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EE 101B
Spring 2017 Midterm
Wednesday, May 3, 2017

Name: ______________________________

Student ID Number: ______________________________

Honor Pledge:

"I have neither given nor received aid on this examination, nor have I concealed any violation of the Honor Code.

Date: __________ Signature: ______________________________

Problem 1:

A 1 GHz left-hand circularly polarized plane wave with an electric field modulus of 100 mV/m is normally incident in air upon a nonmagnetic medium with $\varepsilon_r = 2.25$, $\sigma = 10^{-4}$ S/m and occupies the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

(a) $E_0^i = 0.1$ V/m

LHC $\Rightarrow \delta = \frac{\pi}{2}$

$$\tilde{E} = \left(\hat{x}\frac{\sqrt{2}}{2} + \hat{y}\frac{\sqrt{2}}{2}\cdot e^{\frac{\pi}{2}j}\right) E_0^i e^{-jkz} e^{j\phi}$$

$$k = \omega/c = \frac{20}{3}\pi$$

$$\vec{E}(t) = E_0^i \frac{\sqrt{2}}{2}\left(\hat{x}\cos(\omega t - kz + \phi) + \hat{y}\cos(\omega t - kz + \tfrac{\pi}{2} + \phi)\right)$$

$$\vec{E}\Big|_{t=0,\,z=0} = E_0^i \frac{\sqrt{2}}{2}\left(\hat{x}\cos\phi + \hat{y}\cos(\tfrac{\pi}{2} + \phi)\right)$$

$$= E_0^i \frac{\sqrt{2}}{2}\left(\hat{x}\cos\phi - \hat{y}\sin\phi\right)$$

∵ magnitude of $\vec{E}\Big|_{t=0,z=0}$ is constant,

∴ any ϕ will satisfy, let's set $\phi = 0$

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$$\therefore \tilde{E} = \left(\hat{x}\frac{\sqrt{2}}{2} + j\hat{y}\frac{\sqrt{2}}{2}\right) 0.1\, e^{-j\frac{20\pi}{3}z}$$

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(b) For second medium, $\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega \epsilon_r \epsilon_0} << 1$, ∴ ~~low-loss approximation~~

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = 251\ \Omega$$

$$\eta_1 = 377\ \Omega$$

$$\therefore \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.2$$

reflection coefficient

$$\tau = 1 + \Gamma = 0.8$$

transmission coefficient

→

(c) $\tilde{E}^r = -(\hat{x}\frac{\sqrt{2}}{2} + j\hat{y}\frac{\sqrt{2}}{2})\, 0.02\, e^{+j\frac{20\pi}{3}z}$

$\tilde{E}^t = (\hat{x}\frac{\sqrt{2}}{2} + j\hat{y}\frac{\sqrt{2}}{2})\, 0.08\, e^{-j10\pi z}$

$k_2 = k_1\sqrt{\varepsilon_r} = 10\pi$ ↗

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$\tilde{E}^{tot} = \tilde{E} + \tilde{E}^r$

$= (\hat{x}\frac{\sqrt{2}}{2} + j\hat{y}\frac{\sqrt{2}}{2})\, 0.1\, e^{-j\frac{20\pi}{3}z}$

$-(\hat{x}\frac{\sqrt{2}}{2} + j\hat{y}\frac{\sqrt{2}}{2})\, 0.02\, e^{+j\frac{20\pi}{3}z}$

(d) reflected percentage is

$|\Gamma|^2 = 0.04$

transmitted percentage is

$|\tau|^2 \frac{\eta_1}{\eta_2} = 0.96$

Problem 2

A 1mW/m^2 parallel-polarized electromagnetic wave is incident from air onto glass at the Brewster angle. $\varepsilon_{r-gl} = 2.25$.

a) What is the amplitude of the transmitted electric field? Explain through calculation, how it is not a violation of the conservation of energy that the transmitted field is different from the incident field even though no wave is reflected.

b) If the incident wave is a mixture of 30% parallel polarized wave and 70% perpendicular polarized wave, what portion of the incident power would be transmitted to medium 2?

(a) $|S_{av,i}| = \dfrac{|E_0^i|^2}{2\eta_1} = 10^{-3}$ W/m²

$\therefore |E_0^i| = \sqrt{2\eta_1 \times 10^{-3}} = 0.868$ V/m

Okay, so I forgot the formula for Brewster angle, let's assume its θ_B for now. (I will see if I can find it out later)

OK.

$\therefore \theta_B = \theta_i$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \sqrt{\frac{1}{\epsilon_r}}$$

$\therefore \theta_t = \sin^{-1}\left(\frac{1}{1.5}\sin\theta_B\right)$

$$\tau_{\parallel} = \frac{2\eta_2\cos\theta_t}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$\eta_2 = \sqrt{\dfrac{\mu}{\epsilon_r\epsilon_0}}$ $\quad \eta_1 = \sqrt{\dfrac{\mu}{\epsilon_0}}$

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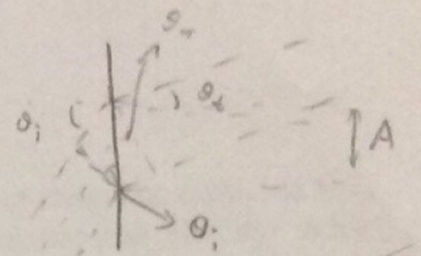
$\therefore \eta_2 = \eta_1 \dfrac{1}{1.5}$

$$\therefore \tau_{\parallel} = \frac{\frac{2}{1.5}\cos\theta_t}{\frac{1}{1.5}\cos\theta_t + \cos\theta_i} = \frac{4/3\cos\theta_t}{\frac{2}{3}\cos\theta_t + \cos\theta_i}$$

→

$\therefore |E_0^t| = \tau_{\parallel} |E_0^i|$



$$S_{av1} = \frac{|E_0^i|^2}{2\eta_1} \Rightarrow P_{av1} = \frac{|E_0^i|^2}{2\eta_1} A_1 = \frac{|E_0^i|^2}{2\eta_1} A\cos\theta_i$$

$$S_{av2} = \frac{|\tau E_0^i|^2}{2\eta_2} \Rightarrow P_{av2} = \frac{|\tau E_0^i|^2}{2\eta_2} A_2 = \frac{|\tau E_0^i|^2}{2\eta_1} A\cos\theta_t$$

$$\frac{P_{av2}}{P_{av1}} = |\tau|^2 \frac{\eta_1}{\eta_2} \frac{\cos\theta_t}{\cos\theta_i}$$

$\therefore$ even if $\tau \neq 1$, P_{av2} still can be equal to P_{av1} due to $\frac{\eta_1}{\eta_2}\frac{\cos\theta_t}{\cos\theta_i}$

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(b) parallel-polarized wave will all be transmitted,
only part of perpendicular-polarized wave will be reflected

find $T_{\perp} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$

$\therefore$ portion of reflected wave is $0.7|T_{\perp}|^2$

$\therefore$ portion of transmitted wave is $1 - 0.7|T_{\perp}|^2$ ×

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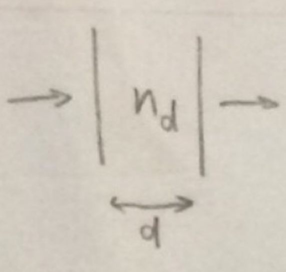
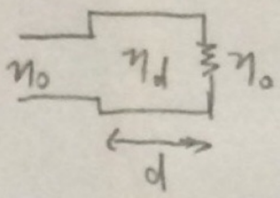
Problem 3

An optical beam at a 600 nm wavelength is normally incident on a non-magnetic dielectric slab. Both sides of the dielectric slab are filled with air. Determine the thickness and refractive index of the dielectric slab that allow transmission of 75% of the incident optical beam through the slab into air.





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$$\eta_{in} = \eta_d \frac{\eta_0 + j\eta_d \tan\beta d}{\eta_d + j\eta_0 \tan\beta d} \quad 5'$$

$$\Gamma = \frac{\eta_{in} - \eta_0}{\eta_{in} + \eta_0} \quad 3'$$

75% transmission $\Rightarrow$ 25% reflection $\Rightarrow \Gamma = \pm\frac{1}{2}$

$\therefore$ if $\Gamma = -\frac{1}{2}$

$$\eta_{in} - \eta_0 = -\frac{1}{2}\eta_{in} - \frac{1}{2}\eta_0$$

$$\Rightarrow \eta_{in} = \frac{1}{3}\eta_0$$

$$\eta_d = \frac{\eta_0}{\sqrt{\epsilon_r}}$$

$$\frac{1}{\sqrt{\epsilon_r}} \frac{1 + j\frac{1}{\sqrt{\epsilon_r}}\tan\beta d}{\frac{1}{\sqrt{\epsilon_r}} + j\tan\beta d} = \frac{1}{3}$$

$$\Rightarrow \frac{1}{\sqrt{\epsilon_r}} + \frac{1}{\epsilon_r} j\tan\beta d = \frac{1}{3}\frac{1}{\sqrt{\epsilon_r}} + \frac{1}{3}\frac{1}{\sqrt{\epsilon_r}} j\tan\beta d$$

For two sides to approach to be equal, $\frac{1}{\epsilon_r} = \frac{1}{3}\frac{1}{\sqrt{\epsilon_r}}$, $\tan\beta d \to \infty$

$$\Rightarrow \begin{cases} \epsilon_r = 9 \\ \beta d = \frac{\pi}{2} + n\pi \end{cases}$$

$$\beta = \frac{2\pi}{\lambda_d}$$

$$\therefore \pi\frac{2d}{\lambda_d} = \frac{\pi}{2} + n\pi$$

$$d = \frac{1}{4}\frac{\lambda}{\sqrt{\epsilon_r}} + \frac{n}{2}\frac{\lambda}{\sqrt{\epsilon_r}}$$ ← thickness

$$= 50 + n \cdot 100 \ [nm]$$

for $n = 0, 1, 2, \ldots$

$n_d = \sqrt{\epsilon_r \mu_r} = 3$

↑ refractive index