**ECE102, Spring 2020** **Final Exam**

Signals & Systems Prof. A. Kadambi

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UCLA True Bruin academic integrity principles apply.

This exam is open book, open note and open internet. Collaboration is not allowed.

8:00 am Wednesday, 3 June 2020
- 8:00 am Thursday, 4 June 2020.

**Instructions for submission**

Submit your work with your answers on this question sheet. Please do not work on a separate paper.

State your assumptions and reasoning for all the questions.
No credit without reasoning.
Show all work on these pages.

Name: ____________________

Signature: ____________________

ID#: ____________________

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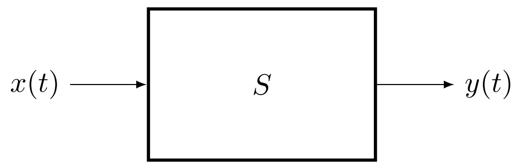
|---|---|---|
| Problem 1 | _____ | / 21 |
| Problem 2 | _____ | / 24 |
| Problem 3 | _____ | / 10 |
| Problem 4 | _____ | / 15 |
| Problem 5 | _____ | / 15 |
| Problem 6 | _____ | / 15 |
| BONUS | _____ | / 6 bonus points |
| Total | _____ | / 100 points + 6 bonus points |

1. **Signal and Systems Basics** (21 points)

(a) (12 points) **System properties.** Consider the following system **S** as shown below where $x(t)$ and $y(t)$ are the input and output pairs of the system, respectively.




For an input $x(t)$, the output $y(t)$ is given by,

$$y(t) = 5x(t+1) + 3\int_{-\infty}^{t-1} cos(t-\tau)x(\tau)d\tau$$

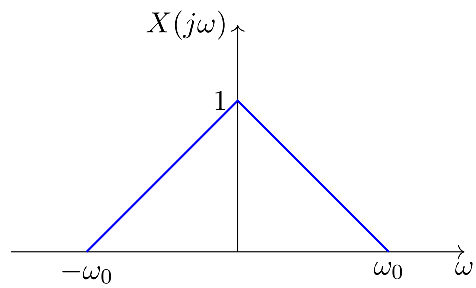
i. Is the system linear? Justify your answer.

ii. Is the system time invariant? Justify your answer.

iii. Is the system casual? Justify your answer.

iv. Find the unit step response of the system, $s(t)$ .

(b) (9 points) **Sampling basics.** A signal $x(t)$ has a band-limited spectrum $X(j\omega)$ as shown below, and zero phase at all frequencies. The signal $x(t)$ is sampled by a unit impulse train with a period $T$ to generate a sampled signal $x_s(t)$, with the spectrum $X_s(j\omega)$.



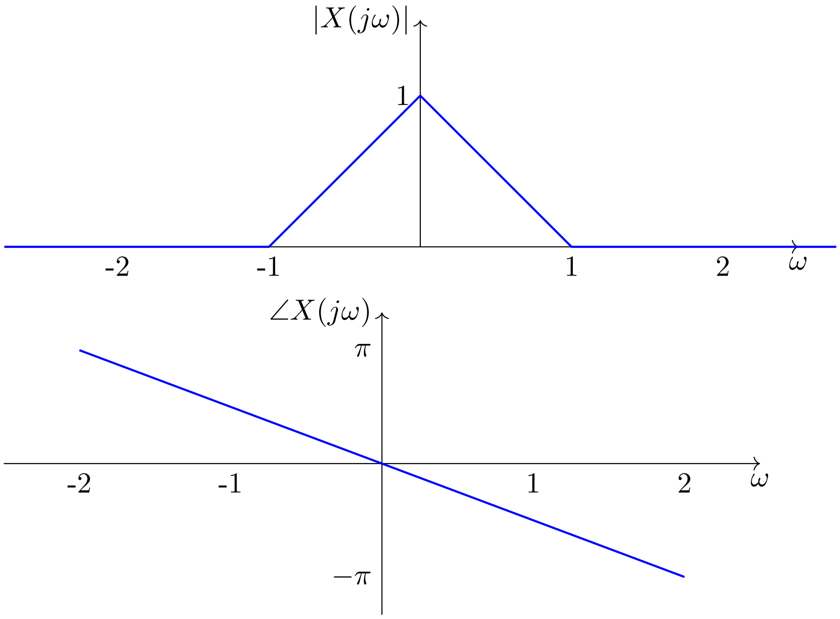

i. Plot the spectrum of $X_s(j\omega)$ when $\frac{2\pi}{T} = 2\omega_0$.

ii. Plot the spectrum of $X_s(j\omega)$ when $\frac{2\pi}{T} = \omega_0$.

iii. Find $x_s(t)$ in the case where $\frac{2\pi}{T} = \omega_0$.

2. **Fourier transform** (24 points)

(a) (12 points) A signal x(t) has has the following the Fourier Transform.




Plot the magnitude and phase plots for the Fourier Transform of the following signals:

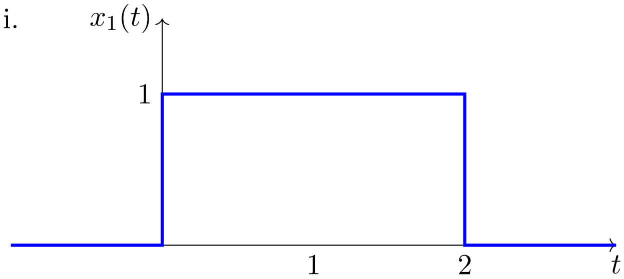
i. $x(2t)$

ii. $x(t - \frac{\pi}{2})$

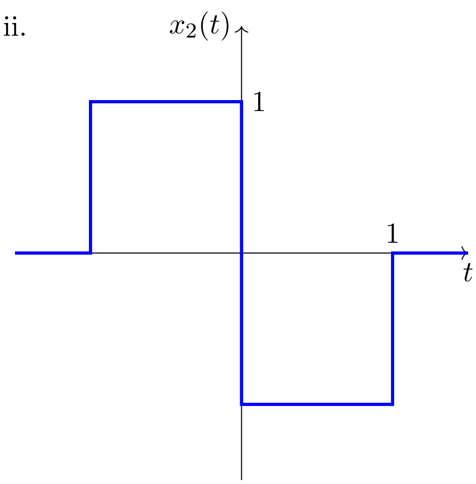
iii. $x(t) * x(t)$

(b) (12 points) Find the expression of the Fourier transform $X_i(j\omega)$ for the following $x_i(t)$:

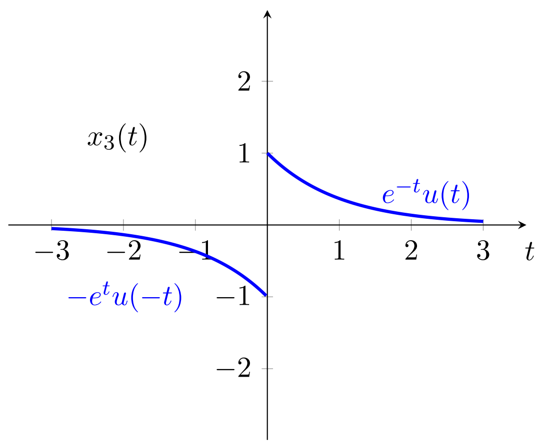
i.



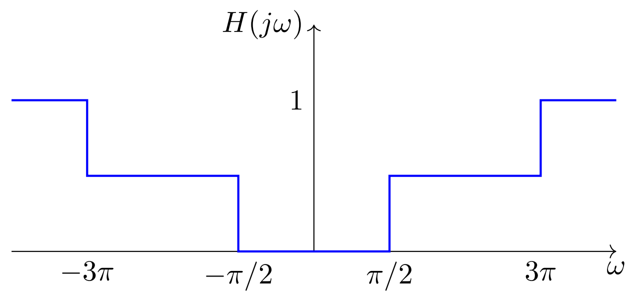

ii.



iii.



3. **LTI system** (10 points)
We have the following signal $x(t) = sin(\pi t) + cos^2(2\pi t)$. The $x(t)$ is input the into the LTI which is described by $H(j\omega)$ as shown below.

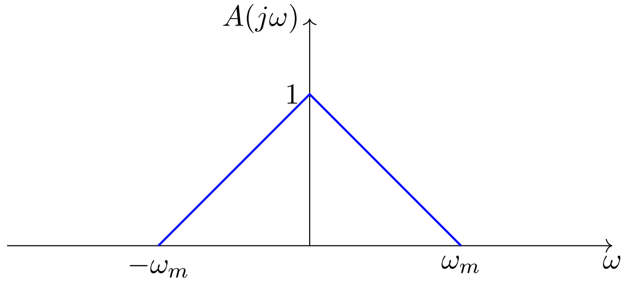


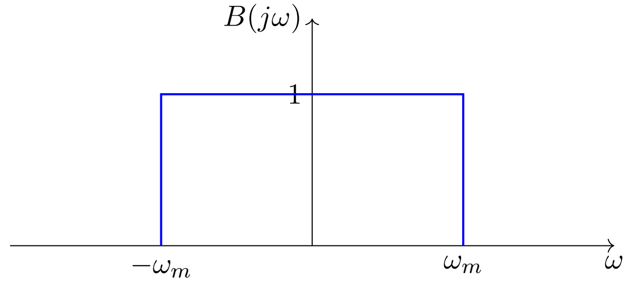

(a) (3 points) Determine the period of $x(t)$.

(b) (3 points) Determine the Fourier series coefficients, $c_k$ of $x(t)$.

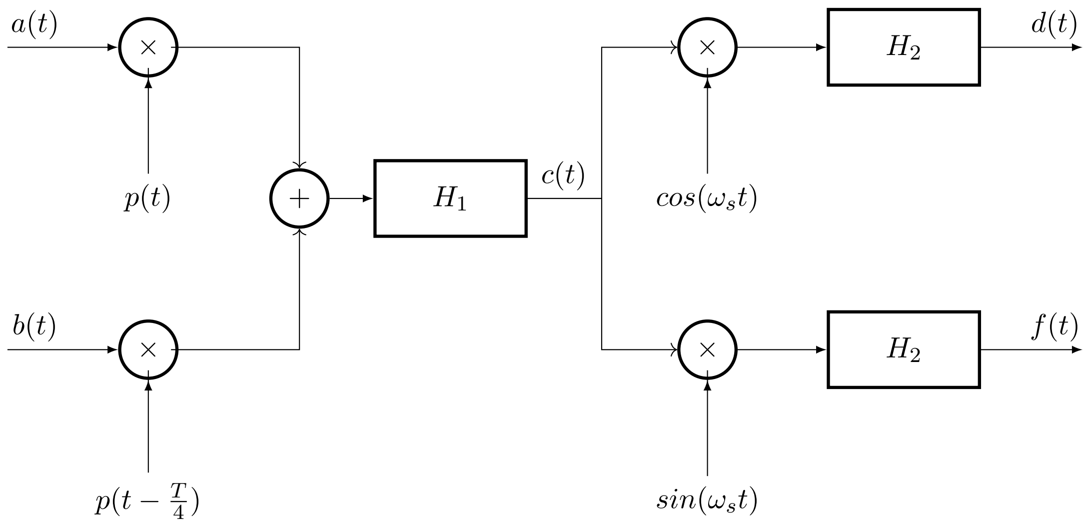
(c) (4 points) Determine the output of $y(t)$.

4. **Modulation and demodulation** (15 points)
It is given that input signals a(t) and b(t) are real and even, with the Fourier Transforms shown below.




Consider the system below:




Where we define the following filters:

$$H_1(j\omega) = \begin{cases} T, & \omega_s - \omega_m \leq |\omega| \leq \omega_s + \omega_m \\ 0, & otherwise. \end{cases}$$

$$H_2(j\omega) = \begin{cases} 1, & |\omega| \leq \omega_m \\ 0, & |\omega| > otherwise. \end{cases}$$

$$p(t) = \sum_{-\infty}^{\infty} \delta(t - nT)$$

$$\frac{2\pi}{T} = \omega_s > 2\omega_m$$

**Note:** You may not need to consider the amplitude scaling in this question.

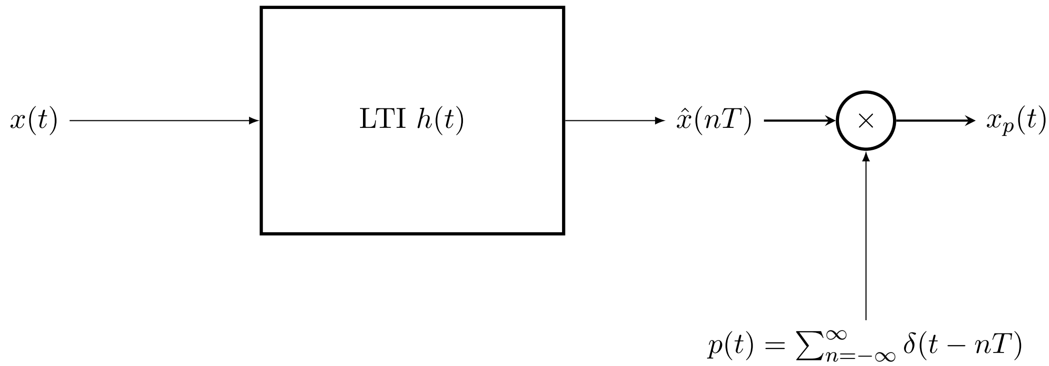
(a) (4 points) In class, we found the Fourier Transform of the impulse train using the Fourier Series. Using a similar argument, derive the Fourier Transform of the shifted impulse train, $p(t - \frac{T}{4})$.

(b) (4 points) Plot the Real and Imaginary components of $C(j\omega)$ (the Fourier Transform of the output signal of the bandpass filter $H_1$).

(c) (4 points) Plot the Real and Imaginary Components of $D(j\omega)$ and $F(j\omega)$

(d) (3 points) Describes d(t) in terms of either a(t) or b(t).

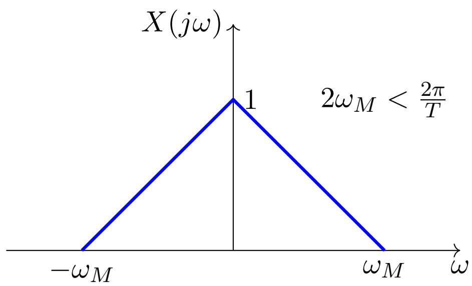
5. **Non-ideal sampling** (15 points)
For the ideal sampling system, the area under the impulse at $t = nT$ would be $x(nT)$. In this non-ideal sampling system the output is an impulse train $x_p(t)$ where the area under the impulse at $t = nT$ is the average value of the input $x(t)$ on the interval $nT - \Delta \leq t \leq nT + \Delta$. The non-ideal sampling system is further illustrated in the following diagram and equations:




$$x_p(t) = \sum_{n=-\infty}^{n=\infty} \hat{x}(nT)\delta(t - nT)$$
$$\hat{x}(nT) = \frac{1}{2\Delta} \int_{t=nT-\Delta}^{nT+\Delta} x(\tau)d\tau$$

(a) Determine $h(t)$.

(b) Suppose $X(j\omega)$, the Fourier transform of the band-limited input $x(t)$ is as shown below.



Express $X_p(j\omega)$, the Fourier transform of $x_p(t)$, in terms of $X(j\omega)$.

(c) Find the maximum value for $\Delta$, such that no frequency component of $x(t)$ is lost due to the non-ideal sampling process.

6. **Laplace transform** (15 points)

A casual LTI system can be described by the following differential equation:

$$y''(t) + 2y'(t) + y(t) = 2x(t) + 3x''(t).$$

You may assume resting initial conditions (y(0)= 0, y'(0) =0, y"(0) = 0)

(a) Find the transfer function $H(s)$.

(b) What is the impulse response h(t) of this system?

(c) Given that $\int_{-\infty}^{\infty} x(t)dt = 2.5$, compute $Y(s)$ at $s = 0$.

**Bonus question** (6 points)

(a) (sound) You work at a company that has flown in a famous pianist by the name of Bruinhoven who will play a high-pitched version of the UCLA Fight song. The lowest note that the score asks for is 1000 Hz. In the digital recording the song sounds very low pitched, and you hear off-key notes that are in the hundreds of Hertz - much less than the lowest note in the score. Your boss believes Bruinhoven just had a bad day and played poorly. However, when you were listening in the room to Bruinhoven, the song sounded perfect. When you analyze the recording you also find something curious - these off-key notes are at unusual frequencies that do not map to the specific keys of the piano. What do you think is the most likely explanation, and what is your evidence?

(b) (light) Can you rediscover the intuition behind a Nobel Prize in Physics? Lasers are like superpowered lightbulbs and tools for humanity. We can shoot lasers far into space or use powerful lasers to perform eye surgery. Unfortunately, lasers do not come in many colors. In the early days of its invention, lasers used to be available at infrared frequencies (topping out at 250 THz). This was not as useful since humans can only perceive frequencies of light from 400 THz to 700 THz. Look up the frequency range of what we perceive as "green". Given the infrared laser tech you have on hand and the tools from this class, how might you generate a green laser?

(c) (light) In this question, you will play with a simple while very effective trick (or philosophy) in signal and systems. The application scenario of this trick is photoplethysmogram (PPG). A PPG is an optically obtained plethysmogram that can be used to detect blood volume changes in the microvascular bed of tissue. A PPG is often obtained by using a pulse oximeter which illuminates visible light onto the skin and measures changes in light absorption using a camera. The blood volume changes can be further utilized for determining the vital signals like heartbeat rate. In industry, some companies try to use the blinking LED instead of continuous LED as illumination source for PPG. How do you think it might help improve the performance of PPG (i.e., what is the trick behind it)?