Midterm November 5, 2020
Deadline October 6, 2020 by 2pm Pacific Time

Problem 1 (10 points)

How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

a: Decimal digits in BCD
b: Hexadecimal representation

which representation is more efficient and why?

Problem 2 (10 points)

Fill in the missing entries

Radix	Value	Value in decimal
16	(5 1 7)	
8	(5 1 7)	

True or False:
a: A bubble on the output of a logic gate indicates that the output is active HIGH
b: Bubbles on the input lines of logic gates indicate that these inputs are active LOW
c: The Boolean expression for a logic circuit is A' + B' = X (A' is complement A)
This tells us both inputs are active LOW and the output is active HIGH.

Problem 3 (10 points)

Given $E(a, b, c, d) = (ab + c)'(ac + (b' + c' + a'cd)') + a((b + c)(b + d) + c)'$, which of the following represents the same function as E(a, b, c, d)? Show all your work.

1. $a+b+c+d'$ 2. $a' + b + c$ 3. $b + c' + d$

4. $a'b'c'd$ 5. $ab'c'$ 6. $b'cd'$

Problem 4: (10 points)

a: Explain Gray Codes and their characteristics and explain conversion of GC to Binary with an example
b: What is X-3 code and explain the self complementary property of it.
c: Subtract $(-25)_8$ from $(25)_{10}$ using 2's complement and explain the results in octal and decimal

d: realize XOR gate using NOR gates only.

Problem 5: (10 points)

Given the following simplification of a boolean expression, answer the following.

$$
\begin{aligned}
& (ab' + c')'(b' + c)(a + bc') && (1)\\
= \; & (ab')'c'(b' + c)(a + bc') && (2)\\
= \; & (a' + b)c'(b' + c)(a + bc') && (3)\\
= \; & (a' + b)c'(ab + ac + bb'c' + bcc') && (4)\\
= \; & (a' + b)c'(ab + ac) && (5)\\
= \; & (a' + b)(ab + ac)c' && (6)\\
= \; & (aa'b + abb + aa'c + abc)c' && (7)\\
= \; & abc' + abcc' && (8)\\
= \; & abc' && (9)
\end{aligned}
$$

a: (4 points) There is at least one mistake in this simplification. Find all steps that are derived incorrectly from its previous step (for example, write (8)→(9) if equation (9) is derived incorrectly from (8)).

b: (6 points) Show the correct simplification of (1)

Problem 6: (20 points)

a: Express the given expression in a product of maxterms:

F = xy+ x'z

b: simplify the following boolean function and realize it using NOR gates

F = (A,B,C,D) = A'B'D' + A'CD + A'BC
d(A,B,C,D) = A'BC'D + ACD + AB'D'

c: simplify the Boolean function in SOP and POS form
F = $\sum$ (0,1,2,5,8,9,10)

Problem 7: (20 points)

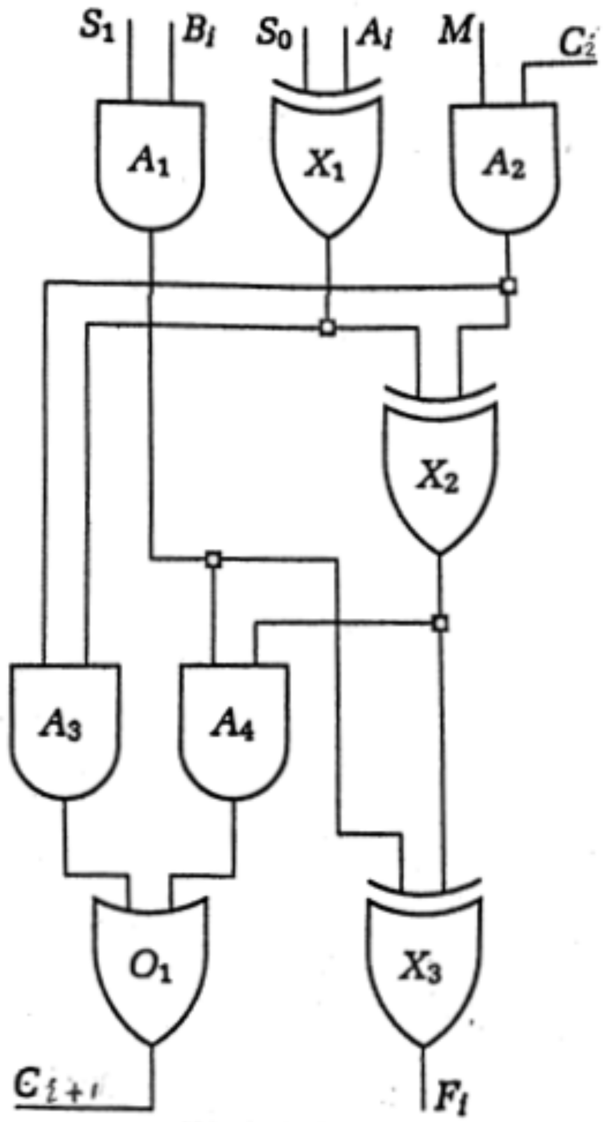
a: Draw the logic diagram for a four-bit comparator and explain the $\geq$ block.

b: explain the BCD-to-7-segment decoding logic for “g” segment

c: explain Decimal to BCD priority encoder, draw it partially and show the truth table for inputs 3 and 6.

Problem 8: (10 points)

Consider the following gate network:



Determine the switching expressions for the outputs F_i and C_{i+1}. Using AND, OR, NOT, XOR expressions as appropriate fill in the following table:

M	S_1S_0	Expression for F_i	Expression for $CI+1$
0	00		
0	01		
0	10		
1	10		
1	11		

Problem 1

a) 16 million colors $\Rightarrow$ 16,000,000 → 0 .... color 1
1 ... color 2
⋮
15 999 999 ... color 16,000,000

each decimal value represents 1 unique color

8 digits in decimal

We need to represent 8 digits in BCD code. BCD code requires 4 bits to represent 1 digit in decimal. Thus, we need $4 \times 8 = \boxed{32}$ total bits to encode the color spectrum of 16,000,000 colors.

b) We need $\boxed{24}$ total bits: $2^{24} = 16,777,216$ unique values that we are able to encode using 24 bits. (23 bits is only $2^{23} \approx 8$ million values)

The hexadecimal representation is more efficient because we not only need fewer bits to represent 16,000,000 colors but also waste fewer states. This is because in BCD, states from 1010 (10) to 1111 (16) aren't used since the 4 bits are used only to represent decimal digits 0-9, whereas in hexadecimal, we can use all 16 states for 4 bits (0-9 and A-F).

Problem 2

Radix	Value	Value in decimal
16	(5 1 7)	1303
8	(5 1 7)	335

$5 \cdot 16^2 + 1 \cdot 16^1 + 7 \cdot 16^0 = 1303$

$5 \cdot 8^2 + 1 \cdot 8^1 + 7 \cdot 8^0 = 335$

a) False; A bubble on the output of a logic gate does <u>not</u> indicate that the output is active HIGH.

b) True; Bubbles on the input lines of logic gates indicate that these inputs are active LOW.

c) True; The inputs are both active LOW, and the output is active HIGH

Problem 3

$$\overline{ab+c}\left(ac+\overline{\bar{b}+\bar{c}+\bar{a}cd}\right)+a\overline{(b+c)(b+d)+c}=\overline{ab+c}\left(ac+bc\left(\overline{\bar{a}cd}\right)\right)+a\overline{b+cb+cd+bd+c}$$

$$=\overline{ab+c}\left(ac+bc\left(a+\bar{c}+\bar{d}\right)\right)+a\overline{b(\underset{1}{\cancel{1+c+d}})+c(\underset{1}{\cancel{1+d}})}=\overline{ab+c}\left(ac+abc+\overset{0}{\cancel{bc\bar{c}}}+bc\bar{d}\right)+a\overline{b+c}$$

$$=\overline{ab}\cdot\bar{c}\left(ac(\overset{1}{\cancel{1+b}})+bc\bar{d}\right)+a\bar{b}\bar{c}=\underbrace{\overset{0}{\cancel{\overline{ab}\bar{c}\cdot ac}}}_{\bar{c}\cdot c=0}+\underbrace{\overset{0}{\cancel{\overline{ab}\bar{c}bc\bar{d}}}}_{\bar{c}\cdot c=0}+a\bar{b}\bar{c}$$

$$=\boxed{a\bar{b}\bar{c}}\quad(\#5)$$

Problem 4

a) Gray code is a binary number system in which any two successive values differ by exactly one bit. Gray code has a cyclic property in that the maximum decimal integer represented in Gray code can be changed into the minimum decimal integer represented in Gray code by flipping a single bit.

You can convert from Gray code to binary by first keeping the most significant bit and then summing the current Gray code bit with the neighboring corresponding bit in binary. An example:

9 in Gray code = 1101

1001

$\Rightarrow$ 9 in binary = 1001

b) X-3 code is an unweighted BCD code in which you add 3 to each digit in the decimal representation of an integer and then convert each resulting sum to binary.

The self-complementary property of X-3 code is that the one's complement of the X-3 representation of any decimal integer is equal to the X-3 representation of the nine's complement of that decimal integer.

Problem 4 (cont.)

c) $(25)_{10} - (-25)_8$

$(25)_{10} = $
$2\,|\,25 \to 1$
$12 \to 0$
$6 \to 0$
$3 \to 1$
$1 \to 1$
0
$= 011001$

$(25)_8 = 2_8\ 5_8 = 010\ 101$

$(-25)_8 = \sim 010101 + 1$
$= 101010 + 1$
$= 101011$

$(25)_{10} - (-25)_8 = 011001 - (101011) = 011001 + (\sim 101011 + 1)$

$= 011001 + (010100 + 1) = 011001 + 010101 =$

$$\begin{array}{r} 011001 \\ +010101 \\ \hline 101110 \end{array}$$

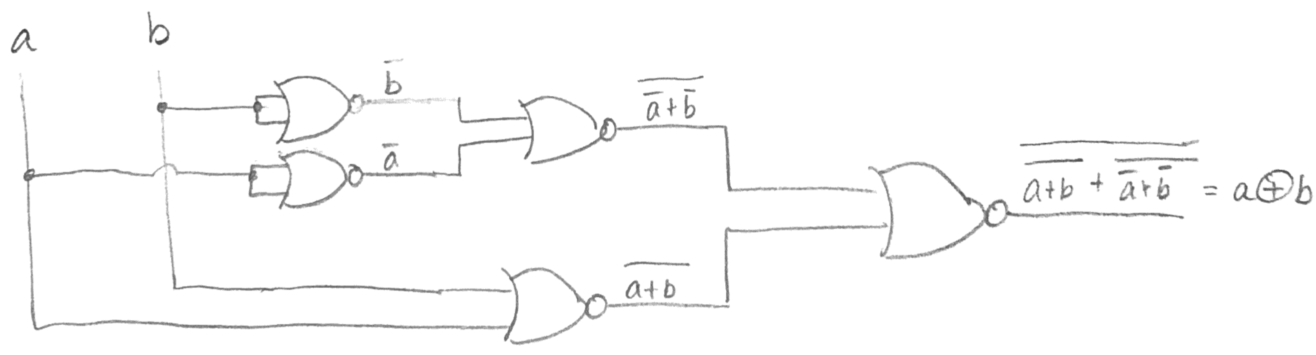
$101110 \to$ $\boxed{\begin{array}{l} 46 \text{ in decimal} \\ 56 \text{ in octal} \end{array}}$

d) XOR gate:

a	b	Y
0	0	0
0	1	1
1	0	1
1	1	0

$Y = \bar{a}b + a\bar{b} = \bar{a}b + a\bar{b} + a\bar{a} + b\bar{b}$
$= (a+b)(\bar{a}+\bar{b}) = \overline{\overline{a+b} + \overline{\bar{a}+\bar{b}}}$

$[\,\overline{a+b} = a \text{ NOR } b\,]$



Problem 5

a) (1) $\rightarrow$ (2) When DeMorgan's is being applied, $(c')'$ should be c, not c'.

(3) $\rightarrow$ (4) When multiplying out $(b'+c)(a+bc')$, the first term in the product should be ab', not ab.

b) $(ab'+c')' \cdot (b'+c)(a+bc')$

$= (ab')'c\,(b'+c)(a+bc')$

$= (a'+b)\,c\,(b'+c)(a+bc')$

$= (a'+b)\,c\,(ab'+ac+bb'c'+bcc')$

$= (a'+b)\,c\,(ab'+ac)$

$= (a'+b)(ab'+ac)\,c$

$= (aa'b'+abb'+abc+aa'c)\,c$

$= \boxed{abc}$

Problem 6

a) $F = xy + \bar{x}z$

$= xy(\bar{z}+z) + \bar{x}z(y+\bar{y})$

$= xy\bar{z} + xyz + \bar{x}\bar{y}z + \bar{x}yz$ ← sum of minterms currently

product of maxterms = complement of sum of minterms

$$\overline{xy\bar{z} + xyz + \bar{x}\bar{y}z + \bar{x}yz} = (\overline{xy\bar{z}})(\overline{xyz})(\overline{\bar{x}\bar{y}z})(\overline{\bar{x}yz})$$

$$= \boxed{(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})(x+y+\bar{z})(x+\bar{y}+\bar{z})}$$

b) $F(A,B,C,D) = \bar{A}\bar{B}\bar{D} + \bar{A}CD + \bar{A}BC = \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BCD + \bar{A}BC\bar{D} + \cancel{\bar{A}BCD}$ repeat

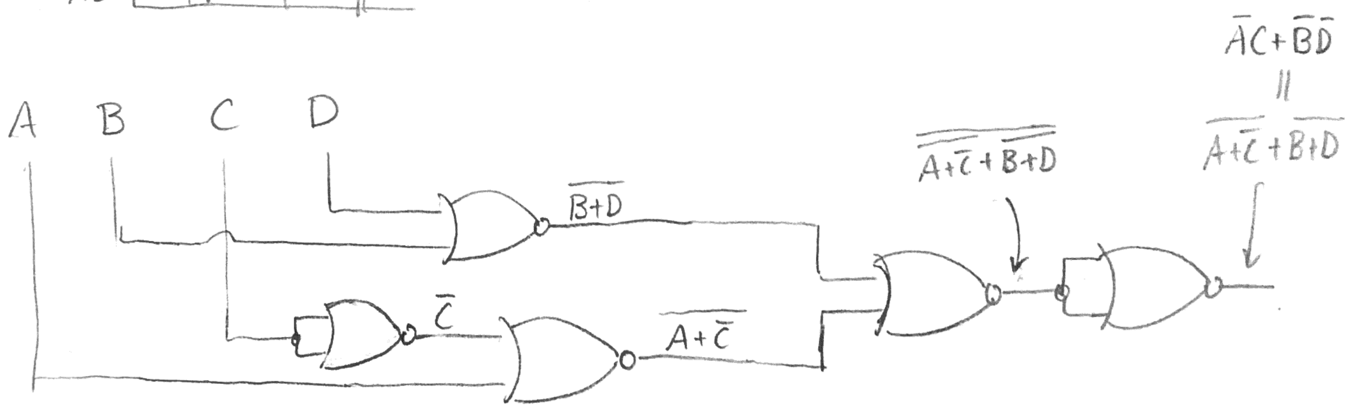
$d(A,B,C,D) = \bar{A}B\bar{C}D + ACD + A\bar{B}\bar{D} = \underbrace{\bar{A}B\bar{C}D + A\bar{B}CD + ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}}_{\text{"don't cares"}}$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1		1	1
$\bar{A}B$		X	1	1
AB			X	
$A\bar{B}$	X		X	X

① $\bar{A}C$
② $\bar{B}\bar{D}$

$\bar{A}C + \bar{B}\bar{D} = \overline{A+\bar{C}} + \overline{B+D}$

$= \overline{\overline{\overline{A+\bar{C}} + \overline{B+D}}}$



Problem 6 (cont.)

c) $F = \Sigma(0,1,2,5,8,9,10)$

SOP: $F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D}$

0000	0001	0010	0101	1000	1001	1010
0	1	2	5	8	9	10

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

① $\bar{B}\bar{D}$
② $\bar{B}\bar{C}$
③ $\bar{A}\bar{C}D$

$\bar{A}\bar{C}D + \bar{B}\bar{C} + \bar{B}\bar{D}$

In SOP: $\boxed{\bar{A}\bar{C}D + \bar{B}\bar{C} + \bar{B}\bar{D}}$

POS:

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

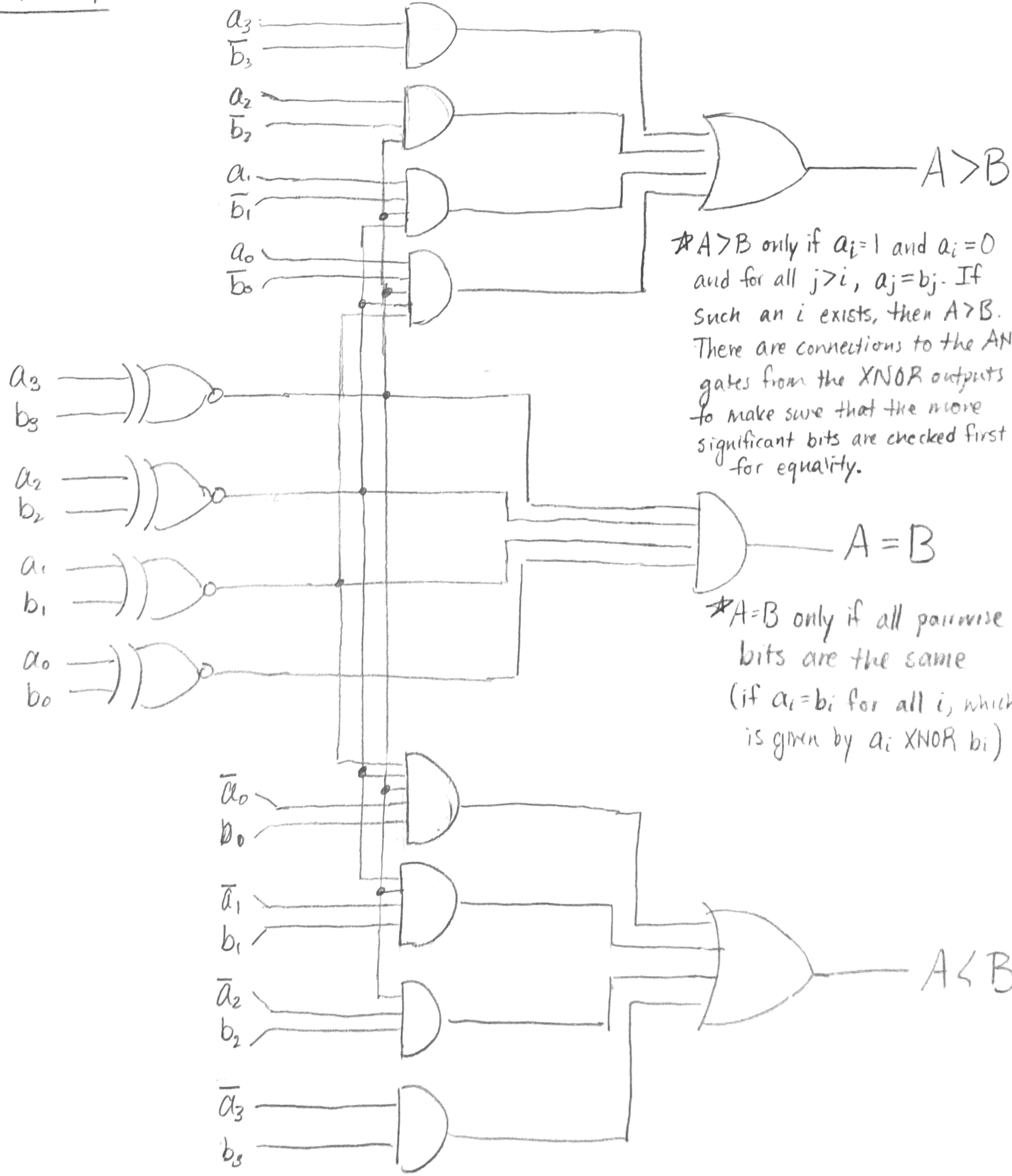
① $\bar{B}+D$
② $\bar{C}+\bar{D}$
③ $\bar{A}+\bar{B}$

$(\bar{A}+\bar{B})(\bar{B}+D)(\bar{C}+\bar{D})$

In POS: $\boxed{(\bar{A}+\bar{B})(\bar{B}+D)(C+\bar{D})}$

Problem 7

a)



*A>B only if $a_i=1$ and $a_i=0$ and for all $j>i$, $a_j=b_j$. If such an i exists, then A>B. There are connections to the AND gates from the XNOR outputs to make sure that the more significant bits are checked first for equality.

*A=B only if all pairwise bits are the same (if $a_i=b_i$ for all i, which is given by a_i XNOR b_i)

Problem 7 (cont.)

b)

"g" lights up for numbers

$$g = \Sigma(2,3,4,5,6,8,9)$$

Assuming inputs ABCD (A being most significant bit)...

$$g = \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D$$

0010	0011	0100	0101	0110	1000	1001
2	3	4	5	6	8	9

Thus, "g" segment's logic should be based on the SOP above. However, we likely want "g" segment to be active low, so

$$g = \overline{\bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D}$$

Each product represents the logic requirement for the inputs for "g" to light up, so we OR them together since "g" will light up for any of the specific combinations of inputs. The complement is to account for "g" being active low.

Problem 7 (cont.)

c) A decimal-to-BCD priority encoder takes 10 input lines, each one for one decimal digit, and outputs the BCD representation of the digit that is currently high. If more than one input line is high at the same time, the encoder will take the high input line corresponding to the greatest decimal digit (9 being greatest, 0 being least), and effectively ignore the other input lines.

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1

Outputs: A, B, C, D (A being MSB)

Inputs: $X_0, X_1, \ldots X_9$

$A = \sum(8,9)$

$B = \sum(4,5,6,7)$

$C = \sum(2,3,6,7)$

$D = \sum(1,3,5,7,9)$

handles case when X_9 is not pressed

$$A = X_9 + \bar{X}_9 X_8 = X_9 + X_8$$

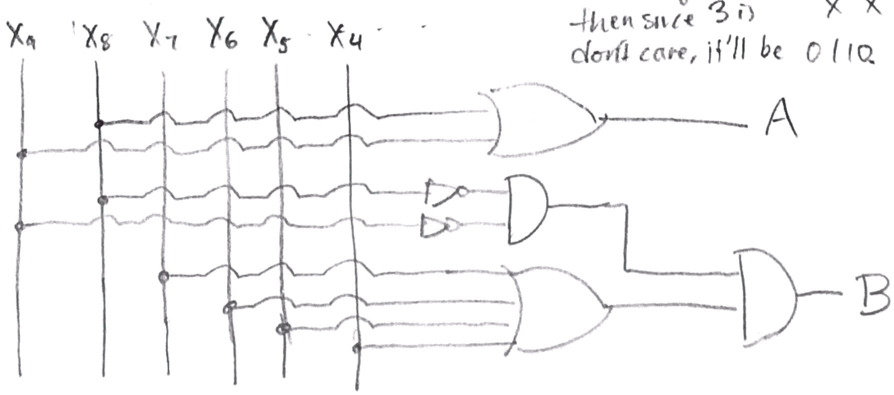
$$B = \bar{X}_9\bar{X}_8X_7 + \bar{X}_9\bar{X}_8\bar{X}_7X_6 + \bar{X}_9\bar{X}_8\bar{X}_7\bar{X}_6X_5 + \bar{X}_9\bar{X}_8\bar{X}_7\bar{X}_6\bar{X}_5X_4$$

$$= \bar{X}_9\bar{X}_8X_7 + \bar{X}_9\bar{X}_8X_6 + \bar{X}_9\bar{X}_8X_5 + \bar{X}_9\bar{X}_8X_4 = \bar{X}_9\bar{X}_8(X_7 + X_6 + X_5 + X_4)$$

$C = \ldots$

If both 3 and 6 are high, then since 3 is don't care, it'll be 0110

X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	A B C D
X	X	X	X	X	X	H	L	L	L	0110
X	X	X	H	L	L	L	L	L	L	0011



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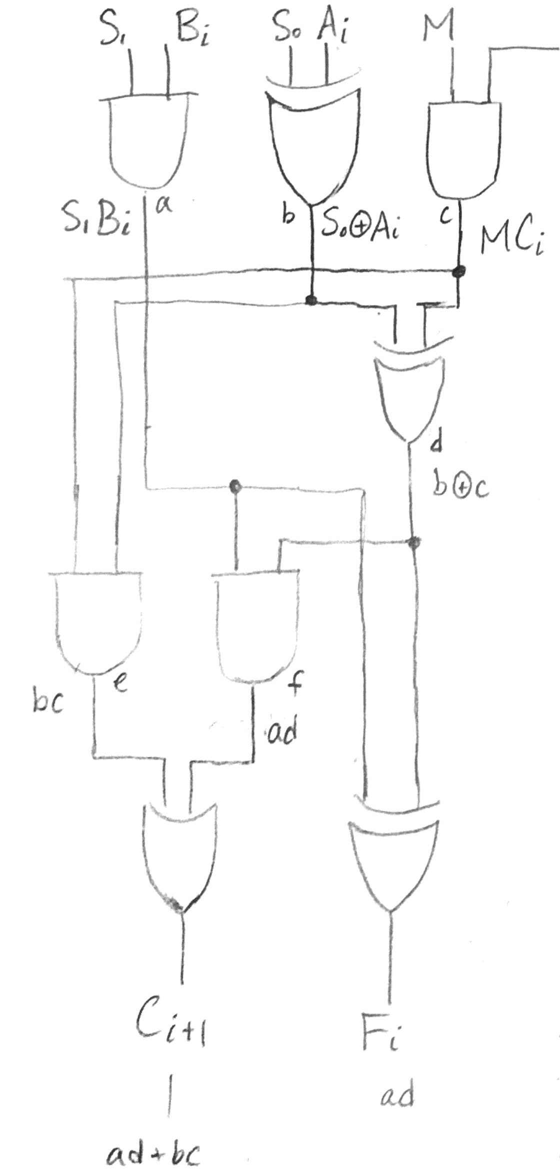
Problem 8



$S_0 \oplus A_i = \overline{S_0}A_i + S_0\overline{A_i}$

$b \oplus c = \overline{b}c + b\overline{c}$

M	S_1S_0	a	b	c	d	e	f
0	00	0	A_i	0	A_i	0	0
0	01	0	$\overline{A_i}$	0	$\overline{A_i}$	0	0
0	10	B_i	A_i	0	A_i	0	A_iB_i
1	10	B_i	A_i	C_i	$A_i \oplus C_i$	A_iC_i	$B_i(A_i \oplus C_i)$
1	11	B_i	$\overline{A_i}$	C_i	$\overline{A_i} \oplus C_i$	$\overline{A_i}C_i$	$B_i(\overline{A_i} \oplus C_i)$

$C_{i+1} = ad + bc = f + bc: \; 0 + 0A_i \cdots 0$

$0 + 0\overline{A_i} \cdots 0$

$A_iB_i + 0\overline{A_i} \cdots A_iB_i$

$B_i(A_i \oplus C_i) + A_iC_i$

$B_i(\overline{A_i} \oplus C_i) + \overline{A_i}C_i$

$F_i = ad = f$ (see table above)

M	S_1S_0	Expression for F_i	Expression for C_{i+1}
0	00	0	0
0	01	0	0
0	10	A_i AND B_i	A_i AND B_i
1	10	B_i AND (A_i XOR C_i)	B_i AND (A_i XOR C_i) OR (A_i AND C_i)
1	11	B_i AND (NOT A_i XOR C_i)	B_i AND (NOT A_i XOR C_i) OR (NOT A_i AND C_i)