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UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Midterm Examination I

February 6, 2018

Duration: 1 hr 50 mins.

INSTRUCTIONS:

- The exam has 6 problems and 14 pages.
- The exam is closed-book.
- One cheat sheet of A4 size is allowed.
- Calculator is NOT allowed.
- Write your discussion session in the top-right corner. ↗↗

Your name: [redacted]

Student ID: [redacted]

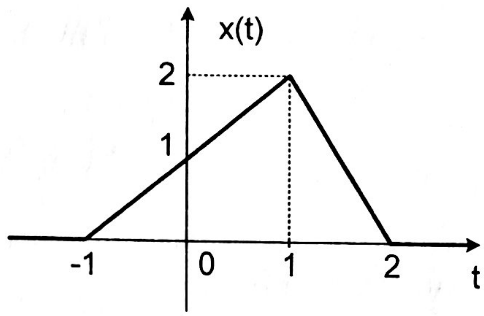
Table 1: Score Table

Problem	a	b	c	d	Score	
1	4	4	4		12	12
2	2	2	8		12	12
3	6	2	4	6	18	18
4	8	8			16	16
5	10	6			16	16
6	5	5	6		16	8
Total					90	[illegible]

82 G.H

12

Problem 1 (12 pts) Consider the following signal $x(t)$ for (a), (b)

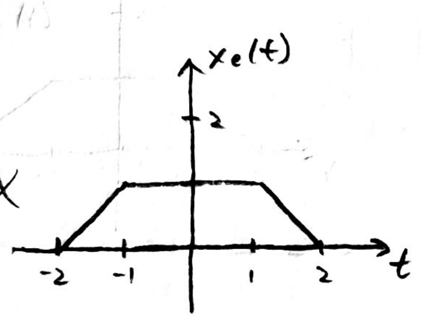


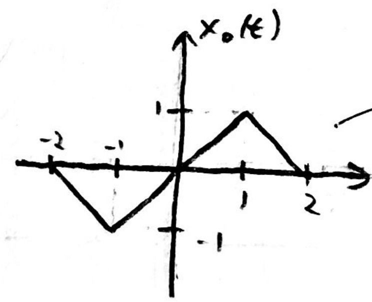
(a) (4 pts) Sketch even and odd decompositions $x_e(t)$ and $x_o(t)$.

(b) (4 pts) Sketch $x(-2t+3)$.

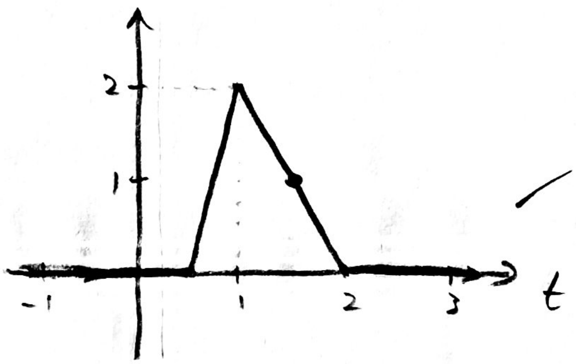
(c) (4 pts) Sketch $x(t/3+2)$.

a)

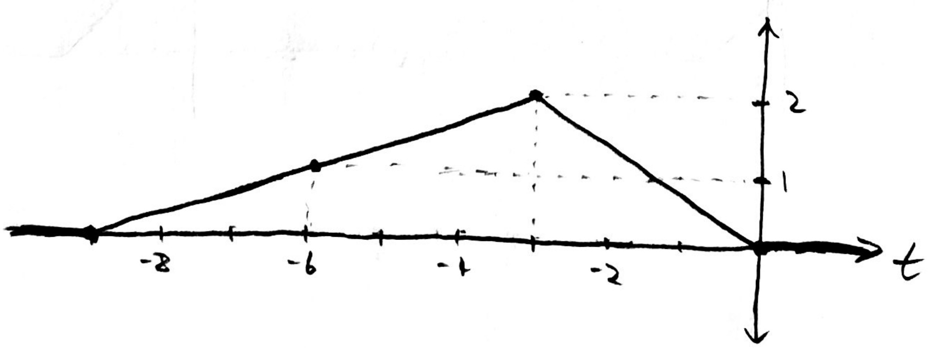




b) $x(-2t+3)$



c) $x(t/3+2)$



12

$t-\tau=\theta$

$t=\tau+\theta$

$h(\theta) = e^{-(\theta)} u(\theta) u(\theta+\tau)$

Problem 2 (12 pts) In this problem, we identify system properties from the impulse response function:

$$h(t,\tau) = e^{-(t-\tau)}u(t-\tau)u(t) \tag{1}$$

(a) (2 pts) Is the system TV or TI? Explain.

(b) (2 pts) Is it C or NC? Explain.

(c) (8 pts) Find the output $y(t)$ if the input is $x(t) = (t-2)u(t-2)$.

a) The system is TV,

as $h(t,\tau)$ is not dependent on $(t-\tau)$

2

b) Causal, since

$h(t,\tau) = 0$ when $t < \tau$

2

c) $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t,\tau)\, d\tau$

$= \int_{-\infty}^{\infty} (\tau-2)u(\tau-2)\, e^{-(t-\tau)} u(t-\tau) u(t)\, d\tau$

$= u(t)\, u(t-2) \int_{2}^{t} (\tau-2)\, e^{-(t-\tau)}\, d\tau$

$= u(t-2) \int_{2}^{t} \tau e^{\tau-t}\, dt \; - \; 2u(t-2) \int_{2}^{t} e^{\tau-t}\, d\tau$

$= u(t-2)\left(\tau e^{\tau-t} - e^{\tau-t}\right)\Big|_{2}^{t} - 2u(t-2)\left(e^{\tau-t}\Big|_{2}^{t}\right)$

$= u(t-2)\left(t-1-(2e^{2-t}-e^{2-t})\right) - 2u(t-2)\left(1-e^{2-t}\right)$

$= u(t-2)\left(t-1-2e^{2-t}+e^{2-t}-2+2e^{2-t}\right)$

$= u(t-2)\left(t-3+e^{2-t}\right)$

$$\boxed{y(t) = u(t-2)\left(t-3+e^{2-t}\right)}$$

8

18

Problem 3 (18 pts)
Consider IPOP relation for an LTI system S:

$$y(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} \left[\cos(t)\cos(\tau) + \sin(t)\sin(\tau)\right] x(\tau) d\tau$$

where $x(t)$ and $y(t)$ are input and output of the system, respectively.

(a) (6 pts) Find the impulse response function $h(t)$.

(b) (2 pts) Is the system C or NC? Provide justification.

(c) (4 pts) Is this system BIBO stable? Provide justification.

(d) (6 pts) Find Laplace transform $H(s)$ and ROC.

(Hint: Use the identity $\cos(\underset{A-B}{A+B}) = \cos(A)\cos(B) \underset{+}{-} \sin(A)\sin(B)$)

a) $y(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} \cos(t-\tau) x(\tau) d\tau$

$h(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} \cos(t-\tau) \delta(\tau) d\tau = e^{-t} \int_{-\infty}^{t} e^{0} \cos(t-0) \delta(\tau) d\tau$

$= e^{-t} \cos(t) \int_{-\infty}^{t} \delta(\tau) d\tau = e^{-t} \cos(t) u(t)$

$$\boxed{h(t) = e^{-t} \cos(t) u(t)}$$

b) $\boxed{\text{Causal}}$, as $y(t)$ does not depend on future inputs

also, $h(t) = 0$ for $t < 0$

also, if $x(t) = 0$, $y(t) = 0$

c) $\boxed{\text{stable,}}$ as $\int_{-\infty}^{\infty} |e^{-\tau}\cos(\tau)u(\tau)|\, d\tau < \infty$

due to the e^{-t} in $h(t)$

d) $h(t) = e^{-t}\cos(t)u(t)$ $\Rightarrow$ causal signal,

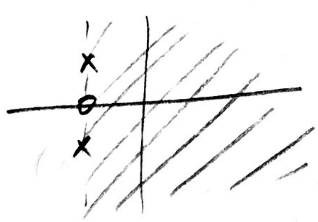
since $L_s[e^{-at}f(t)] = F(s+a)$,

$$\boxed{H(s) = \frac{s+1}{(s+1)^2+1}}$$

$(s+1)^2+1 = 0$

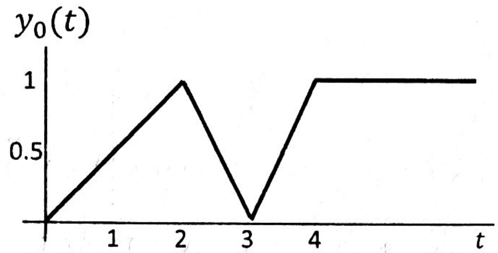
$s = -1 \pm i$

$\boxed{\text{ROC}: s \geq -1}$



16

Problem 4 (16 pts) Consider an LTI system S_0 with input $x_0(t)$ and the impulse response function $h_0(t)$. The corresponding output $y_0(t)$ is shown below:



(a) (8 pts) Consider an LTI system S_1 with input $x_1(t) = x_0(t+2)$ and IRF $h_1(t) = h_0(t-1)$. Express the output $y_1(t)$ as a function of $y_0(t)$ and then plot it.

(b) (8 pts) Consider an LTI system S_2 with input $x_2(t) = x_0(-t)$ and IRF $h_2(t) = h_0(-t)$. Express the output $y_2(t)$ as a function of $y_0(t)$ and then plot it.

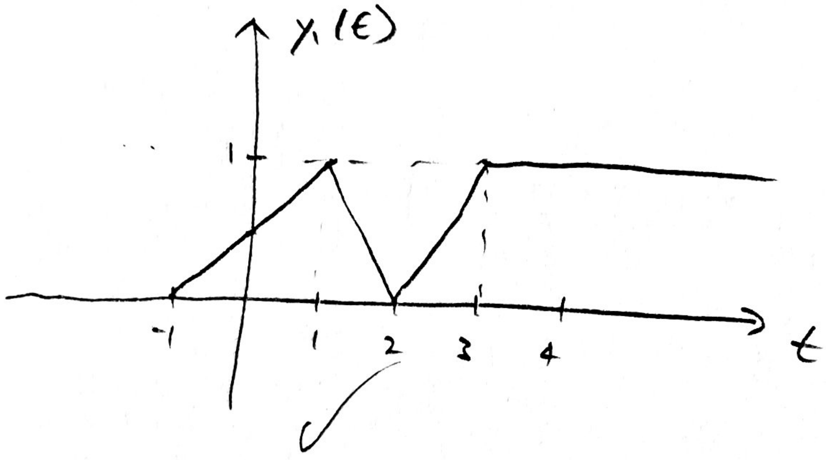
a) $$y_0(t) = \int_{-\infty}^{\infty} x_0(\tau) h_0(t-\tau)\, d\tau$$

$$y_1(t) = \int_{-\infty}^{\infty} x_1(\tau) h_1(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x_0(\tau+2) h_0(t-1-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x_0(\tau+2)\, h_0((t+1)-(\tau+2))\, d\tau$$

$$= \int_{-\infty}^{\infty} x_0(\tau)\, h_0((t+1)-\tau)\, d\tau = y_0(t+1)$$

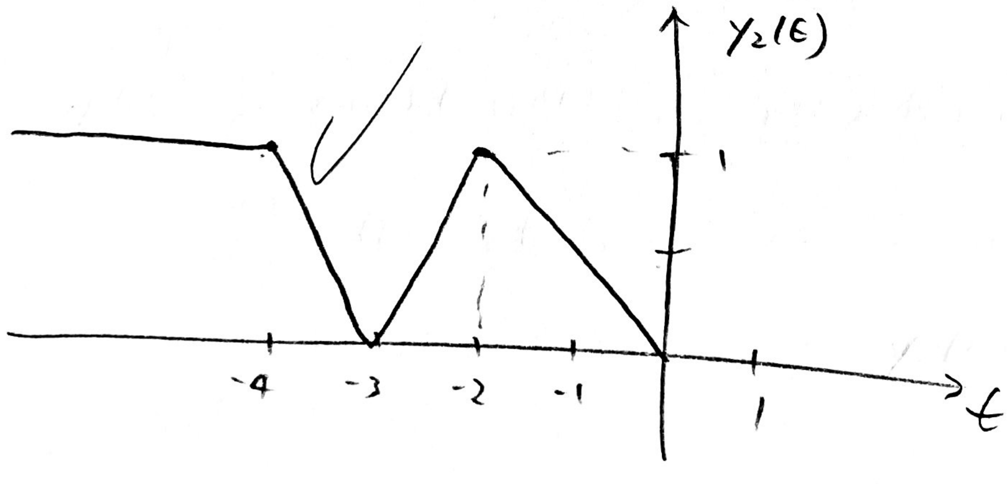
a) cont.



b) $$y_2(t) = \int_{-\infty}^{\infty} x_2(\tau)\, h_2(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x_0(-\tau)\, h_0(-(t-\tau))\, d\tau$$

$$= \int_{-\infty}^{\infty} x_0(-\tau)\, h_0(-t-(-\tau))\, d\tau$$

$$= y_0(-t)$$



16 **Problem 5** (16 pts) Consider a cascade combination of two systems S_1 and S_2: $x(t)$ is input to S_1 and $y(t)$ is the output, while the output of S_2 is $z(t)$.

$$x(t) \to [S_1] \to y(t) \to [S_1] \to z(t)$$

The IPOP relation for S_1 and S_2 is:

$$S_1 : y(t) = e^{-t}x(t)u(t),$$
$$S_2 : z(t) = \int_0^t e^{-(t-\sigma)}y(\sigma)u(\sigma)d\sigma.$$

10 (a) (10 pts) Compute impulse response function $h_{12}(t,\tau)$ of the cascaded system S_1S_2.

6 (b) (6 pts) Compute the output $z(t)$ if the input is $x(t) = e^{-3t}[u(t) - u(t-3)]$.

a) $h_1(t,\tau) = e^{-t}\delta(t-\tau)u(t) = e^{-\tau}\delta(t-\tau)u(\tau)$

$$h_{12}(t,\tau) = \int_0^t e^{-(t-\theta)}\left(e^{-\tau}\delta(\theta-\tau)u(\tau)\right)u(\theta)\,d\theta$$

$$= e^{-\tau}e^{-(t-\tau)}u(\tau)\int_0^t \delta(\theta-\tau)\,d\theta$$

$$= e^{-t}u(\tau)u(t-\tau)$$

$$\boxed{h_{12}(t,\tau) = e^{-t}u(\tau)u(t-\tau)}$$

b) $z(t) = \int_{-\infty}^{\infty} x(\tau)\, h_{12}(t,\tau)\, d\tau$

2 $= \int_{-\infty}^{\infty} e^{-3\tau}\left(u(\tau) - u(\tau-3)\right) e^{-t} u(\tau) u(t-\tau)\, d\tau$

$= u(t)\left(\int_0^t e^{-3\tau} e^{-t} u(\tau)\, d\tau - \int_0^t e^{-3\tau} e^{-\tau} u(\tau-3)\, d\tau\right)$

$= e^{-t} \cancel{u(t)} \int_0^t e^{-3\tau}\, d\tau - e^{-t} u(t) u(t-3) \int_3^t e^{-3\tau}\, d\tau$

$= e^{-t} u(t) \cdot \left(-\frac{e^{-3\tau}}{3}\Big|_0^t\right) - e^{-t} u(t-3)\left(-\frac{e^{-3\tau}}{3}\Big|_3^t\right)$

$= e^{-t} u(t) \cdot \left(-\frac{e^{-3t}}{3} + \frac{1}{3}\right) - e^{-t} u(t-3) \cdot \left(-\frac{e^{-3t}}{3} + \frac{e^{-9}}{3}\right)$

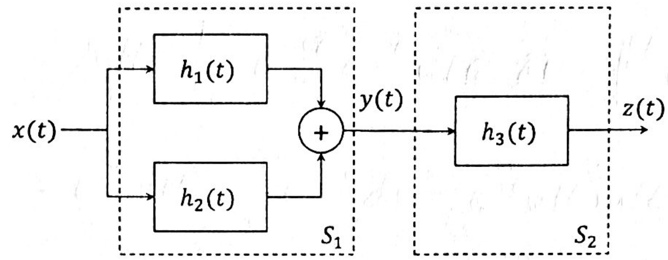
$$\boxed{z(t) = \frac{1}{3} e^{-\tau}\left(\cancel{u(t)}\left(1 - e^{-3t}\right) - u(t-3)\left(e^{-9} - e^{-3t}\right)\right)}$$

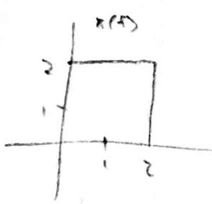
$z(t) = u(t)\left(\frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t}\right) - u(t-3)\left(\frac{1}{3}e^{-t-9} - \frac{1}{3}e^{-4t}\right)$

8 **Problem 6** (16 pts)
Consider a cascaded LTI system S_1S_2 as follows

$$x(t) \to [S_1] \to y(t) \to [S_2] \to z(t)$$

The cascade system is shown below.





where $h_1(t) = \delta(t-1)$, $h_2(t) = \delta(t-2)$, and $h_3(t) = \delta(t-1) - \delta(t-2) + \delta(t-3)$.
Let $x(t) = 2(u(t) - u(t-2))$, then

5 (a) (5 pts) Find the IPOP between $x(t)$ and $y(t)$. Plot $y(t)$.

2 (b) (5 pts) Write the impulse response of the cascade system S_1S_2.

1 (c) (6 pts) Compute and plot $z(t)$ for the specified input.

a) $h_{S1}(t) = h_1(t) + h_2(t) = \delta(t-1) + \delta(t-2)$

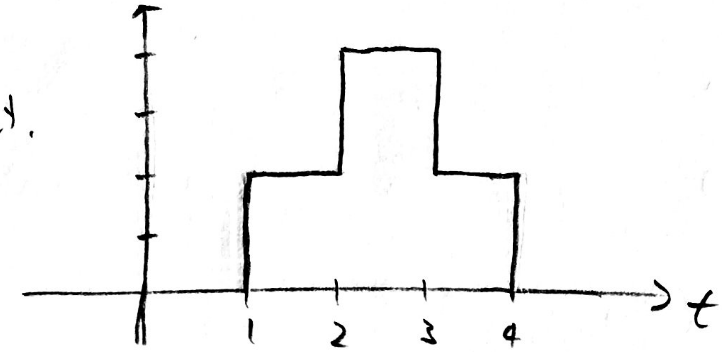
$$y(t) = \int_{-\infty}^{\infty} 2(u(\tau) - u(\tau-2))(\delta(t-(1+\tau)) + \delta(t-(2+\tau)))\,d\tau$$

$$= 2\int_0^2 \delta(-\tau + (t-1))\,d\tau + 2\int_0^2 \delta(-\tau + (t-2))\,d\tau$$

$$= 2(u(t-1) - u(t-1-2)) + 2(u(t-2) - u(t-2-2))$$

$$= 2(u(t-1) - u(t-3)) + 2(u(t-2) - u(t-4))$$

a) con't.



b)

$$h_{s2}(t) = h_{12}(t) = h_{s1}(t) * h_3(t)$$

$$\underset{2}{h_{s2}(t)} = \int_{-\infty}^{\infty} \big(\delta(\tau-1)+\delta(\tau-2)\big)\big(\delta(t-\tau-1)-\delta(t-\tau-2)+\delta(t-\tau-3)\big)\,d\tau$$

$$= \int_{-\infty}^{\infty} h_{s1}(\tau)\,\delta(-\tau+(t-1))\,d\tau + \int_{-\infty}^{\infty} h_{s1}(\tau)\,\delta(-\tau+(t-2))\,d\tau$$

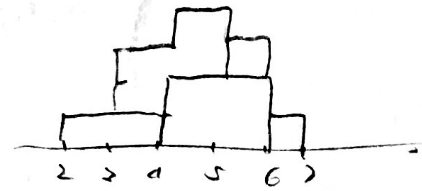
$$\cancel{+\int_{-\infty}^{\infty} h_{s1}(\tau)\,\delta(-\tau+(t-3))\,d\tau}$$

$$= h_{s1}(t-1) + h_{s1}(t-2) \cancel{+ h_{s1}(t-3)}$$

$$= \big(\delta(t-2)+\delta(t-3)\big) + \big(\delta(t-3)+\delta(t-4)\big) + \big(\delta(t-4)+\delta(t-5)\big)$$

$$= \delta(t-2) + 2\delta(t-3) + 2\delta(t-4) + \delta(t-5)$$

$$\boxed{h_{s2}(t) = \delta(t-2) + 2\delta(t-3) + 2\delta(t-4) + \delta(t-5)}$$



c) $z(t) = \int_{-\infty}^{\infty} x(\tau)\, h_{s2}(t-\tau)\, d\tau$

$$= 2\int_{-\infty}^{\infty} \left(u(\tau) - u(\tau-2)\right) h_{s2}(t-\tau)\, d\tau$$

$$= 2\int_{0}^{2} \left(\delta(-\tau \cancel{+(t-2)}) + 2\delta(-\tau+(t-3)) + 2\delta(-\tau+(t-4)) + \delta(-\tau+(t-5))\right) d\tau$$

$$z(t) = 2\left(u(t-2)-u(t-4)\right) + 4\left(u(t-3)-u(t-5)\right) + 4\left(u(t-4)-u(t-6)\right) + 2\left(u(t-5)-u(t-7)\right)$$