UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Midterm Examination II
March 7, 2017

INSTRUCTIONS:

- The exam has 5 problems and 13 pages.
- The exam is closed-book.
- Two cheat sheets of A4 size are allowed.
- Calculator is NOT allowed.
- Put your discussion session in the top-right corner ↗↗

Your name:______________________________

Student ID:______________________________

Table 1: Score Table

Problem	a	b	c	d	Score
1	4	4			8
2	6	6			12
3	8	4	3		15
4					15
5	8	3	5	4	20
Total					70

Problem 1 (8 pts)

State whether each of the following statements is **TRUE** or **FALSE** and justify your answers.

(a) (4 pts) Let $h(t)$ be a real function with finite energy. The convolution of $h(t)$ with $\cos(2\pi t)$ can be written as $A\cos(4\pi t - \theta)$ for some constants A and θ.

(b) (4 pts) The LTI causal system with transfer function

$$H(s) = \frac{se^{-s}}{s^2 - 2s + 2}$$

is BIBO stable.

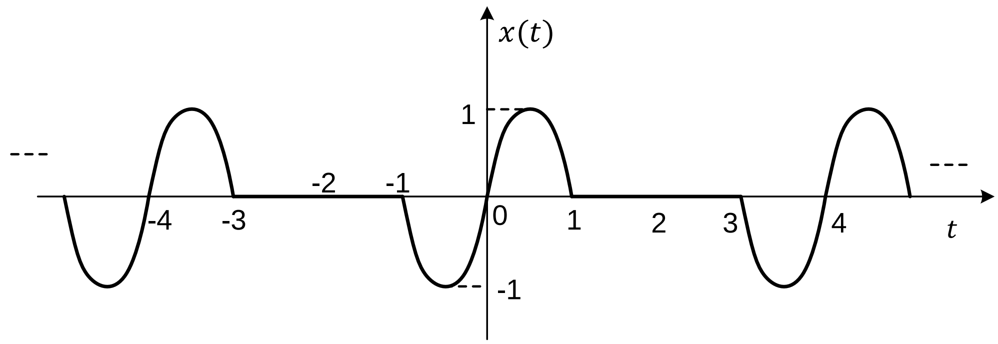
Problem 2 (12 pts)

Compute Laplace transform and corresponding region of convergence (ROC) of the following signals. Sketch pole-zero plot in s-plane.

(a) (6 pts) $x(t) = \cos(3t)u(t - 2\pi)$

(b) (6 pts) $y(t) = \int_0^t (t - \tau)^3 \cos(3\tau) d\tau$, $t > 0$

Problem 3 (15 pts)
Consider following periodic signal as shown in the figure. The signal can be expressed as $\sin(\pi t)$ for $t \in [-1, 1]$.



(a) (8 pts) Compute complex Fourier series coefficients X_k that satisfy

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\Omega_0 t}.$$

Hint: Use Euler's identity $\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$.

(b) (4 pts) Evaluate magnitude and phase of X_1 and X_{-1}.

(d) (3 pts) Express the signal as trigonometric Fourier series, i.e., find coefficients a_k and b_k such that

$$x(t) = X_0 + 2\sum_{k=1}^{\infty} a_k \cos(k\Omega_0 t) - 2\sum_{k=1}^{\infty} b_k \sin(k\Omega_0 t).$$

Problem 4 (15 pts)
The following information is given for a continuous-time periodic signal $x(t)$ with period $T_0 = 2$, where X_k is its Fourier coefficient

- $X_k = X_{-k}$
- $X_k = 0$ for $|k| \geq 3$
- $\frac{1}{2}\int_{-1}^{1} x(t)dt = 1$
- The value of the signal at time instant $t = 0.5$ is 3, i.e., $x(0.5) = 3$
- The power of signal is $\frac{1}{T_0}\int_0^{T_0} |x(t)|^2 dt = 3$

Find its Fourier series coefficients X_k and determine the time domain signal $x(t)$.

Problem 5 (20 pts)
Consider a cascaded LTI causal system S_1S_2 as follows

$$x(t) \to [S_1] \to y(t) \to [S_2] \to z(t)$$

The IPOP relation for S_1 is given by

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{d^2x(t)}{dt^2} + x(t), t > 0$$
$$y(0) = 0, x(0) = 0, y'(0) = 0, x'(0) = 0$$

The IPOP relation for S_2 is given by

$$z(t) = \int_0^\infty [\cos(\tau) + \sin(\tau)]\, y(t-\tau)d\tau$$

(a) (8 pts) Find the impulse response function of system S_1 and S_2, namely $h_1(t)$ and $h_2(t)$

(b) (3 pts) Find the transfer function of cascaded system $H_{12}(s)$

(c) (5 pts) Let $z(t)$ be the steady-state response due to periodic input signal

$$x(t) = 1 + 2\sin(t) + 2\cos(3t).$$

Find complex Fourier series coefficients of $z(t)$, namely Z_k.

(d) (4 pts) Evaluate the power of output signal $z(t)$ in part (c).
Hint: Use Parseval's power relation.