(s) - f(0-)$
Second derivative	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0-) - f^{(1)}(0)$
Integral	$\int_{0-}^{t} f(t')dt$	$\frac{F(s)}{s}$
Expansion/contraction	$f(\alpha t)\ \alpha \neq 0$	$\frac{1}{\lvert\alpha\rvert}F\left(\frac{s}{\alpha}\right)$
Initial value	$f(0+) = \lim_{s\to\infty} sF(s)$	
Final value	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$	

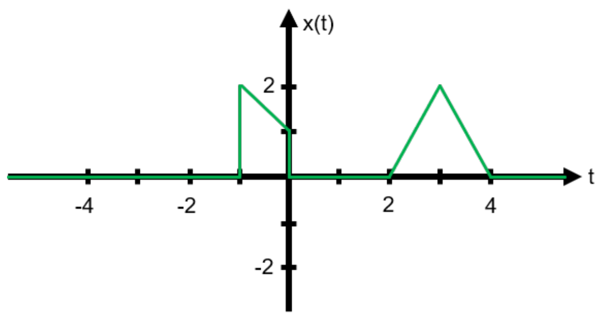
Simple Real Poles

If $X(s)$ is a proper rational function

$$X(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{\prod_k (s-p_k)} \tag{3.21}$$

Problem 1 (15 pts)

Consider signal $x(t)$ depicted in the figure below

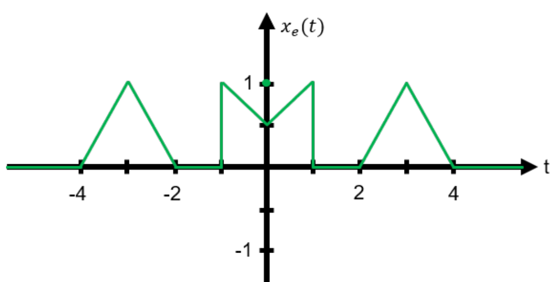


(a) (8 pts) Sketch even and odd components of $x(t)$. Label the value of the even and odd components at $t = 0$. Assume $x(0) = 1$.

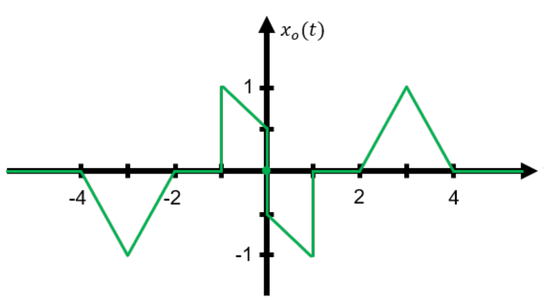
(b) (7 pts) Sketch $-2x(-\frac{1}{2}t + 1)$.

Solutions:

(a) Even component of the signal: $x_e(0) = 1$

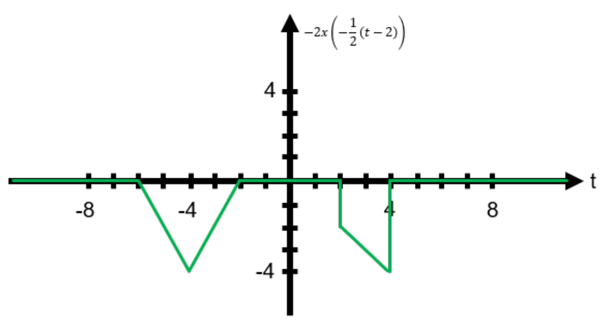


Odd component of the signal: $x_e(0) = 0$



(b) This function can be rewritten as $-2x(-\frac{1}{2}(t-2))$, showing that the following 5 operations should be used on the original $x(t)$:

1) Scale amplitude by 2.
2) Flip in amplitude (over the x-axis).
3) Flip in time (over the y-axis).
4) Scale time by 0.5 (expanded in time).
5) Shift in time by 2 units (delayed) after scaling time.



Problem 2 (13 pts)

The system S is given by the following relation

$$y(t) = x(t) + e^{-t}x(t)u(t+1), \qquad -\infty < t < \infty$$

Using the system relation, answer the following questions about the system. You need to justify your answer and show your work.

(a) (3 pts) Is the system linear? Prove it.

(b) (3 pts) Is the system time-varying or time-invariant? Justify your answer.

(c) (2 pts) Is the system causal or not causal? Justify your answer.

(d) (4 pts) Find the impulse response function of the system.

Solutions:

(a) It is linear. To prove it, we will introduce two arbitrary input signals with their corresponding outputs:

$$y_1(t) = S[x_1(t)] = x_1(t) + e^{-t}x_1(t)u(t+1)y_2(t) = S[x_2(t)] = x_2(t) + e^{-t}x_2(t)u(t+1)$$

We then define an arbitrary linear combination of these two signals:

$$x_3(t) = \alpha x_1(t) + \beta x_2(t)$$

Then the output from this signal is:

$$\begin{aligned} y_3(t) = S[x_3(t)] &= x_3(t) + e^{-t}x_3(t)u(t+1) \\ &= (\alpha x_1(t) + \beta x_2(t)) + e^{(} - t)(\alpha x_1(t) + \beta x_2(t)) \\ &= \alpha(x_1(t) + e^{-t}x_1(t)u(t+1)) + \beta(x_2(t) + e^{-t}x_2(t)u(t+1)) \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

Since $y_3(t) = \alpha y_1(t) + \beta y_2(t)$, S must be linear.

(b) S is time varying. To show this, we will use a delayed version of $x_1(t)$ as a new input signal: $x_4(t) = x_1(t-\tau)$. The output from this new input is then:

$$\begin{aligned} y_4(t) = S[x_4(t)] &= x_4(t) + e^{-t}x_4(t)u(t+1) \\ &= x_1(t-\tau) + e^{-t}x_1(t-\tau)u(t+1) \end{aligned}$$

The output for $x_1(t)$ delayed by τ, however, is:

$$y_1(t-\tau) = x_1(t-\tau) + e^{-(t-\tau)}x_1(t-\tau)u(t-\tau+1)$$

Since $y_4(t) \neq y_1(t-\tau)$, this system cannot be time invariant (i.e. is time varying).

(c) S is causal. We can see this by evaluating the system at a specified time:

Let $t = \sigma$ where σ is some constant value on the range $(-\infty, \infty)$. The output of S at this time for any input $x(t)$ will be:

$$y(\sigma) = x(\sigma) + e^{-\sigma}x(\sigma)u(\sigma+1)$$

Notice that $y(\sigma)$ only depends on the input at time σ. Thus, since the output only depends on the input at the current time, it does not depend on the input at any future time and is causal.

Note that the unit step term does NOT count as part of this input, although it does affect the value of the output depending on the value of σ. This impact of the unit step really shows S's time varying property and not its lack of causality.

(d) To find the impulse response, we will just use the signal $x(t) = \delta(t-\tau)$:

$$\begin{aligned} h(t,\tau) = S[\delta(t-\tau)] &= \delta(t-\tau) + e^{-t}\delta(t-\tau)u(t+1) \\ &= \delta(t-\tau) + e^{-\tau}u(\tau+1) \end{aligned}$$

Problem 3 (15 pts)

The impulse response function for a linear system is given below:

$$h(t,\tau) = \sqrt{\frac{1}{\pi}} e^{-t^2+2t\tau-\tau^2}[u(t-\tau+2) - u(t-\tau-2)]$$

(a) (2 pts) Is the system time varying or time invariant? Justify your answer.

(b) (2 pts) Is the system causal or not causal? Justify your answer.

(c) (4 pts) Is this system BIBO stable? Justify your answer.

(d) (3 pts) Write the system input-output relation with $x(t)$ as the input and $y(t)$ as the output. Simplify your answer so it does not include unit step functions.

(e) (4 pts) Compute the step response of this system.

Hint:

$$\int_a^b e^{-x^2} dx = \int_{t+a}^{t+b} e^{-(t-x)^2} dx = \frac{\sqrt{\pi}}{2}(f(b) - f(a))$$

where $f(x)$ is the "error function", defined for all x with output range $[-1, 1]$. Thus, this integral will evaluate to a value in the range $[-\sqrt{\pi}, \sqrt{\pi}]$ for any a and b. You do not need to simplify error functions.

Solutions:

(a) Since we know the system is linear and we are given the impulse response of the system, we can determine if the system is time invariant by determining if the IRF is only a function of $t - \tau$:

$$h(t,\tau) = \sqrt{\frac{1}{\pi}} e^{-(t-\tau)^2}[u((t-\tau)+2) - u((t-\tau)-2)] \quad = h(t-\tau, 0)$$

By simplifying the IRF, we find that the IRF is indeed only a function of $t - \tau$ and thus the system must be time invariant.

Now that we know the system is LTI, we will refer to the impulse response as:

$$h(t) = \sqrt{\frac{1}{\pi}} e^{-t^2} [u(t+2) - u(t-2)]$$

(b) To check if this LTI system is causal, we can check if the impulse response is nonzero at any time $t < 0$. We notice that:

$$h(t) \neq h(t)u(t)$$

Thus, the system is not causal.

(c) Since the system is LTI, we can determine if it is BIBO stable by determining if the IRF is absolutely integrable:

$$\begin{aligned}
\int_{-\infty}^{\infty} |h(\tau)| \, d\tau &= \int_{-\infty}^{\infty} \left| \sqrt{\frac{1}{\pi}} e^{-\tau^2} [u(\tau+2) - u(\tau-2)] \right| d\tau \\
&= \sqrt{\frac{1}{\pi}} \int_{-\infty}^{\infty} e^{-\tau^2} [u(\tau+2) - u(\tau-2)] d\tau \\
&= \sqrt{\frac{1}{\pi}} \int_{-2}^{2} e^{-\tau^2} d\tau \\
&= \sqrt{\frac{1}{\pi}} \left(\frac{\sqrt{\pi}}{2} \left(f(2) - f(-2) \right) \right)
\end{aligned}$$

Using the hint, we know that this will evaluate to a value on the range $[-1, 1]$, a finite value. Thus, the system must be BIBO stable.

(d) Since the system is LTI, we can just use a convolution integral to determine the system IPOP:

$$\begin{aligned}
y(t) &= x(t) * h(t) \\
&= x(t) * \left(\sqrt{\frac{1}{\pi}} e^{-t^2} [u(t+2) - u(t-2)] \right) \\
&= \int_{-\infty}^{\infty} x(\tau) \sqrt{\frac{1}{\pi}} e^{-(t-\tau)^2} [u((t-\tau)+2) - u((t-\tau)-2)] d\tau \\
&= \sqrt{\frac{1}{\pi}} \int_{t-2}^{t+2} x(\tau) e^{-(t-\tau)^2} d\tau
\end{aligned}$$

Alternatively, we can swap the functions inside the convolution and find:

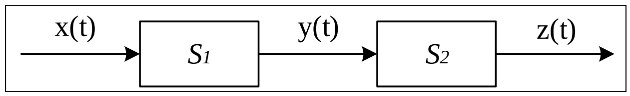
$$y(t) = \sqrt{\frac{1}{\pi}} \int_{-2}^{2} x(t-\tau) e^{-\tau^2} d\tau$$

(e) To find the step response, we use $x(t) = u(t)$:

$$\begin{aligned}
y(t) &= \sqrt{\frac{1}{\pi}} \int_{t-2}^{t+2} u(\tau) e^{-(t-\tau)^2} d\tau \\
&= \begin{cases} \sqrt{\frac{1}{\pi}} \int_{t-2}^{t+2} u(\tau) e^{-(t-\tau)^2} d\tau, & t-2 \geq 0 \\ \sqrt{\frac{1}{\pi}} \int_{0}^{t+2} u(\tau) e^{-(t-\tau)^2} d\tau, & t+2 \geq 0, t-2 \leq 0 \\ 0 & t+2 \leq 0 \end{cases} \\
&= \begin{cases} \frac{1}{2} \left(f(2) - f(-2) \right), & t \geq 2 \\ \frac{1}{2} \left(f(2) - f(-t) \right), & -2 \leq t \leq 2 \\ 0 & t \leq -2 \end{cases}
\end{aligned}$$

Problem 4 (20 pts)

Consider a cascade of two systems $S_{12} = S_1 S_2$.



The first system S_1 is described by:

$$y(t) = \int_{-\infty}^{\infty} \delta(t - \sigma - 1)x(\sigma) - \delta(t - \sigma - 3)x(\sigma)d\sigma$$

where $x(t)$ and $y(t)$ are the input and the output, respectively. The second system is described by:

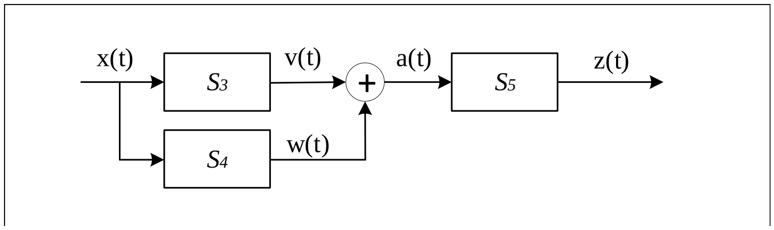
$$z(t) = \int_{t-1}^{t} y(\sigma)e^{t-\sigma}d\sigma$$

where $y(t)$ and $z(t)$ and the input and the output, respectively.

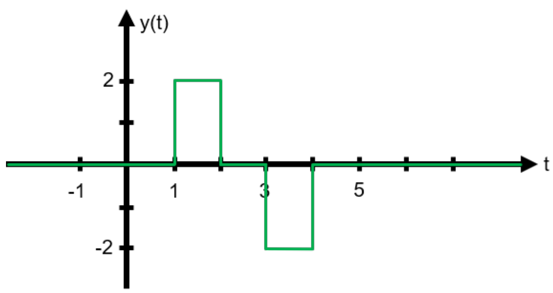
(a) (6 pts) Find the impulse response functions $h_1(t, \tau)$ and $h_2(t, \tau)$ of S_1 and S_2 respectively.

(b) (6 pts) Find the impulse response function $h_{12}(t, \tau)$ of the cascaded system S_{12}. Show your work.

(c) (4 pts) We can express this same cascaded system as a parallel cascaded system, as shown in the figure below. Determine the input-output relationships for each of the systems in this topology, S_3, S_4, and S_5. Do not leave any unit impulse or unit step functions in your equations. Describe how you arrived at this solution.



(d) (2 pts) While using this system, we probed $y(t)$ and found the following signal. What was the input $x(t)$?



Solutions:

(a) We notice that these IPOP equations look very similar to convolutions and reorganize them:

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} \delta(t-\sigma-1)x(\sigma) - \delta(t-\sigma-3)x(\sigma)d\sigma \\
&= \int_{-\infty}^{\infty} x(\sigma)\left[\delta((t-\sigma)-1) - \delta((t-\sigma)-3)\right]d\sigma \\
&= \int_{-\infty}^{\infty} y(\sigma)h_1(t-\sigma)d\sigma
\end{aligned}
$$

$$
\begin{aligned}
z(t) &= \int_{t-1}^{t} y(\sigma)e^{t-\sigma}d\sigma \\
&= \int_{-\infty}^{\infty} y(\sigma)e^{t-\sigma}u(\sigma-(t-1))u(t-\sigma)d\sigma \\
&= \int_{-\infty}^{\infty} y(\sigma)h_2(t-\sigma)d\sigma
\end{aligned}
$$

From these reorganized equations, we recognize that both systems are LTI and that their IRFs must be:

$$
\begin{aligned}
h_1(t) &= \delta(t-1) - \delta(t-3) \\
h_2(t) &= e^t u(1-t)u(t)
\end{aligned}
$$

(b) Knowing now that these systems are LTI, we can find the impulse response for the cascaded system by convolving the IRFs of the two systems:

$$
\begin{aligned}
h_{12}(t) &= h_1(t) * h_2(t) \\
&= [\delta(t-1) - \delta(t-3)] * \left[e^t u(1-t)u(t)\right] \\
&= \int_{-\infty}^{\infty} [\delta(\sigma-1) - \delta(\sigma-3)] \left[e^{t-\sigma}u(1-(t-\sigma))u(t-\sigma)\right] d\sigma \\
&= \int_{-\infty}^{\infty} \delta(\sigma-1) \left[e^{t-\sigma}u(1-(t-\sigma))u(t-\sigma)\right] d\sigma \\
&\quad - \int_{-\infty}^{\infty} \delta(\sigma-3) \left[e^{t-\sigma}u(1-(t-\sigma))u(t-\sigma)\right] d\sigma \\
&= e^{t-1}u(2-t)u(t-1) - e^{t-3}u(4-t)u(t-3)
\end{aligned}
$$

(c) To make the system fit this new parallel cascaded system model, we notice that we could just split the original first system S_1. Remember that the IRF we found for S_1 was $h_1(t) = \delta(t-1) - \delta(t-3)$. We also remember that the IRF for the two parallel systems (between input $x(t)$ and output of the sum $a(t)$) will just be the sum of the two IRFs for S_3 and S_4. Thus the IRFs can be:

$$\begin{aligned}
h_34(t) &= h_3(t) + h_4(t) \\
&= \delta(t-1) - \delta(t-3) \\
&\implies h_3(t) = \delta(t-1) \\
&\implies h_4(t) = -\delta(t-3) \\
h_5(t) &= h_2(t) = e^t u(1-t)u(t)
\end{aligned}$$

We need the IPOPs for these systems, so we can just use convolution integrals and find:

$$\begin{aligned}
v(t) &= S_3[x(t)] = \int_{-\infty}^{\infty} x(\sigma)(\delta(t-\sigma-1))d\sigma \\
&= x(t-1) \\
w(t) &= S_4[x(t)] = \int_{-\infty}^{\infty} x(\sigma)(-\delta(t-\sigma-3))d\sigma \\
&= -x(t-3) \\
a(t) &= v(t) + w(t) \\
z(t) &= S_5[a(t)] = \int_{-\infty}^{\infty} a(\sigma)\left(e^{t-\sigma}u(1-(t-\sigma))u(t-\sigma)\right)d\sigma \\
&= \int_{t-1}^{t} a(\sigma)e^{t-\sigma}d\sigma
\end{aligned}$$

(d) We first notice that the combination of S_3 and S_4 just add delayed versions of the input $x(t)$. S_3 adds $x(t)$ delayed by 1 time unit while S_4 subtracts $x(t)$ delayed by 3 time units. This conveniently lines up with our output $y(t)$ if the input was merely a scaled rectangular window function. Thus, $x(t) = 2u(t) - 2u(t-1)$.

Problem 5 (8 pts) Find the Laplace transform and ROC of the following functions. Show all of your work and specify any Laplace identities or properties you use.

(a) (3 pts)

$$f(t) = sin(5t - 15)sin(3t - 9)u(t - 3) + u(t)$$

Hint: $sin(\theta)sin(\phi) = \frac{1}{2}[cos(\theta - \phi) - cos(\theta + \phi)]$.

(b) (5 pts)

$$f(t) = \int_{0^-}^{\infty} e^{-\sigma} sin(2\sigma - 8)u(\sigma - 4)u(t - \sigma)d\sigma$$

Solutions:

(a) First we will simplify the equation using the hint and reorganize:

$$\begin{aligned} f(t) &= \frac{1}{2}[cos(2t - 6) - cos(8t - 24)]u(t - 3) + u(t) \\ &= \frac{1}{2}cos(2(t - 3))u(t - 3) - \frac{1}{2}cos(8(t - 3))u(t - 3) + u(t) \end{aligned}$$

Now we can use Laplace transform properties and identities to solve:

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\mathcal{L}\{cos(2(t - 3))u(t - 3)\} - \frac{1}{2}\mathcal{L}\{cos(8(t - 3))u(t - 3)\} + \mathcal{L}\{u(t)\} \tag{1}$$

$$= \frac{e^{-3s}}{2}\mathcal{L}\{cos(2t)u(t)\} - \frac{e^{-3s}}{2}\mathcal{L}\{cos(8t)u(t)\} + \mathcal{L}\{u(t)\} \tag{2}$$

$$= \frac{e^{-3s}}{2}\left(\frac{s}{s^2 + 2^2}\right) - \frac{e^{-3s}}{2}\left(\frac{s}{s^2 + 8^2}\right) + \frac{1}{s} \tag{3}$$

The steps use the following properties/identities:

(1) Linearity property

(2) Time shift property

(3) Identities for $cos(\Omega_0 t)u(t)$ and $u(t)$

The ROC is $Re\{s\} > 0$ from the identities in (3).

(b) For this problem, we will go straight to using properties and identities:

$$
\begin{aligned}
\mathcal{L}\{f(t)\} &= \mathcal{L}\left\{\int_{0^-}^{\infty} e^{-\sigma} sin(2\sigma - 8)u(\sigma - 4)u(t-\sigma)d\sigma\right\} \\
&= \mathcal{L}\left\{\int_{0^-}^{t} e^{-\sigma} sin(2\sigma - 8)u(\sigma - 4)d\sigma\right\} & (4) \\
&= \frac{1}{s}\mathcal{L}\left\{e^{-t} sin(2(t-4))u(t-4)\right\} & (5) \\
&= \frac{1}{s}\left.\mathcal{L}\left\{sin(2(t-4))u(t-4)\right\}\right|_{s'=s+1} & (6) \\
&= \frac{1}{s}\left.\left[e^{-4s}\mathcal{L}\left\{sin(2t)u(t)\right\}\right]\right|_{s'=s+1} & (7) \\
&= \frac{1}{s}\left.\left[e^{-4s'}\left(\frac{2}{s'^2 + 2^2}\right)\right]\right|_{s'=s+1} & (8) \\
&= \frac{1}{s}\left[e^{-4(s+1)}\left(\frac{2}{(s+1)^2 + 4}\right)\right] & (9) \\
&= \frac{2e^{-4(s+1)}}{s((s+1)^2 + 4)}
\end{aligned}
$$

The steps use the following properties/identities:

(4) Just removing the unit-step (not a Laplace property)

(5) Integral property

(6) Frequency shift property

(7) Time shift property

(8) Identity for $sin(\Omega_0 t)u(t)$

(9) Just substitution (not a Laplace property)

The ROC is $Re\{s\} > 0$. Although the poles give us two possible ROCs ($Re\{s\} > 0$ from the pole at $s = 0$ and $Re\{s\} > -1$ from the pole at $s = -1$), the union of these two ROCs is the stricter region (i.e. $Re\{s\} > 0$ is contained in $Re\{s\} > 0$).

Problem 6 (12 pts)

The transfer function for a causal system S is:

$$H(s) = \frac{3s^2 + 12s + 15}{(s^2 + 2s + 5)(s + 3)}$$

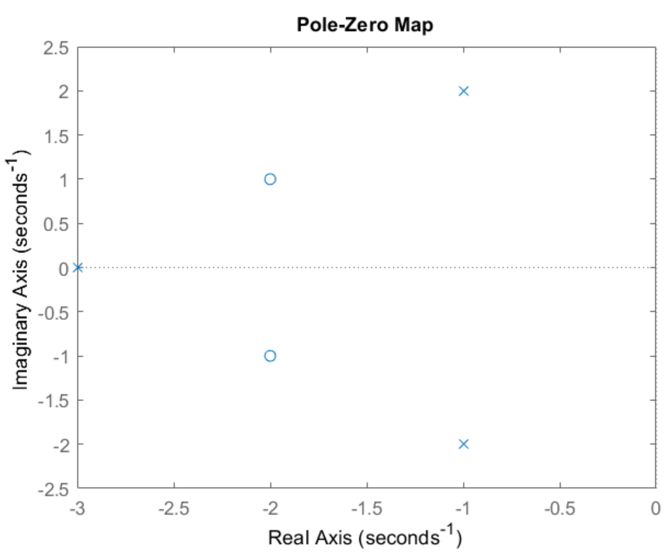
(a) (4 pts) Sketch the zero-pole plot of $H(s)$. Describe how you found the poles and zeros.

(b) (8 pts) Find the impulse response function of system S by finding the Inverse Laplace transform of $H(s)$. Show your work and specify any Laplace identities or properties you use.

Solutions:

(a) We first find the poles and zeros of this transfer function. The poles are the roots of the denominator while the zeros are the roots of the numerator. Solving using the quadratic formula for the quadratic term in the numerator gives us the zeros at $s = -2 \pm j$. Similarly solving the quadratic term in the denominator gives us poles at $s = -1 \pm 2j$. Additionally, the $s + 3$ term gives us a third pole at $s = -3$.

We then plot these poles and zeros on the complex plane, resulting in the following pole-zero plot:



(b) To find the IRF, we take the Inverse Laplace transform of the transfer function. To do this, we use partial fraction decomposition:

$$\begin{aligned} H(s) &= \frac{3s^2 + 12s + 15}{(s^2 + 2s + 5)(s + 3)} \\ &= \frac{As + B}{s^2 + 2s + 5} + \frac{C}{s + 3} \end{aligned}$$

Solving using one of the partial fractions methods (for which you should show work), we can find that $A = \frac{9}{4}$, $B = \frac{15}{4}$, $C = \frac{3}{4}$.

$$\begin{aligned} H(s) &= \frac{\left(\frac{9}{4}\right)s + \left(\frac{15}{4}\right)}{s^2 + 2s + 5} + \frac{\left(\frac{3}{4}\right)}{s + 3} \\ &= \left[\left(\frac{9}{4}\right)\left(\frac{s + 1}{(s + 1)^2 + 4}\right) + \left(\frac{3}{4}\right)\left(\frac{2}{(s + 1)^2 + 4}\right)\right] + \left(\frac{3}{4}\right)\left(\frac{1}{s + 3}\right) \end{aligned}$$

Thus, using the Laplace identities for $e^{-at}cos(\Omega_0 t)u(t)$, $e^{-at}sin(\Omega_0 t)u(t)$, and $e^{-at}u(t)$, we find that the IRF is:

$$\mathcal{L}^{-1}\{H(s)\} = \frac{9}{4}e^{-t}cos(2t)u(t) + \frac{3}{4}e^{-t}sin(2t)u(t) + \frac{3}{4}e^{-3t}u(t)$$