EC ENGR101A-1
Mid-term (Total: 40 points)
Due: 2/9/21, 11:59 pm PST

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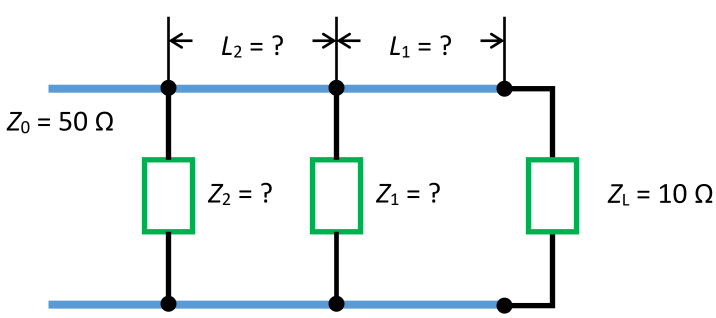
I, [redacted]________, hereby, agree with the code of honor specified above.

Question #1 (8 points)

Please describe in your own words what key concepts have you learned for transmission lines.

Question #2 (8 points)

A 10-Ω load can be matched to a 50-Ω lossless transmission line by placing two shunt components Z_1 ($\neq \infty$) and Z_2 ($\neq \infty$) at distances L_1 ($\neq 0$) and $L_1 + L_2$ ($L_2 \neq 0$) respectively from the load. Determine their values using the Smith chart. (*Note: there are infinitely many solutions; any one of such solutions will do providing that you clearly indicate how your solution is obtained on the Smith chart.*)



Question #3 (8 points)

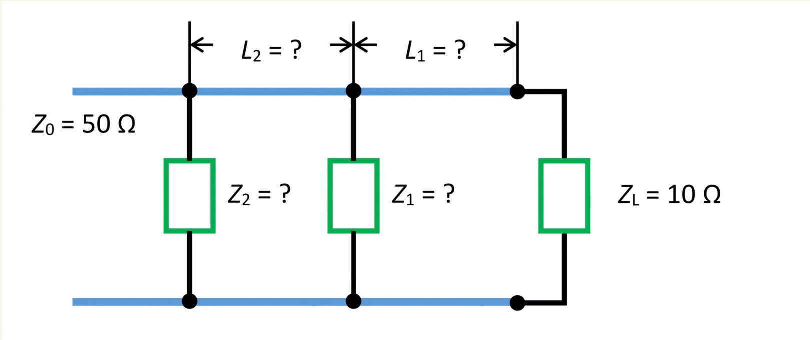
Given the vector field $\mathbf{A} = \hat{\boldsymbol{\phi}} \sin(\phi/2)$, verify Stoke's theorem over the hemispherical surface and its circular contour that are shown in the following figure.

1) Please describe in your own words what key concepts have you learned for transmission lines.

A transmission line is something used in order to connect two different pieces of equipment, or in more general terms two points, and the transmission line is used to transfer energy or information between those two points.
Note: A transmission line will have two ports or terminals. One end of the transmission is connected to a source, which is the provider of energy, while the other is connected to the load, which takes in the energy. The transmission line transfers the energy. Some examples where we can see transmission lines used are power amplifiers, antenna rays, and filters. In a power amplifier we can see transmission lines used in order to guide the signal through the power amplifier. With a transmission line the information flowing through will travel in a sinusoidal wave shape. The wave can be modeled using a general expression that includes an attenuation constant, phase constant, angular frequency and a reference phase. An important factor to note is that for a pair of wires to be considered a transmission line, the wire line should be about 10 percent of the operating wavelength.

There are two main types of transmission lines, traverse electromagnetic(TEM) and higher order lines. The TEM moves in the x-y plane (where it propagates from) while the high order line has a component in all 3 xyz directions. When analyzing a transmission line we can use a lumped element model. Some parameters that can be seen in a transmission lines are the resistance, inductance, conductance and capacitance. As we find these parameters, we need to adjust for to see if the transmission line is coaxial, two-wire, or a parallel plate. As we analyze the transmission line, we can the wave equation for both voltage and current. These can be found with the help of Kirchhoff's voltage and current laws. As the wave travels in a sinusoidal time-domain we need to use phasors in order to accurate pinpoint the voltage and current of the wave. The voltage and current are related by a characteristic called the impedance. The impedance can have both real and imaginary parts but in lossless lines, the impedance is reduced to a real number. A lossless line is a line with no line resistance and no dielectric loss. The impedance can be found using a smith chart and it allows for transmission lines to be manipulated. The smith chart has both the impedance and admittance grads which also us to check values for circuits. In order to use a smith chart we have to understand how to normalize the impedance and use a reflection coefficient, which is portrayed by gamma. We can rotate the smith chart and it rotates in a clockwise fashion.

2)



Note: $Z_0 = 50\ \Omega$

$Z_L = 10\ \Omega$

Normalized

$$\bar{Z}_L = \frac{Z_L}{Z_0} = \frac{10\ \Omega}{50\ \Omega} = 0.2$$

↑ on smith chart

$\bar{Z}_{L,1} = 5.0$ rotate 0.25λ

↓

$Z_{L,1} = Z_0(\bar{Z}_{L,1})$

$Z_{L,1} = (50\ \Omega)(5.0)$

$= 250\ \Omega$

if we use $Z_1 = 375$

$$Z_{in,1} = \left(\frac{1}{Z_1} + \frac{1}{Z_{L,1}}\right)^{-1} = \left(\frac{1}{375} + \frac{1}{250}\right)^{-1} = 150$$

$$\bar{Z}_{in,1} = \frac{Z_{in,1}}{Z_0}$$

$$= \frac{150\ \Omega}{50\ \Omega}$$

$= 3$

↓

rotate by 0.5λ

$\bar{Z}_{in,1} = \bar{Z}_{L,2} = 3.0$

$Z_{L,2} = (50\ \Omega)(3) = 150\ \Omega$

↓

$Z_{in,2} = Z_0 = 50\ \Omega$

$$Z_{in,2} = \left(\frac{1}{Z_2} + \frac{1}{Z_{L,2}} \right)^{-1}$$

$$\frac{1}{50} = \frac{1}{Z_2} + \frac{1}{150}$$

$$\frac{1}{50} - \frac{1}{150} = \frac{1}{Z_2}$$

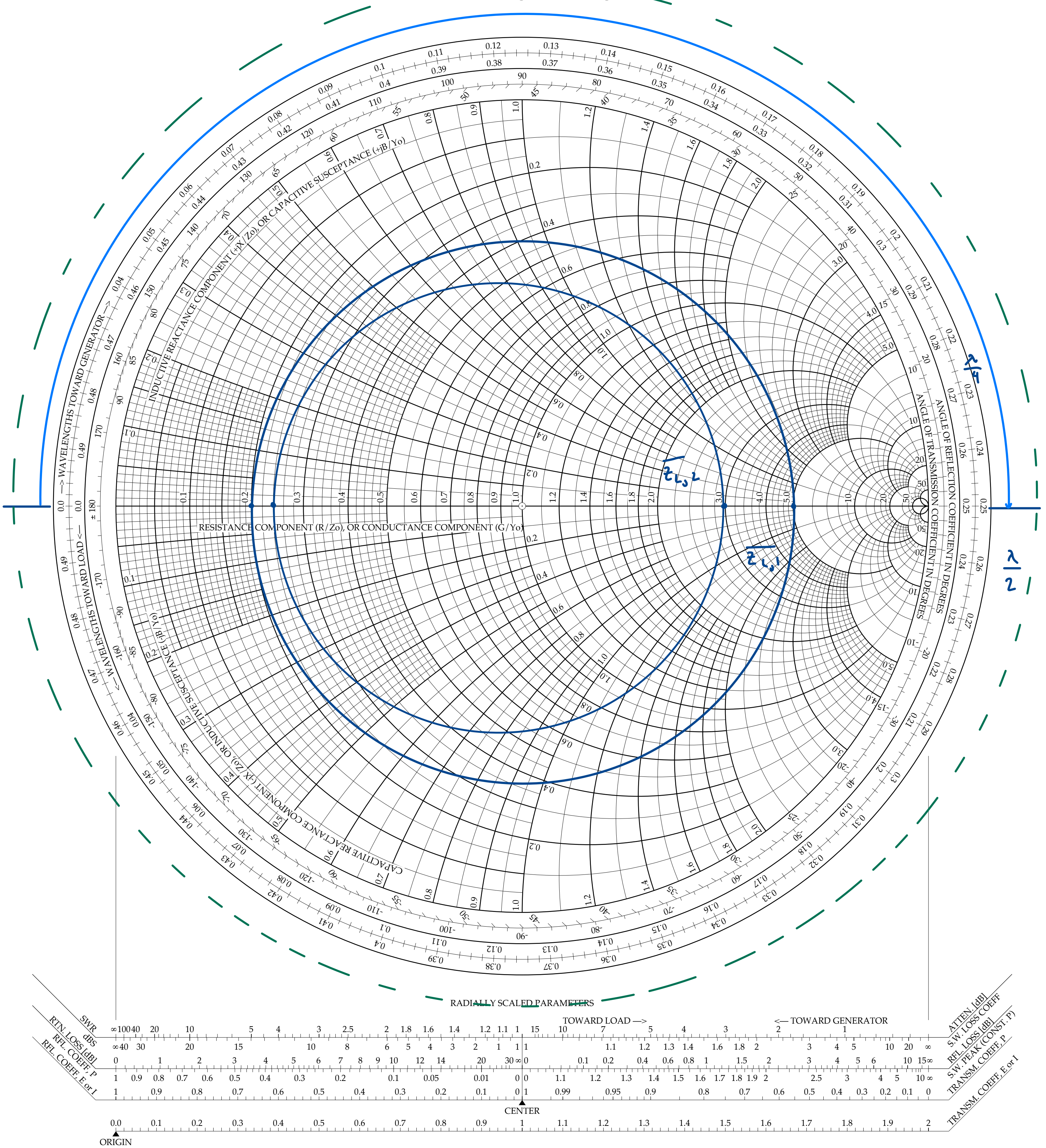
$$\frac{3}{150} - \frac{1}{150} = \frac{1}{Z_2}$$

$$Z_2 = \frac{150}{2} = 75\,\Omega$$

$$\boxed{\begin{aligned} l_1 &= 0.25\lambda \\ Z_1 &= 150\,\Omega \\ l_2 &= 0.5\lambda \\ Z_2 &= 75\,\Omega \end{aligned}}$$

The Complete Smith Chart

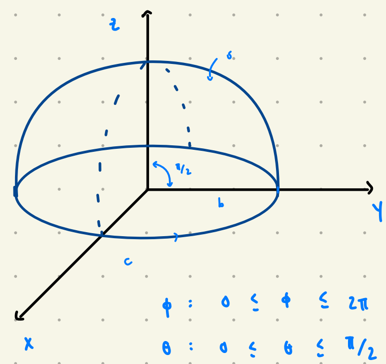
Black Magic Design



#3) $A = \hat{\phi}\sin(\phi/2)$ verify stokes theorem

stokes theorem

$$\int_S (\nabla \times \vec{B}) \, d\vec{S} = \oint_C \vec{B} \cdot d\vec{l}$$

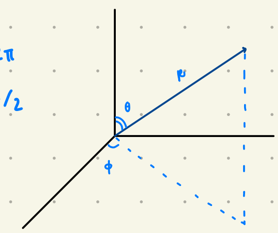


WTS $A = \hat{\phi} \sin\left(\frac{\phi}{2}\right)$

$\phi : 0 \leq \phi \leq 2\pi$

$\theta : 0 \leq \theta \leq \pi/2$

$r : b$



RHS

$$\oint_C \vec{B} \, d\vec{l}$$

$$\vec{B} = \hat{\phi} \sin\left(\frac{\phi}{2}\right)$$

$$d\vec{l} = \hat{r}\,dr + \hat{\theta}\,r\,d\theta + \hat{\phi}\,r\sin\theta\,d\phi$$

$$\vec{B}\,d\vec{l} = \left(\hat{\phi}\sin\left(\frac{\phi}{2}\right)\right)\left(\hat{r}\,dr + \hat{\theta}\,r\,d\theta + \hat{\phi}\,r\sin\theta\,d\phi\right)$$

$$= 0 + 0 + r\sin\theta \sin\left(\frac{\phi}{2}\right) d\phi$$

$$= r\sin\theta \sin\left(\frac{\phi}{2}\right) d\phi$$

$$\oint_C \vec{B}\,d\vec{l} = \int_{\phi=0}^{2\pi} r\sin\theta \sin\frac{\phi}{2}\,d\phi \Big|_{\theta=\pi/2} \Big|_{r=b}$$

$$= \int_{\phi=0}^{2\pi} r\,\sin(\pi/2)\,\sin\left(\frac{\phi}{2}\right) d\phi \Big|_{r=b}$$

$$= \int_{\phi=0}^{2\pi} r\,(1)\,\sin\left(\frac{\phi}{2}\right) d\phi \Big|_{r=b}$$

$$= \int_{\phi=0}^{2\pi} b \sin\left(\frac{\phi}{2}\right) d\phi$$

$$= b\int_{\phi}^{2\pi} \sin\left(\frac{\phi}{2}\right) d\phi$$

$$= -b\cos\left(\frac{\phi}{2}\right)\left(\frac{1}{1/2}\right)\Big|_0^{2\pi}$$

$$= -2b\cos\left(\frac{\phi}{2}\right)\Big|_0^{2\pi}$$

$$= -2b\left(\cos\left(\frac{2\pi}{2}\right) - \cos(0)\right)$$

$$= -2b(-1 - 1)$$

$$= -2b(-2)$$

$$= 4b$$

$$\boxed{\therefore RHS = 4b}$$

LHS

$$\int_S (\nabla \times \vec{B})\, d\vec{s}$$

$$\nabla \times \vec{B} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & \hat{\theta}\, r & \hat{\phi}\, r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \underset{A_r}{0} & \underset{rA_\theta}{r(0)} & \underset{r\sin\theta\, A_\phi}{r\sin\theta\, \sin\left(\frac{\phi}{2}\right)} \end{vmatrix}$$

Note: $A = \hat{\phi}\, \sin\left(\frac{\phi}{2}\right)$

$$= \hat{r}\, \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta} A_\phi \sin\theta - \frac{\partial}{\partial\phi} A_\theta\right] + \hat{\theta}\, \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi} A_r - \frac{\partial}{\partial r} r A_\phi\right]$$

$$+ \hat{\phi}\, \frac{1}{r}\left[\frac{\partial}{\partial r} r A_\theta - \frac{\partial A_r}{\partial\theta}\right]$$

Note: $r = b$

$$= \hat{r}\, \frac{1}{b\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\left(\frac{\phi}{2}\right)\sin\theta - \cancel{\frac{\partial}{\partial\phi} 0}^{\,0}\right) + \hat{\theta}\, \frac{1}{b}\left[\cancel{\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}(2)}^{\,0} - \cancel{\frac{\partial}{\partial r} b \sin\left(\frac{\phi}{2}\right)}^{\,0}\right]$$

$$+ \hat{\phi}\, \frac{1}{b}\left[\cancel{\frac{\partial}{\partial r} b\,(0)}^{\,0} - \cancel{\frac{\partial}{\partial\theta}(0)}^{\,0}\right]$$

$$= \hat{r}\, \frac{1}{b\sin\theta}\sin\left(\frac{\phi}{2}\right)\cos\theta + 0 + 0$$

$$= \hat{r}\, \frac{1}{b\sin\theta}\sin\frac{\phi}{2}\cos\theta$$

$$d\vec{s} = \hat{r}\, r^2 \sin\theta\, d\theta\, d\phi + \hat{\theta}\, r\sin\theta\, dr\, d\phi + \hat{\phi}\, r\, dr\, d\theta$$

$\downarrow$

$$(\nabla \times \vec{B})\, d\vec{s} = r^2 \cancel{\sin\theta}\, d\theta\, d\phi \cdot \frac{1}{b\cancel{\sin\theta}} \sin\frac{\phi}{2}\cos\theta$$

$$= \frac{r^2}{b}\sin\frac{\phi}{2}\cos\theta\, d\theta\, d\phi$$

Note: $r = b$

$$= \frac{b^{\cancel{2}}}{\cancel{b}}\sin\frac{\phi}{2}\cos\theta\, d\theta\, d\phi$$

$$= b\sin\frac{\phi}{2}\cos\theta\, d\theta\, d\phi$$

$$\int_S (\nabla \times \vec{B}) \, d\vec{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} b \sin \frac{\phi}{2} \cos\theta \, d\theta \, d\phi$$

$$= \int_{\phi=0}^{2\pi} b \sin\left(\frac{\phi}{2}\right) \left(\int_{\theta=0}^{\pi/2} \cos\theta \, d\theta \right) d\phi$$

Aside: $\int_{\theta=0}^{\pi/2} \cos\theta \, d\theta = \sin\theta \Big|_0^{\pi/2}$

$= \sin\left(\frac{\pi}{2}\right) - \sin(0)$

$= (1 - 0)$

$= 1$

$$= \int_{\phi=0}^{2\pi} b \sin\left(\frac{\phi}{2}\right) (1) \, d\phi$$

$$= b \int_{\phi=0}^{2\pi} \sin \frac{\phi}{2} \, d\phi$$

$$= b(-2) \cos \frac{\phi}{2} \Big|_{\phi=0}^{2\pi}$$

$$= -2b(\cos\pi - \cos 0)$$

$$= -2b(-1 - 1)$$

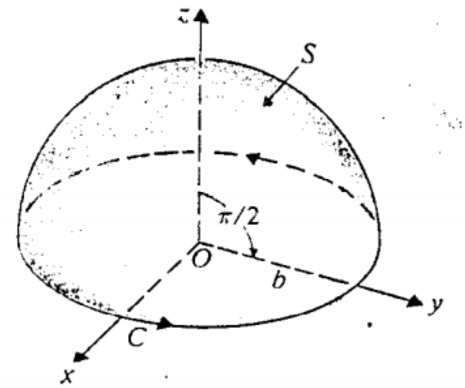
$$= -2b(-2)$$

$$= 4b$$

$\therefore$ LHS $= 4b$

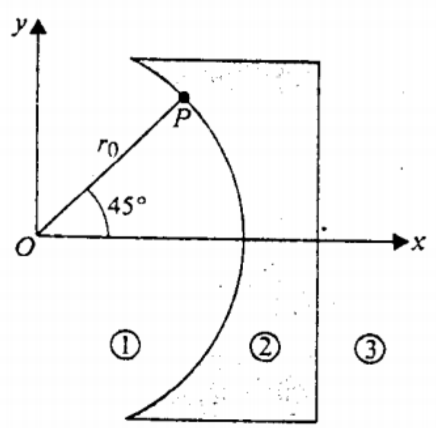
B/c LHS = RHS

↳ $\therefore$ we have verified stoke's theorem



Question #4 (8 points)

Dielectric lenses can be used to collimate electromagnetic fields. In the following figure, the left surface of the lens is that of a circular cylinder, and the right surface is a plane. If $\mathbf{E}_1$ at point $P(r_0, 45^\circ, z)$ in region 1 is $\hat{\mathbf{r}}7 - \hat{\boldsymbol{\phi}}3$, what must be the dielectric constant of the lens in order that $\mathbf{E}_3$ in region 3 is parallel to the x-axis?

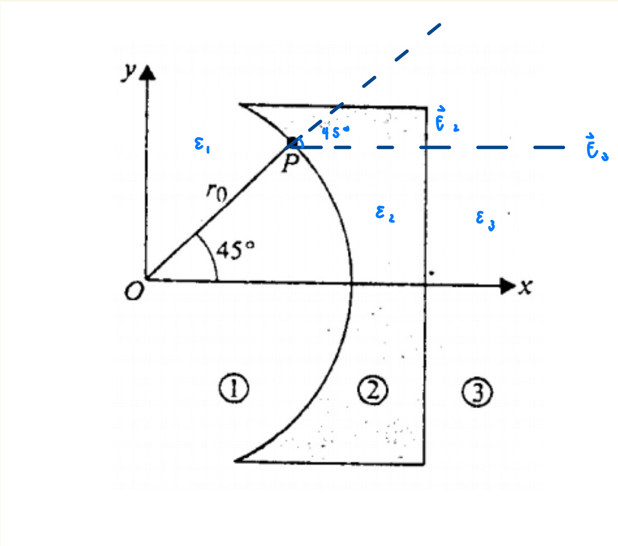


Question #5 (8 points)

Given a uniform sphere of charge (centered at origin) of radius R_0 and volume charge density ρ. In addition, the point at infinity acts as zero potential reference.

a) Find the potential at a point outside the sphere ($R > R_0$).
b) Find the potential at a point inside the sphere ($R \leq R_0$).
c) Verify the solution obtained in part a) satisfies Laplace's equation.
d) Verify the solution obtained in part b) satisfies Poisson's equation.

4)



$P(r_0, 45°, z)$ $\quad E_1 = \hat{r}7 - \hat{\phi}3$

want $E_3 \parallel$ x-axis

↳ parallel

$$\vec{E}_1 = \hat{r}E_{1r} + \hat{\phi}E_{1\phi}$$
$$\vec{E}_2 = \hat{r}E_{2r} + \hat{\phi}E_{2\phi}$$
$$\vec{E}_3 = \hat{r}E_{3r} + \hat{\phi}E_{3\phi}$$

$\longrightarrow$ $\vec{E}_1 = \hat{r}7 - \hat{\phi}3$

↳ $E_{1r} = 7$

↳ $E_{1\phi} = -3$

Due to permitivity

$$\varepsilon_3 = \varepsilon_1 = \varepsilon_0$$

$$\varepsilon_2 = \varepsilon_{lens} \cdot \varepsilon_0$$

boundary and surface of dielectric lense

if $\vec{E}_3 \parallel$ x-axis

↳ $\vec{E}_2 \parallel$ x-axis

$$\vec{E}_3 \cdot \hat{y} = 0$$
$$\vec{E}_2 \cdot \hat{y} = 0$$

] b/c parallel to x axis

↳ in regions 2 and 3

converting

$$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$$
$$\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$$

$$(\hat{r}\, E_{2r} + \hat{\phi}\, E_{2\phi}) \cdot (\hat{r} \sin\phi + \hat{\phi} \cos\phi) =$$

$$\downarrow$$

$$E_{2r} \sin\phi + E_{2\phi} \cos\phi = 0$$

point $P(r_0, 45°, z)$

$$\phi = 45°$$

$$\downarrow$$

$$E_{2r} \sin 45° + E_{2\phi} \cos 45° = 0$$

$$E_{2r} \frac{\sqrt{2}}{2} + E_{2\phi} \frac{\sqrt{2}}{2} = 0$$

$$\frac{\sqrt{2}}{2} (E_{2r} + E_{2\phi}) = 0$$

$$E_{2r} + E_{2\phi} = 0$$

$$E_{2r} = -E_{2\phi}$$

By observation of region 1 and 2

$\hookrightarrow$ $E_{1\phi} = E_{2\phi}$

$P(r_0, 45°, z)$ $\quad \vec{E}_1 = \hat{r}7 - \hat{\phi}3$

$$\downarrow$$

$$E_{1\phi} = -3$$

$E_{2\phi} = -3$ $\quad$ B/c $E_{2\phi} = -E_{2r}$

$$E_{2r} = -(-3) = 3$$

electric flux density

$$\varepsilon_1 E_1 \hat{r} = \varepsilon_2 E_2 \hat{r}$$

$\varepsilon_1 E_{1r} = \varepsilon_2 E_{2r}$ $\qquad \varepsilon_2 = \varepsilon_{lens} \cdot \varepsilon_0$

$$\varepsilon_1 E_{1r} = \varepsilon_{lens}\, \varepsilon_0\, E_{2r}$$

$$\varepsilon_{lens} = \frac{E_{1r}}{E_{2r}} = \frac{7}{3}$$

$$\boxed{\varepsilon_{lenses} = \frac{7}{3} \text{ or } 2.33}$$

uniform sphere of charge of radius R_0 and volume charge density ρ

5a) potential at point outside sphere $R > R_0$

$$\oint_S E_R \, ds = E_R \cdot 4\pi R^2 = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$Q_{\text{enclosed}} = \rho \, V_{\text{enclosed}}$

$$\downarrow$$

$$Q_{\text{encl}} = \rho \, \frac{4}{3} \pi R_0^3$$

$$\oint_S E_R \, ds = E_R \cdot 4\pi R^2 = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\Big(\underbrace{\rho \, \frac{4}{3}\pi R_0^3}_{Q_{\text{encl}}}\Big)$$

$$\oint_S E_R \, ds = \frac{\rho \, 4\pi R_0^3}{3\varepsilon_0} = E_R \, 4\pi R^2$$

$$\downarrow$$

$$E_R = \frac{\rho \, \cancel{4\pi} R_0^3}{3\varepsilon_0 \, \cancel{4\pi} R^2} \hat{r} = \frac{\rho R_0^3}{3\varepsilon_0 R^2} \hat{r}$$

$$V_R = -\int_\infty^R \frac{\rho R_0^3}{3 R^2 \varepsilon_0} \, dR$$

$$= (-1) \frac{\rho R_0^3}{3\varepsilon_0} \int_\infty^R \frac{1}{R^{-2}} \, dR$$

Aside: $\int_\infty^R \frac{1}{R^{-2}} = \frac{R^{-1}}{-1} \Big|_\infty^R = (-1)\left(\frac{1}{R} - \cancel{\frac{1}{\infty}}^{\,0}\right) = -\frac{1}{R}$

$$= \cancel{(-1)} \frac{\rho R_0^3}{3\varepsilon_0} \frac{\cancel{(-1)}}{R}$$

$$\boxed{V_R = \frac{\rho R_0^3}{3\varepsilon_0 R}, \quad (R > R_0)}$$

5b) potential a point inside the sphere ($R \le R_0$)

$$\oint_S E_R \, ds = E_R \cdot 4\pi R^2 = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$Q_{encl} = \rho V_{encl} = \rho \frac{4}{3} \pi R^3$$

$$\frac{Q_{encl}}{\varepsilon_0} = \frac{\rho 4\pi R^3}{3 \varepsilon_0} = \oint E_R \, ds$$

$$E_R 4\pi R^2 = \oint E_R \, ds$$

$$\Downarrow$$

$$E_R 4\pi R^2 = \frac{\rho 4\pi R^3}{3\varepsilon_0}$$

$$E_R = \frac{\rho 4\pi R^3}{3\varepsilon_0} \frac{1}{4\pi R^2}$$

$$= \frac{\rho R}{3\varepsilon_0}$$

$$V_R = -\int E_R \, dR = -\int \frac{\rho R}{3\varepsilon_0} dR$$

$$= \frac{-\rho}{3\varepsilon_0} \int R \, dR$$

$$= \frac{-\rho}{3\varepsilon_0} \frac{R^2}{2}$$

$$\boxed{V_R = \frac{-\rho R^2}{6\varepsilon_0} \; ; \quad R \le R_0}$$

5c) verify part a satisfies Laplace's Eqn

Laplace Eqn

$$\nabla^2 V = 0$$

$$\downarrow$$

$$V = \frac{\rho R_0^3}{3 \varepsilon_0 R} = \frac{\rho R_0^3}{3 \varepsilon_0} R^{-1}$$

$$\frac{\partial}{\partial R} V = \frac{\rho R_0^3}{3 \varepsilon_0} (-1) R^{-2} \longrightarrow \frac{(-1) \rho R_0^3}{3 \varepsilon_0 R^2}$$

$$\frac{\partial^2}{\partial R^2} V = \frac{\rho R_0^3}{3 \varepsilon_0} (-1)(-2) R^{-3} \longrightarrow \frac{2 \rho R_0^3}{3 \varepsilon_0 R^3}$$

$$\nabla \cdot \nabla V = \frac{\partial^2}{\partial R} V + \frac{2}{R} \frac{\partial}{\partial R} V$$

$$= \frac{2 \rho R_0^3}{3 \varepsilon_0 R^3} + \frac{2}{R} \cdot \frac{(-1) \rho R_0^3}{3 \varepsilon_0 R^2}$$

$$= \frac{2 \rho R_0^3}{3 \varepsilon_0 R^3} - \frac{2 \rho R_0^3}{3 \varepsilon_0 R^3}$$

$$= 0$$

$\therefore \nabla^2 V = 0$

Laplace eqn is satisfied

5a) verify part b satisfy's poisson equation

poisson eqn

$$\nabla^2 V = \frac{-\rho_v}{\varepsilon} = \frac{-\rho}{\varepsilon_0}$$

$$V = \frac{-\rho R^2}{6\varepsilon_0}$$ from part b

$$\frac{\partial}{\partial R} V = \frac{-\rho}{6\varepsilon_0} 2R \rightarrow \frac{-\rho R}{3\varepsilon_0}$$

$$\frac{\partial^2}{\partial R^2} V = \frac{-\rho}{3\varepsilon_0}$$

$$\nabla(\nabla V) = \frac{\partial^2}{\partial R} V + \frac{2}{R} \frac{\partial V}{\partial R}$$

$$= \frac{-\rho}{3\varepsilon_0} + \frac{2}{\cancel{R}} \frac{-\rho \cancel{R}}{3\varepsilon_0}$$

$$= \frac{-\rho}{3\varepsilon_0} - \frac{2\rho}{3\varepsilon_0}$$

$$= (-1) \cancel{\frac{(1+2)}{3}}^{1} \frac{\rho}{\varepsilon_0}$$

$$\boxed{\therefore \nabla^2 V = \frac{-\rho}{\varepsilon_0}}$$ poisson's eqn is satisfied