SOLUTION

UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Winter 2015
Quiz 1, January 26 2015, (20 minutes)

Name ______________________________ Student number_________________________

This is a closed book quiz – no notes or equations.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Electric Field	50	
Problem 2	Capacitance	50	
Total		100	

Maxwell's Equations:

$$\nabla\cdot\mathbf{D}=\rho_f$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\cdot\mathbf{B}=0$$

$$\nabla\times\mathbf{H}=\mathbf{J}_f+\frac{\partial\mathbf{D}}{\partial t}$$

Auxillary Fields:

$$\mathbf{D}=\varepsilon_0\mathbf{E}+\mathbf{P}$$

$$\mathbf{H}=\frac{\mathbf{B}}{\mu_0}-\mathbf{M}$$

In linear media:

$$\mathbf{P}=\varepsilon_0\chi_e\mathbf{E} \qquad \mathbf{D}=\varepsilon\mathbf{E} \qquad \varepsilon=\varepsilon_0(1+\chi_e)$$

$$\mathbf{M}=\chi_m\mathbf{H} \qquad \mathbf{B}=\mu\mathbf{H} \qquad \mu=\mu_0(1+\chi_m)$$

Electrostatic Potential: $\mathbf{E}=-\nabla V$ Vector potential: $\mathbf{B}=\nabla\times\mathbf{A}$

Gradient Theorem: $\int_a^b(\nabla f)\cdot d\mathbf{l}=f(b)-f(a)$

Divergence Theorem: $\int_V(\nabla\cdot\mathbf{A})\,dV=\oint_S\mathbf{A}\cdot d\mathbf{S}$

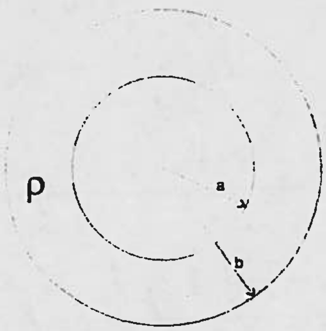
Stokes's Theorem: $\int_S(\nabla\times\mathbf{A})\cdot d\mathbf{S}=\oint_C\mathbf{A}\cdot d\mathbf{l}$

Electric energy density: $W_e=\frac{1}{2}\mathbf{E}\cdot\mathbf{D}$ or $W_e=\frac{1}{2}\varepsilon E^2$ (in linear media)

Magnetic energy density: $W_m=\frac{1}{2}\mathbf{B}\cdot\mathbf{H}$ or $W_m=\frac{1}{2}\mu H^2$ (in linear media)

Capacitance: $C=\frac{Q}{V}$ Inductance: $L=\frac{\Lambda}{I}=N\frac{\Phi}{I}$

1.



Consider a spherical shell of charge (volume density ρ). $\varepsilon=\varepsilon_0$ everywhere. When giving answers, don't forget the vector direction.

(a) What is the E-field for $R<a$?

$$\oint_S \vec{E}\cdot d\vec{S} = \int_V \frac{\rho}{\varepsilon_0} dV = 0 \Rightarrow \boxed{E=0 \text{ for } R<a}$$

(b) What is the E-field field for $R>b$?

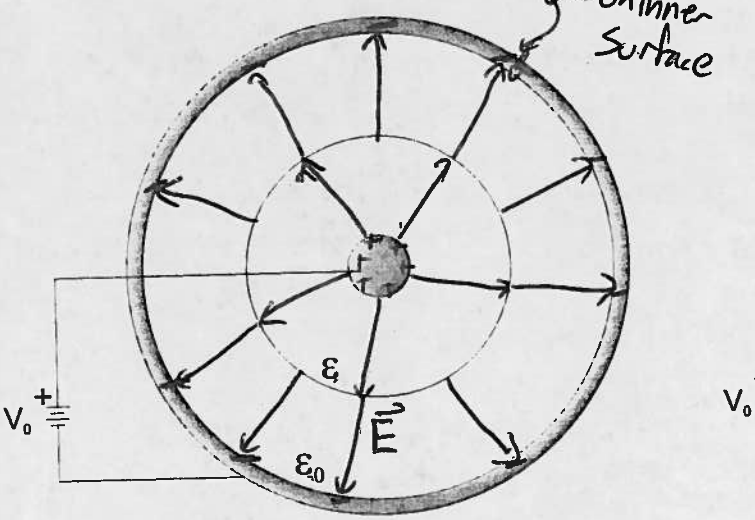
$$\oint_S \vec{E}\cdot d\vec{S} = \int_V \frac{\rho}{\varepsilon_0} dV$$

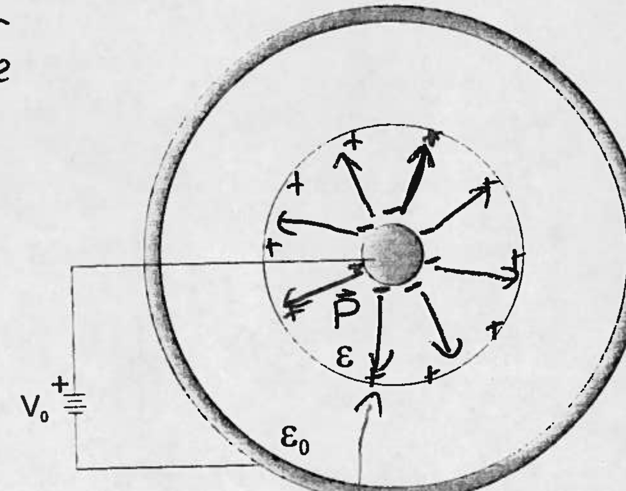
Choose spherical Gaussian surface $R>b$

$$E_r 4\pi R^2 = \int_a^b dR \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\, R^2 \sin\theta\, \rho/\varepsilon_0 = \frac{4}{3}\pi(b^3-a^3)\rho/\varepsilon_0$$

$$\boxed{\vec{E} = \hat{R}\,\frac{(b^3-a^3)\rho}{3\varepsilon_0 R^2}}$$

2. Consider a coaxial capacitor with a potential difference V_0 applied between the center and outer conductor. In between the conductors, there is a dielectric core surrounding the inner wire of permittivity ε, which is itself surrounded by vacuum. On the left figure, sketch the electric field lines **E** inside the dielectric, and the location and sign of the free charge. On the right side, sketch the polarization field lines **P**, and sketch the location and sign of the bound charge. **Please be precise and neat!**





E-field is stronger in vacuum at interface between regions since εE_n is continous at interface between $\varepsilon/\varepsilon_0$.

$\varepsilon_1 E_{1,n} = \varepsilon_2 E_{2,n}$

Bound surface charge on dielectric. $\vec{P}=0$ in vacuum.

GRADIENT, DIVERGENCE, CURL & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$