Winter 2010

EE102 Midterm Exam

1	10
2	10
3	5/10
4	10/10

35

1. Given the IO: $\frac{dv(t)}{dt} - 4v(t) = u(t)$

   a) Calculate the total response to the unit step input.

   b) Calculate the steady state response if any.

2. Given the second order IO: $\frac{d^2v(t)}{dt^2} + 2\frac{dv(t)}{dt} + v(t) = u(t)$

   a) Find the response to Zero input for given initial conditions: $v(0) = 0, v'(0) = 1$

   b) For what value of t is the response at its maximum?

   c) Is the system stable?

3. $\frac{d^2v(t)}{dt^2} + 5\frac{dv(t)}{dt} + 6v(t) = u(t)$

   a) Calculate the total response to the sinusoid of angular frequency w: $\sin(wt), \quad t \geq 0$

   b) Calculate the steady state response.

4. In problem 3,

   a) Express the amplitude of the steady state response in terms of w, at what value of w is the amplitude of the steady state response a maximum?

   b) In problem 3 what is the corresponding phase shift in terms of w?

(1) (a) $\frac{dV(t)}{dt} - 4V(t) = u(t)$.

Take the laplace transform of the two sides.

$sV(s) - V(0) - 4V(s) = U(s)$.

Assuming $V(0) = 0$.

we have. and $U(s) = \frac{1}{s}$ since $U(t)$ is the unit step.

$V(s)(s-4) = \frac{1}{s}$.

$\Rightarrow V(s) = \frac{1}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$.

$A = \frac{1}{s-4}\Big|_{s=0} = \frac{-1}{4}$

$B = \frac{1}{s}\Big|_{s=4} = \frac{+1}{4}$

$V(s) = -\frac{1}{4}\left(\frac{1}{s}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{s-4}\right)$ Take the inverse laplace Transform.

$\boxed{V(t) = -\frac{1}{4} + \frac{1}{4}e^{4t}}$ ✓ $\boxed{t \geq 0}$.

(b). For the steady state response we take $\lim_{t\to\infty} V(t)$

$V(t)_{t\to\infty} = \lim_{t\to\infty}\left(-\frac{1}{4} + \frac{1}{4}e^{4t}\right)$

$= -\frac{1}{4} + \frac{1}{4}\underbrace{\lim_{t\to\infty} e^{4t}}_{\to\infty} = \infty$. The limit

doesn't converge, it blows up.

$\Rightarrow$ There is no steady state response.

(2) (a) $\frac{d^2 v(t)}{dt^2} + 2\frac{dv(t)}{dt} + v(t) = u(t)$.

$s^2 V(s) - s\underbrace{v(0)}_{0} - \underbrace{v'(0)}_{1} + 2sV(s) - 2\underbrace{v(0)}_{0} + V(s) = 0$

$V(s)(s^2+2s+1) = 1$

$\Rightarrow V(s) = \frac{1}{s^2+2s+1} = \frac{1}{(s+1)^2}$

$t f(t) \longleftrightarrow -\frac{dF(s)}{ds}$

$e^{-t} \longleftrightarrow \frac{1}{s+1}$

$te^{-t} \longleftrightarrow \frac{1}{(s+1)^2}$

$\Rightarrow v(t) = te^{-t}$ This only include the zero input response because the input is zero. This is due to the initial response of the system (transient response).

(b) To find the maximum we take the derivative and set it to be zero.

$v'(t) = e^{-t} - t^2 e^{-t} = 0$.

$e^{-t}(1 - t^2) = 0$

we just consider the positive time

$\Rightarrow 1 - t^2 = 0 \Rightarrow t = \mp 1$ we just take the positive time

$\Rightarrow$ we have maximum response at $\boxed{t = 1}$.