# 22W-EC ENGR-100-LEC-1 Midterm Exam

TOTAL POINTS

**92.5 / 100**

QUESTION 1

1 Q1 **4 / 4**

✓ **- 0 pts Correct**

**- 2 pts** 1.1 wrong

**- 2 pts** 1.2 wrong

**- 4 pts** wrong

**- 0.5 pts** one blank of a part wrong

QUESTION 2

2 Q2 **3 / 6**

**- 0 pts** Correct

**- 1 pts** Part 1: Partially incorrect node - voltage or mesh analysis

✓ **- 1 pts Part 2: Rth wrong, Partially Incorrect Thevenin equivalent circuit**

✓ **- 1 pts Part 2: Vth calculation wrong, Partially Incorrect Thevenin equivalent circuit**

✓ **- 1 pts Part 3: R_N calculation wrong , Partially Incorrect Norton equivalent circuit**

**- 1 pts** Part 3: I_N calculation wrong, Partially Incorrect Norton equivalent circuit

**- 2 pts** Part 1 solution missing

**- 2 pts** Part 2 solution missing

**- 2 pts** Part 3 solution missing

QUESTION 3

3 Q3 **6 / 6**

✓ **- 0 pts Correct**

**- 1 pts** Part 1: partially incorrect - incorrectly identified series and parallel combinations or calculation mistake

**- 1 pts** Part 2: partially incorrect - incorrectly identified series and parallel combinations or calculation mistake

**- 1 pts** Part 3: partially incorrect - incorrectly identified series and parallel combinations or calculation mistake

**- 2 pts** Part 1 solution missing

**- 2 pts** Part 2 solution missing

**- 2 pts** Part 3 solution missing

QUESTION 4

4 Q4 **10 / 10**

✓ **- 0 pts Correct**

**- 10 pts** solution missing

**- 5 pts** incorrect KVL / KCL analysis

**- 2 pts** partially incorrect : calculation mistake, wrong input

QUESTION 5

5 Q5 **10 / 10**

✓ **- 0 pts Correct**

**- 10 pts** No submission

**- 3 pts** incorrect KCL / KVL application

**- 1 pts** incorrect or not determined relation between V1 and V2 : V1 - V2 = 10V

**- 1 pts** minor mistake, not calculated the final ans

**- 5 pts** incomplete

QUESTION 6

6 Q6 **10 / 10**

✓ **- 0 pts Correct**

**- 10 pts** no submission

**- 2 pts** Part 1: i(0) wrong or missing

**- 2 pts** Part 2: i(infinity) wrong or missing

**- 2 pts** Part 3: partially incorrect or incomplete

**- 4 pts** Part 3: completely incorrect or missing major steps

**- 6 pts** Part 3: no submission

QUESTION 7

7 Q7 **10 / 10**

✓ **- 0 pts Correct**

**- 10 pts** no submission

**- 2 pts** Part 1: incorrect or incomplete KCL / KVL analysis

**- 2 pts** Part 2: incorrect or missing expression of i_s

**- 2 pts** Part 3: Incorrect or missing expression of power absorbed by R3

**- 3 pts** Part 1: no work shown

**- 3 pts** Part 2: no work shown

**- 3 pts** Part 3: no work shown

QUESTION 8

8 Q8 **14 / 14**

✓ **- 0 pts Correct**

**- 14 pts** no submission or wrong

**- 4 pts** Part 1 : incorrect DC steady state circuit

**- 2 pts** Part 2: incorrect voltage across C1

**- 2 pts** Part 2: incorrect voltage across C2

**- 2 pts** Part 3 : incorrect current through L1

**- 2 pts** Part 3: incorrect current through L2

**- 5 pts** Part 2 missing

**- 5 pts** Part 3 missing

**- 2 pts** Part 1 partially incorrect

**- 1 pts** minor mistake

QUESTION 9

9 Q9 **12 / 14**

**- 0 pts** Correct

**- 2 pts** incorrect or missing value for Vc for t < 1 sec (steady state capacitance)

**- 2 pts** incorrect or missing value of i(t) for t < 1 sec

**- 2 pts** incorrect value of Vc at t -> infinity

**- 2 pts** incorrect total expression of Vc(t)

✓ **- 2 pts incorrect or incomplete expression for i(t) for t > 1**

**- 1 pts** minor mistake in the final expression of current i(t)

**- 14 pts** solution missing (no equation or calculation)

QUESTION 10

10 Q10 **13.5 / 16**

**- 0 pts** Correct

**- 1.5 pts** incorrect v(0+) or missing

✓ **- 1.5 pts incorrect dv(0+)/dt or missing**

**- 0.5 pts** Part 2: incorrect differential equation, KCL equation correct

✓ **- 1 pts incorrect or missing general solution for underdamped system**

**- 3 pts** Part 3) damping type incorrect or solution missing

**- 3 pts** Part 2: no solution

**- 16 pts** no solution : no steps shown

**- 3 pts** major mistakes in v(t) expression or missing major steps

**- 2 pts** minor mistake in v(t) expression or missing final expression

**- 1.5 pts** Part 2: KCL equation incorrect

**- 7 pts** Part 4: no solution

**- 4 pts** Part 4: no calculation of alpha, w, A1, A2 (or all four incorrect)

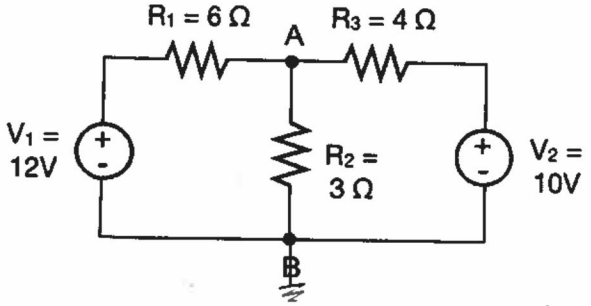
**100 Midterm Exam (4-5:50pm, 10 questions)** Name: [redacted]

Question#1 (4 points, 2 points for each subquestion). Fill in the blanks:

1. The Voltage of a capacitor and the Current of an inductor don't permit a sudden jump.
2. The Voltage of a current source and the Current of a voltage source are undetermined.

Question#2 (6 points, 2 points for each subquestion).

1. Solve for the current $I_{AB}$
2. Find the Thévenin equivalent circuit between A and B
3. Find the Norton equivalents circuit between A and B




D $\frac{(12-V_a)}{6} + \frac{(10-V_a)}{4} = \frac{V_a}{3}$.

$24 - 2V_a + 30 - 3V_a = 4V_a$

$9V_a = 54$

$V_a = 6\,V$

$V_{ab} = 6\,V$

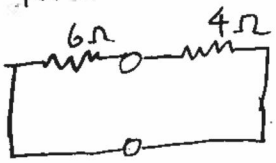
$I_{ab} = \frac{V_{ab}}{3}$

$= \frac{6V}{3} = \boxed{2A}$

$V_{oc}$: $\frac{12-V_a}{6} = \frac{10-V_a}{4}$, $24-2V_a = 30-3V_a$.

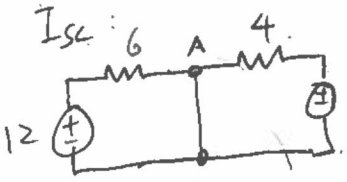
$V_a = \frac{54}{5} = 10.8\,V$ ✓

Rth




$6//4 = \frac{6\cdot4}{6+4} = \boxed{2.4\,\Omega}$

Isc:



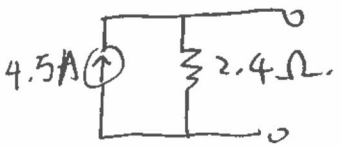

$\frac{12}{6} + \frac{10}{4} = \boxed{4.5\,A}$

$V_{oc} = I_{sc} \cdot R_{th}$

$= \boxed{10.8\,V}$

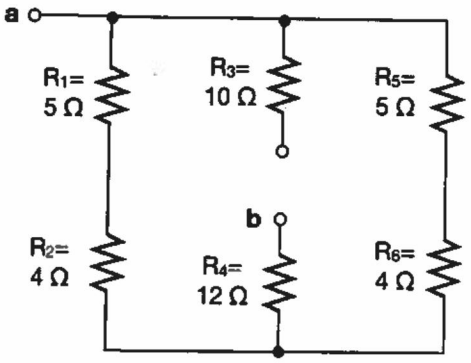
2) Thévenin


3) Norton

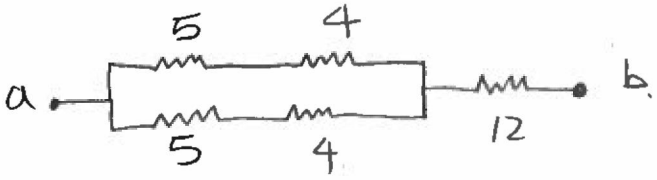


Question#3 (6 points, 2 points for each subquestion)

1. Find the equivalent resistance $R_{ab}$ between points a and b
2. Replace all resistors with inductors (replace Ω with mH too), find the equivalent inductance $L_{ab}$ between points a and b
3. Replace all resistors with capacitors (replace Ω with μF too), find the equivalent inductance $C_{ab}$ between points a and b




1).

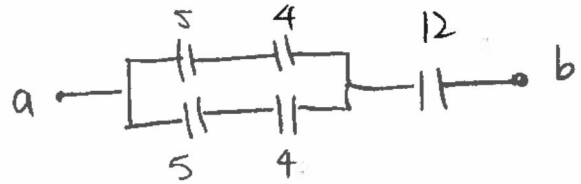



$9 // 9 = \frac{9 \cdot 9}{9+9} = 4.5\Omega$.

$R_{ab} = 12 + 4.5 = \boxed{16.5\Omega}$

2) $\boxed{16.5\ mH}$. (same maths as R)

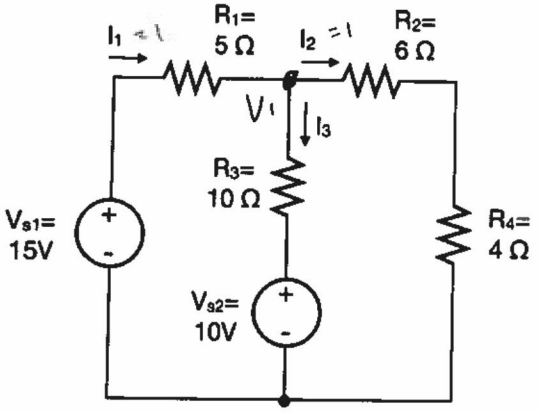
3).




$\left(\frac{5 \cdot 4}{5+4}\right) + \left(\frac{5 \cdot 4}{5+4}\right) = \frac{40}{9}\Omega$

$\frac{\frac{40}{9} \cdot 12}{\frac{40}{9} + 12} = \frac{120}{37} \approx \boxed{3.24\ \mu F}$

Question#4 (10 points). Find $I_3$




$$I_1 = \frac{15 - V_1}{5}$$

$$I_2 = \frac{V_1}{6+4}$$

$$I_3 = \frac{V_1 - 10}{10}$$

$$\frac{15 - V_1}{5} = \frac{V_1}{10} + \frac{V_1 - 10}{10}$$

$$2(15 - V_1) = 2V_1 - 10$$

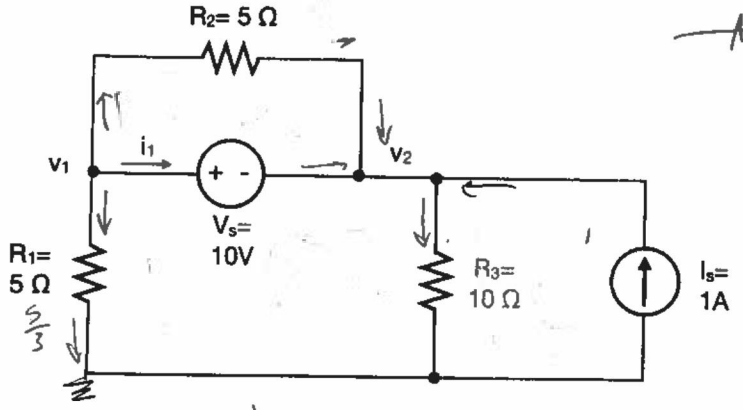
$$30 - 2V_1 = 2V_1 - 10$$

$$4V_1 = 40$$

$$V_1 = 10$$

$$I_3 = \frac{10 - 10}{10} = \boxed{0A}$$

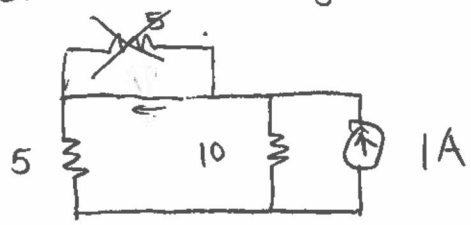
Question#5 (10 points). Find $i_1$




~~Mesh-current~~

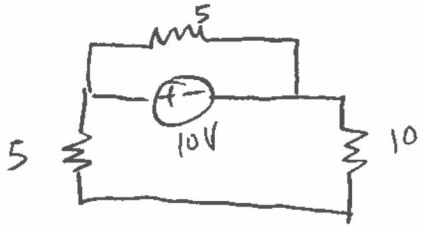
Superposition

zero out Voltage source




$i_1 = 0A$. $\quad i_1 = -1 \cdot \frac{10}{10+5} = -\frac{2}{3} A$

Zero out current source.




$15 // 15 = \frac{15 \cdot 5}{15+5} = \frac{75}{20} = 3.75\Omega$

$i_1 = -\frac{10V}{3.75\Omega} = -\frac{8}{3} A$

$\sum i = -\frac{2}{3} - \frac{8}{3} = \boxed{-\frac{10}{3} A}$

check: $i_1 + \frac{(V_1 - V_2)}{5} + \frac{V_1}{5} = 0$

$i_1 + \frac{(V_1 - V_2)}{5} + 1 = \frac{V_2}{10}$

$V_1 - V_2 = 10V$

$i_1 + 2 + \frac{4}{3} = 0$

$i_1 + 3 = -\frac{1}{3}$

$i_1 = -\frac{10}{3} A$ ✓

$i_1 + 2 + \frac{V_1}{5} = 0$

$i_1 + 4 = \frac{V_1}{10}$

$\frac{V_1}{10} - 4 = -\frac{V_1}{5} - 2$

$V_1 - 40 = -2V_1 - 20$

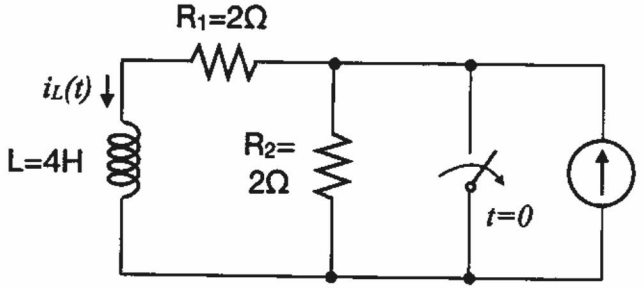
$3V_1 = 20V$

$V_1 = \frac{20}{3} V$

$V_2 = -\frac{10}{3} V$

Question#6 (10 points). The switch shown in the circuit has been closed for a long time prior to t = 0, then it opens at t = 0.

1. Find $i_L(0\text{-})$ (2 points)
2. Find $i_L(\infty)$ (2 points)
3. Find $i_L(t)$ for t > 0 (6 points)

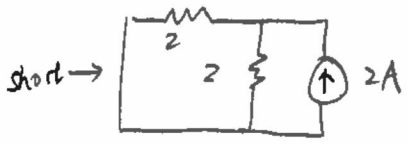



1) $i_L(0^-) = 0A$

current doesn't reach inductor due to short

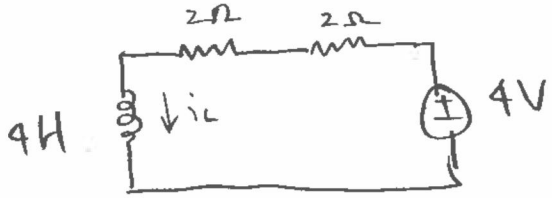
2) $i_L(\infty) = 2 \cdot \frac{2}{2+2} = 1A$

current division




L acts as short in steady state.

3) src. transformation.




$i_L(t) = K_1 + K_2 e^{st}$

$4V = 4 i_L(t) + 4 \frac{di_L(t)}{dt}$

$4V = 4K_1 + 4K_2 e^{st} + 4sK_2 e^{st}$

$K_1 = 1$

$(4s+4) K_2 e^{st} = 0$

$s = -1$

$i_L(0) = 0A$

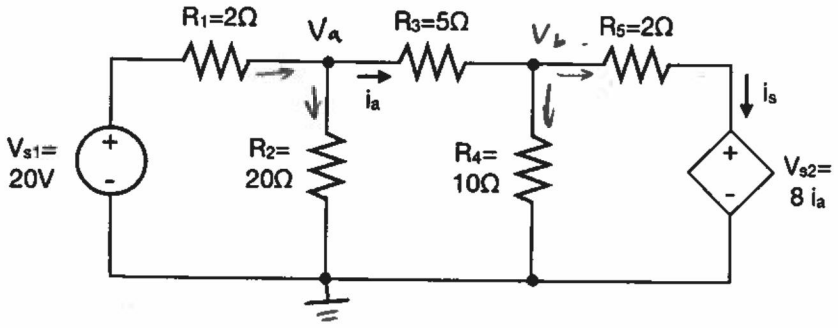
$K_1 + K_2 \cdot 1 = 0$

$K_2 = -1$

$i_L(t) = 1 - e^{-t}$

Question#7 (10 points).

1. Find the voltage across $R_2$ (3 points)
2. Find $i_s$ (3 points)
3. Find the power absorbed by $R_3$ (4 points)




$$\begin{cases} \frac{20-V_a}{2} = \frac{V_a}{20} + \frac{V_a-V_b}{5} \\ \frac{V_a-V_b}{5} = \frac{V_b}{10} + \frac{(V_b-8i_a)}{2} \end{cases}$$

$\rightarrow 2V_a - 2V_b = V_b + 5V_b - 40i_a$

$2V_a = 8V_b - 8(V_a - V_b)$

$10V_a = 16V_b$.

$V_a = 1.6V_b$

$\rightarrow \frac{20-1.6V_b}{2} = \frac{1.6V_b}{20} + \frac{0.6V_b}{5}$

$200 - 16V_b = 1.6V_b + 2.4V_b$.

$20V_b = 200$

$V_b = 10V$.

$V_a = 1.6V_b = 16V$.

$i_a = \frac{V_a - V_b}{5} = \frac{16-10}{5} = 1.2A$.

1) $R_2 = V_a = \boxed{16V.}$

2) $i_s = \frac{V_b - 8i_a}{2\Omega}$

$= \frac{10 - 9.6}{2\Omega}$

$= \boxed{0.2A}$

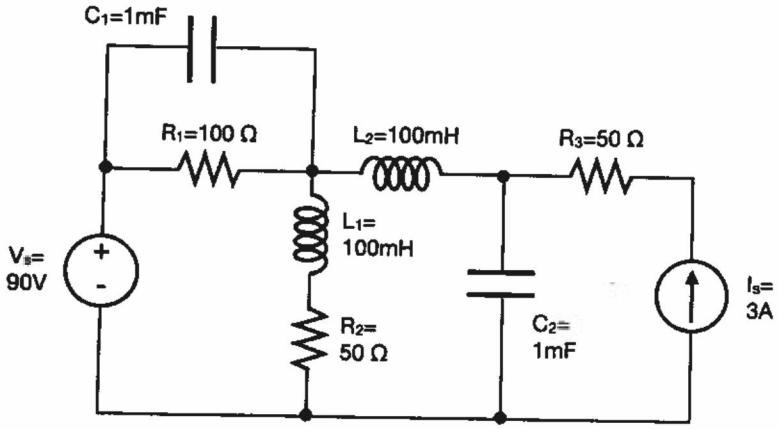
3) $P_{R3} = VI = I^2R$

$= 1.2^2 \cdot 5$

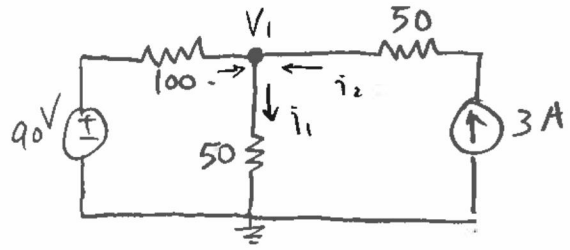
$= \boxed{7.2W}$

Question#8 (14 points). The circuit below is operating in DC steady state.

1. Draw its equivalent circuit in DC steady state (4 points)
2. Find the total energy in all capacitors (5 points)
3. Find the total energy in all inductors (5 points)




1)




2) Solving the s.s. circuit from 1

$$\frac{(90 - V_1)}{100} + 3 = \frac{V_1}{50}$$

$$90 - V_1 + 300 = 2V_1$$

$$3V_1 = 390$$

$$V_1 = 130V$$

$V_{C1} = 130 - 90 = 40V$

$V_{C2} = 130V$

$$\Sigma E_C = \frac{1}{2} \cdot 1mF \cdot 40V^2 + \frac{1}{2} \cdot 1mF \cdot 130V^2$$

$$= \boxed{9.25\ J}.$$

3) $E = \frac{1}{2} L i^2$.

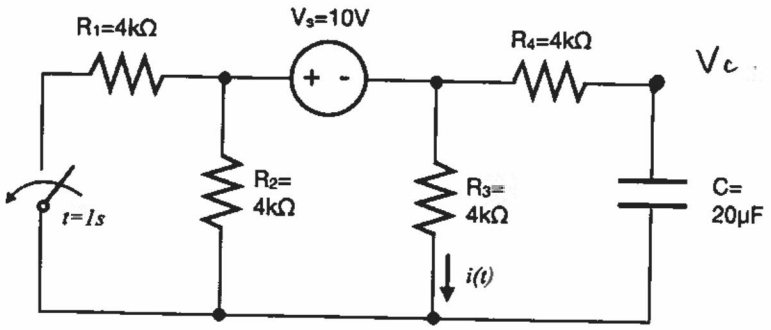
$$i_1 = \frac{130V}{50\Omega} = 2.6A$$

$i_2 = 3A$

$$\Sigma_{EL} = \frac{1}{2} \cdot 100mH \cdot 2.6^2 + \frac{1}{2} \cdot 100mH \cdot 3^2$$

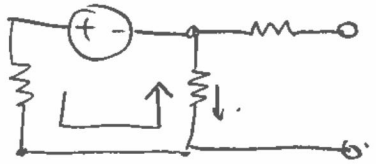
$$= \boxed{0.788\ J}.$$

Question#9 (14 points). The switch in the circuit has been open for a long time before closing at t=1s. Find *i(t)* for t>0.




$V(0) = 1.25 \cdot 4 = 5V.$

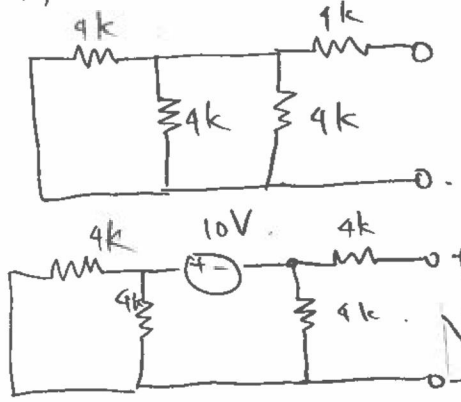
$t<0$, $t \le 1$, steady state



$$i(t) = \frac{-10V}{4k+4k} = -\frac{10}{8000}$$

$$= -1.25\,mA$$

$t>1$, Transient state, use thévenin to simplify




$4k \| 4k = 2k.$

$$2k \| 4k = \frac{2000 \cdot 4000}{6000} = \frac{4000}{3}\,\Omega$$

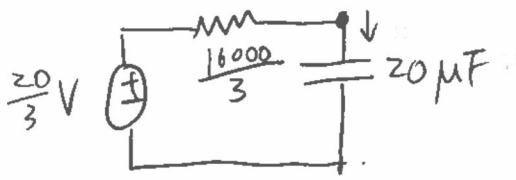
$$R_{th} = \frac{16000}{3}\,\Omega$$

$$V_{oc} = \frac{10 \cdot 4k}{2k+4k} = \frac{20}{3}\,V.$$

$$I_{sc} = \frac{10V}{2k+4k} = 2.5mA$$

$$\frac{2.5}{2} = 1.25\,mA \checkmark$$

Eq. Circuit.




$$\therefore C\frac{dV}{dt} = \frac{V - \frac{20}{3}}{\frac{16000}{3}}$$

$$2 \cdot 10^{-5}\frac{dV}{dt} = \frac{3V-20}{16000} \checkmark$$

Treating start of switch ($t=1$) as $t=0$ for now.

$$V_c(t) = K_1 + K_2 e^{st}.$$

$$2 \cdot 10^{-5} s K_2 e^{st} = \frac{3K_1 + 3K_2 e^{st} - 20}{16000}$$

$$K_1 = \frac{20}{3}, \quad \left(2 \cdot 10^{-5} s - \frac{3}{16000}\right) = 0. \qquad s = -\frac{75}{8}$$

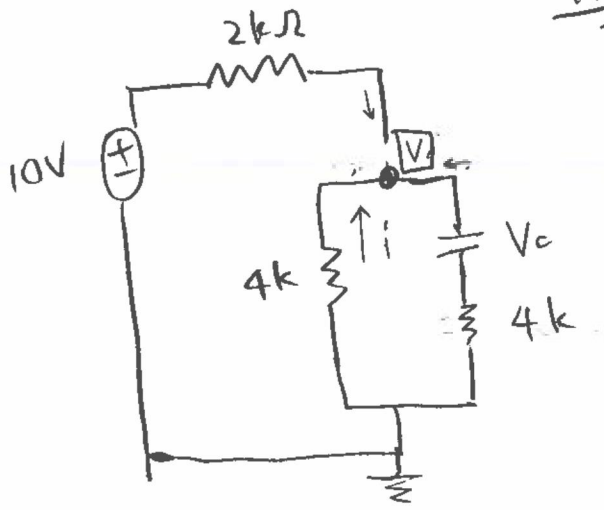
$V(0) = k_1 + k_2 = 5$

$k_2 = 5 - \frac{20}{3}$

$= -\frac{5}{3}$.

$V_c(t) = \frac{20}{3} - \frac{5}{3} e^{-\frac{75}{8}t}$

$\frac{(10-V)}{2k} + \frac{-V_c - V}{4k} = -\frac{V}{4k}$.




$20 - 2V - V_c - V = -V$

$V = 10 - \frac{V_c}{2}$.

$i = \frac{-V}{4k} = \boxed{-\frac{10 - \frac{5}{3} + \frac{5}{6} e^{-\frac{75}{8}t}}{4000}}$

$= -\frac{50 - 5e^{-\frac{75}{8}t}}{24000}$.

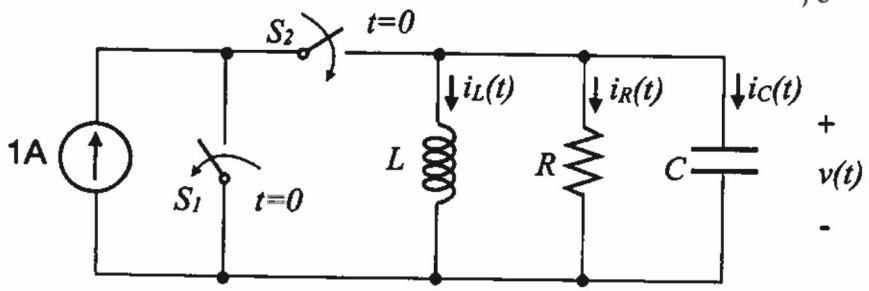
$$\begin{cases} -1.25 \text{ mA} & 0 \le t < 1. \\ -\frac{50 - 5e^{-\frac{75}{8}(t-1)}}{24000} & t \ge 1 \end{cases}$$

Question#10 (16 points). Calculate the transient response v(t) of the RLC circuit. The circuit stays in steady state before the switch turns off at t=0. R = 1Ω, L=$\frac{1}{2\pi}$H, C=$\frac{1}{2\pi}$F. Assume no initial charge on L and C. Note that $S_1$ opens and $S_2$ closes at t=0.

1. Calculate v(0+) and $\frac{dv(0+)}{dt}$ (3 points)
2. Write a differential equation with v(t) as the variable (3 points)
3. What's the damping type? (3 points)
4. Calculate v(t) (7 points)

$i_C = C\frac{dv}{dt}$ $V_C = \frac{1}{C}\int_0^t i_C(t)\,dt$

$I = \frac{1}{L}\int_0^t v(t)\,dt$




1) $v(0^+) = 1\,V$ ← no intial charge on 1 and C

$\frac{dv(0^+)}{dt} = 0V$ ← no instantaneous changes of voltage

2) KCL. $i_L + i_R + i_C = 1A$

$$\frac{1}{L}\int_0^t v(t)\,dt + \frac{v(t)}{R} + C\frac{dv}{dt} = 1A$$

$$\frac{1}{L}v + \frac{1}{R}\dot{V} + C\ddot{V} = 0$$

$$\ddot{V} + \frac{1}{RC}\dot{V} + \frac{1}{LC}V = 0$$

$$\ddot{V} + 2\pi\dot{V} + 4\pi^2 V = 0$$

3) $b^2 - 4ac = 4\pi^2 - 4\cdot 4\pi^2 < 0$.

Underdamped.

4) $s^2V(s) - sV(0) - \dot{v}(0) + 2\pi(sV(s) - V(0)) + 4\pi^2 V(s) = 0$

$(s^2 + 2\pi s + 4\pi^2)V(s) = (s+2\pi)V(0) + \dot{v}(0)$

$$V(s) = \frac{s+2\pi}{s^2+2\pi s+4\pi^2}$$

See next page!

$$= \frac{s+\pi}{(s+\pi)^2+3\pi^2} + \frac{\pi}{(s+\pi)^2+3\pi^2}$$

$$V(s) = \frac{s+\pi}{(s+\pi)^2+3\pi^2} + \frac{\pi\sqrt{3}\cdot\frac{1}{\sqrt{3}}}{(s+\pi)^2+3\pi^2}.$$

$$v(t) = e^{-\pi t}\cos(\sqrt{3}\,\pi t) + \frac{1}{\sqrt{3}}\,e^{-\pi t}\sin(\sqrt{3}\,\pi t).$$