UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Midterm Examination I

February 7, 2017

INSTRUCTIONS:

- The exam has 6 problems and 14 pages.
- The exam is closed-book.
- One cheat sheet of A4 size is allowed.
- Calculator is NOT allowed.
- Put your discussion session in the top-right corner.

Your name:________________________________

Student ID:________________________________

Table 1: Score Table

Problem	a	b	c	d	e	Score
1	6	4	4	6		20
2	2	2	2	3	6	15
3	2	4	4	5		15
4	5	10				15
5	2	10	4	4		20
6	2	4	9			15
Total						100

Problem 1 (20 pts)
Consider the following signal

$$x(t) = \sin(\pi t)\,[u(t+1) - u(t-2)]$$

(a) (6 pts) Sketch $x(t)$, $2x(-t-1)$, and $x\left(\frac{t}{3}+1\right)$.

(b) (4 pts) Compute the energy of signals $x(t), 2x(-t-1)$, and $x\left(\frac{t}{3}+1\right)$.
(*Hint:* You may use the following trigonometric identity:
$\sin^2(x) = 0.5 - 0.5\cos(2x)$.)

(c) (4 pts) What will be the energy of the signal

$$Ax(Bt+C),$$

where A, B, and C are arbitrary non-zero real values?

(d) (6 pts) Sketch the even and odd decompositions of $x(t)$.

Problem 2 (15 pts)
The impulse response function of a linear system S is

$$h(t,\tau) = e^{2\tau - 2t} u(t - \tau).$$

(a) (2 pts) Is the system time-invariant? Justify your answer.

(b) (2 pts) Is the system causal? Justify your answer.

(c) (2 pts) Is the system BIBO stable? Justify your answer.

(d) (3 pts) Find the output $y_1(t)$ corresponding to input $x_1(t) = \delta(t - 1)$.

(e) (6 pts) Find the output $y_2(t)$ corresponding to input $x_2(t) = u(1 - t)$.

Problem 3 (15 pts)

Consider IPOP relation for a system S:

$$y(t) = \int_{-\infty}^{t} e^{t-\tau} \sin\left[2(t+\tau) - 4(1+\tau)\right] x(\tau) d\tau$$

where $x(t)$ and $y(t)$ are input and output of the system, respectively.

(a) (2 pts) Find the impulse response function $h(t, \tau)$.

(b) (4 pts) Is the system C or NC? Provide justification.

(c) (4 pts) Is the system TI or TV? Provide justification.

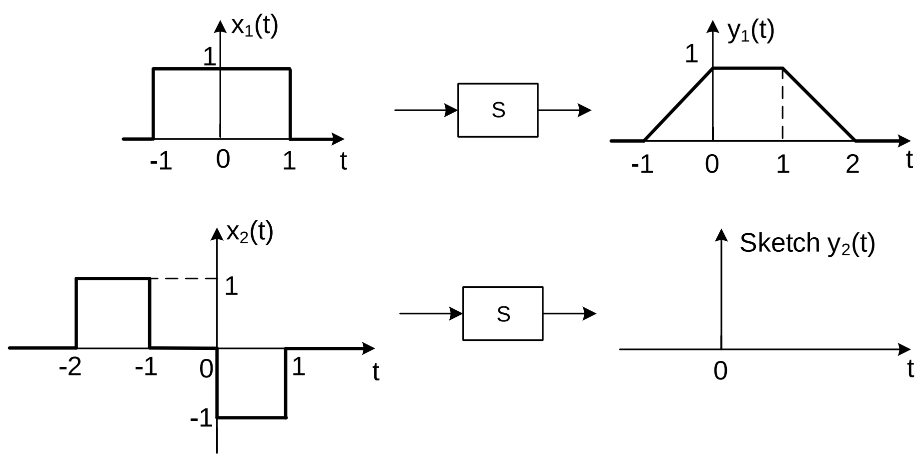
(d) (5 pts) Is this system BIBO stable? Provide justification.

Problem 4 (15 pts)
Consider a linear time-invariant (LTI) system S. If the input $x_1(t)$ is applied to this system, the output $y_1(t)$ is observed as shown in the figure. Using this knowledge, we want to find the output $y_2(t)$ of the system if input $x_2(t)$ is applied.

(a) (5 pts) First, write $x_2(t)$ as sum of scaled and time shifted version of $x_1(t)$. In particular you need to determine a_1, a_2, τ_1, τ_2 such that

$$x_2(t) = a_1 x_1(t - \tau_1) + a_2 x_1(t - \tau_2).$$

(b) (10 pts) Using the properties of LTI systems and the above decomposition of $x_2(t)$, sketch the output $y_2(t)$.



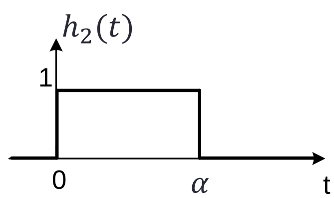
Problem 5 (20 pts)
Consider transmission of an audio signal over the air at radio frequency of $f_0 = 98.7\text{MHz}$. The first step is the modulation of a sinusoid of frequency f_0 using the audio signal $x(t)$. Let us denote the modulation by system S_1 as follows:

$$x(t) \to [S_1] \to y(t) = x(t)\cos(2\pi f_0 t)$$

The second step involves a filtering operation, which can be described using a convolution integral as follows:

$$y(t) \to [S_2] \to z(t) = \int_{-\infty}^{\infty} y(t-\sigma)h_2(\sigma)d\sigma,$$

where $h_2(t)$ is the filter response given as follows:



(α is a positive constant.)

(a) (2 pts) Write the expression for $h_2(t)$ using unit step function.

(b) (10 pts) Find the IRF $h_{12}(t,\tau)$ of the cascaded system S_1S_2.

(c) (4 pts) Is the cascaded system S_1S_2 time invariant? Justify your answer.

(d) (4 pts) Is the cascaded system S_1S_2 causal? Justify your answer.

Problem 6 (15 pts)
The impulse response function of a LTI system is given by

$$h(t) = \cos(2\pi t)u(t) + \sin(4\pi t)u(t).$$

(a) (2 pts) Is the system causal or non-causal? Provide justification.

(b) (4 pts) Write down the Laplace transform of $h(t)$ and region of convergence (ROC).

(c) (9 pts) Find the output $y(t)$ of the system if input is
$x(t) = e^{2t}$, $t \in (-\infty, \infty)$.
(Hint: Use eigen-function property of LTI system.)