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Last First

UCLA COMPUTER SCIENCE DEPARTMENT
MIDTERM EXAMINATION
CS M51A Spring 2015 Section 1 - Logic Design of Digital Systems
May 4, 2015
Dr. Yutao He

Rules:

This is a closed-textbook, closed-note, and independent exam (110 minutes). You may use two-page 8.5"x11" single-sided note. No scratch paper or calculator is allowed. Points are assigned to the problems based on estimates of how long they should take. PACE YOURSELF ACCORDINGLY. The order may not reflect the degree of difficulty. BROWSE THROUGH THE ENTIRE SET first to decide the order you want to follow. READ THE PROBLEM DESCRIPTION CAREFULLY. Be sure to include all final answers at indicated locations. Write down your Student ID at the top of each page. Use provided space for all work. Have fun and good luck!

Honor Code:

I attest that I have not given or received any aid or discussion in relationship to this examination.

Ryan Peterman

Your Signature

Your Score:

No.	Your Score	Maximal Score
#1	10	10
#2	15	15
#3	10	10
#4	15	15
#5	20	20
#6	15	15
#7	15	15
Total	100	100

CS M51A/EEM16 S'15 Sec.1 Midterm

10

Problem No. 1 (10 points)

can we reuse bits?

✓ **Part (a)** By using binary code, at least how many bits would be needed to encode the combination of your gender (M, F), your year (freshman, sophomore, junior, senior, graduate), and the first two-digit of your Student ID (assuming each digit is a decimal number)?

Answer: 11 bits bits. ✓

<u>Show</u> your work below for full credit:

64 32 16 8 4 2 1
128 64 32 16 8 4 2
7 6 5 4 3 2 1 bits
options

M, F → one bit

year → 5 options → 3 bits

2 digit ID → 100 options → 7 bits

(1+3+7) bits = 11 bits

Part (b) An electronic cylinder *iCylinder* measures liquid capacity by one 3-digit mixed-radix number system (*gallon, quart, pint*). The relationships between three capacity units of this number system are: 1 *gallon* = 4 *quarts*, 1 *quart* = 2 *pints*. Assume that radix for digit *gallon* is sixteen.

gallon quart pint
— — —

(b.1) Using binary code, at least how many bits are needed to encode the largest number of *pints* that can be represented by this number system?

Your Answer: 7 bits. ✓

(b.2) How many *pints* are represented by a reading of $X = (11, 2, 1)$?

Your Answer: 93 pints. ✓ 1+6=7

<u>Show</u> all your work below for full credit:

11 × 4 = 44, × 2 = 88; 2 × 2 = 4; + 1

15 × 4 = 60, × 2 = 120

b.2) 11·4·2 + 2·2·1

88 + 4 + 1

= 93 pints

b.1) max pints = 15 gal 3 quart 1 pint = 1 + 3·2 + 15·4·2 = 127 pints max → 7 bits gives us 127 max which can encode it

1111111

Part (c) A binary number 10010010 is stored in a computer memory.

8+4+2+1
64+32+16
96+28+3 → 127

(c.1) If it is used to represent time in seconds of a car parking meter, how many seconds a car has parked in the spot?

Your Answer: 146 seconds. ✓

<u>Show</u> all your work below for full credit:

7 6 5 4 3 2 1 0
1 0 0 1 0 0 1 0

$2^7 + 2^4 + 2^1 =$

128 + 16 + 2 = 146 ✓

Since seconds are only positive, interpret as unsigned int

(c.2) If it is used to represent the temperature of a car thermometer in integer degrees in Celsius, what is the temperature inside a car?

Your Answer: −18° Celsius. ✓

<u>Show</u> all your work below for full credit:

Using Sign Magnitude Representation since temperature can be negative,

[1]0010010
↑ sign bit

$-(2^4+2^1) = -18$

15

Problem No. 2 (15 points)

Part (a) Name the following 2-input switching functions using primitive logic functions:

(a.1) The output of f_1 is 1 **if and only if** both inputs are 1. It is AND function.

(a.2) The output of f_2 is 1 if the inputs **are different**. It is XOR function.

(a.3) The output of f_3 is 1 **if no more than** one input is 1. It is NAND function.

(a.4) The output of f_4 is 1 **only when** both inputs are 0. It is NOR function.

(a.5) The output of f_5 is 1 **if at least** one input is 1. It is OR function.

(a.6) The output of f_6 is 1 if number of 1's in inputs **are even**. It is XNOR function.

Part (b) The logic designer Logik Luv plans to design one single two-input (x,y) "multi-function" logic module with two-output (g_1, g_2) to implement these switching functions in Part (a). To do so, he introduces two additional inputs (a, b) that decide the output functions g_1 and g_2 as follows:

a	b	g_1	g_2
0	0	x'	y'
0	1	f_1	f_2
1	0	f_3	f_4
1	1	f_5	f_6

(b.1) Filling in the following 2-D truth table for the functions g_1 and g_2. Place values of g_1 and g_2 in each cell in order of (g_1, g_2).

		(x,y) 00	01	10	11
(a,b)	00	1,1	1,0	0,1	0,0
	01	0,0	0,1	0,1	1,0
	10	1,1	1,0	1,0	0,0
	11	0,1	1,0	1,0	1,1

g_2

ab \ xy	00	01	10	11
y' →	1	0	1	0
XOR →	0	1	1	0
NOR →	1	0	0	0
XNOR →	1	0	0	1

Zero-set (1,3,4,7, 9,10,11,13,14)

Not Karnaugh, just for clarity

g_1

ab \ xy	00	01	10	11
x' → 00	1	1	0	0
AND → 01	0	0	0	1
NAND → 10	1	1	1	0
OR → 11	0	1	1	1

oneset(0,1,7,8,9,10,13,14)

15/15

(Continued on the next page)

14 → 1110

(Problem No. 2 - Continue)

✓ (b.2) Write the *minterm* expression for $g_1(a,b,x,y)$ in compact form:

$g_1(a,b,x,y) = \sum m\{$ 0, 1, 7, 8, 9, 10, 13, 14} 15

✓ (b.3) The *switching* expression for m_{14} is $a \cdot b \cdot x \cdot y'$

1 → uncomplemented for minterm

✓ (b.4) Write the *maxterm* expression for $g_2(a,b,x,y)$ in compact form:

$g_2(a,b,x,y) = \prod M\{$ 1, 3, 4, 7, 9, 10, 11, 13, 14}

✓ (b.5) The *switching* expression for M_{14} is $a' + b' + x' + y$ ✓

0 → uncomplemented for Maxterm

(End of Problem No. 2)

Problem No. 3 (10 points) 10

Your high school buddy BB Frank is interviewed for an internship position at a startup *Cookle Electronics*. One of his interview questions is the tabular minimization using the Quine-McCluskey algorithm for the following 4-input switching function:

$$f(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,10,14)$$

BB Frank has completed the first step and the Prime Implicant Chart is shown below. You have to help him identify the essential prime implicants and then write the minimal AND-OR (sum-of-product) expression for $f(a,b,c,d)$.

Prime Implicant Chart

Prime Implicants	0	1	2	5	6	7	8	9	10	14
−00−	X	X					X	(X)		
−0−0	X		X				X		X	
−−10			X		X				X	(X)
0−01		X		X						
01−1				X		X				
011−					X	X				

essential since alone in column

only 5, 7 implicants not been covered by essentials, 01−1 covers 5, 7 for us Thus

Part (a) The essential prime-implicants in switching expressions are:

b'c', cd'

Part (b) The minimal switching expression in AND-OR form is:

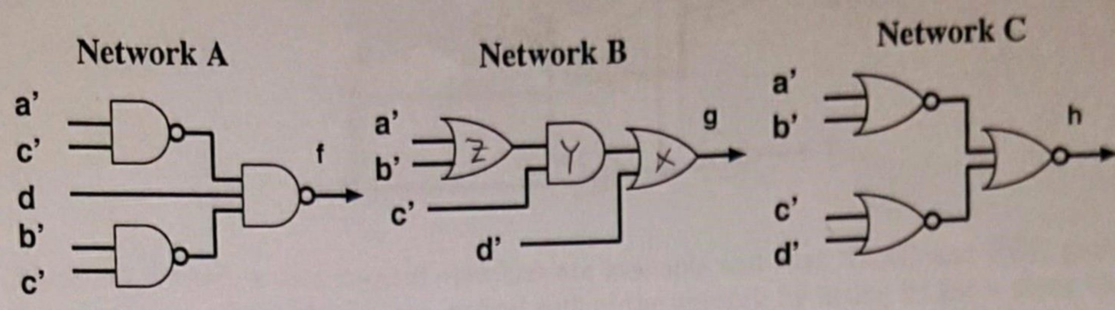
$f(a,b,c,d) =$ b'c' + cd' + a'bd

Show all your work on the *prime implicant chart* above for full credit.

Problem No. 4 (15 points)

15

Three gate networks A, B and C are given below. Tests have shown that two of them are *equivalent*, that is, they implement the same switching function. You are asked to identify the network that <u>is not</u> equivalent.



Part (a) Describe your approach concisely in one sentence.

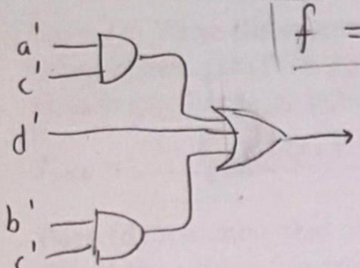
Answer: Either use Truth table or Boolean algebra to show output is the same for all input combinations.

Part (b) The non-equivalent network is C.

✓

<u>Show</u> all your work below for credit:

A) simp using → bubble logic



$f = a' \cdot c' + d' + b' \cdot c'$

A = B

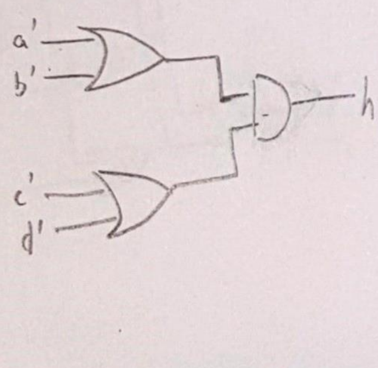
B) $g = x = Y + d' \quad Y = Z \cdot c' \quad Z = a' + b'$

$= Z \cdot c' + d'$

$= (a' + b')c' + d'$

$g = a' \cdot c' + d' + b'c'$

C) bubble → logic



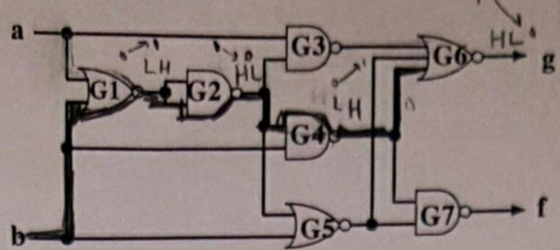
$h = (a' + b') \cdot (c' + d')$

$h = a'c' + a'd' + b'c' + b'd'$

Problem No. 5 (20 points)

20

Given the gate network below, answer the following questions:



Part (a) Assuming that negated variables are available and that NAND and NOR gates have the same delays, identify the *critical path* of the network by listing its gates along the path, starting at the inputs: # gates   largest load   largest Fanin

largest load

b → G1 → G2 → G4 → G6 ✓

Part (b) Assuming that load factors of all gates equal to 1 and that both outputs f and g have the output load value L, list the output load value of every gate in the *critical path* (e.g., G3: 1):

G1: 2, G2: 3, G4: 2, G6: L ✓

Part (c) Write the expression of the longest network propagation delay T_{pHL} in terms of delays of each gate (You do not need to compute the final result but the transition direction at each gate has to be indicated):

$T_{pHL} =$ $t_{pLH}(G1) + t_{pHL}(G2) + t_{pLH}(G4) + t_{pHL}(G6)$ ✓

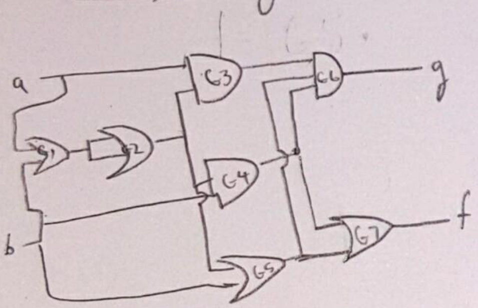
Part (d) Assuming that negated variables are available, find the minimal switching expression of the output f in two-level AND-OR (sum-of-product) form. Show your work below for full credit.

$f(a, b) =$ $a + b$ ✓

Your work for Part (d):

Simplify using bubble logic

(Extra space available on the next page)



$f = G4 + G5$
$= b \cdot (a+b) + (a+b)$
$= (a+b) \cdot (b+1)$
$= a + b$

$G4 = b \cdot G2$
$= b \cdot (a+b)$

$G5 = G2 + b$
$= a + b + b = a + b$

$G2 = G_1 + G_1 = a + b + a + b$
$= a + b$

$G_1 = a + b$