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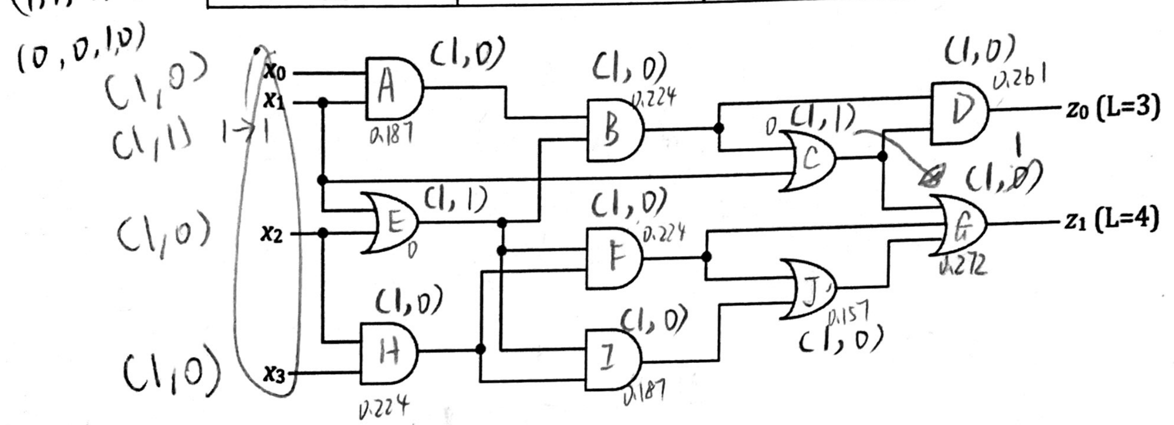
Problem 1 (20 points):

Given the network below, find the critical path and calcuate critical path delay. Assume the values of (x_3, x_2, x_1, x_0) are initially (1,1,1,1) and they change to (0,0,1,0) in the next clock cycle. Now, choose a gate on the citical path which maximally decreases overall delay when the gate decreases its delay by 20%. Finally, find the critical path in the new network and its length.

Gate	Fan-in	t_{pLH} (1,0)	t_{pHL} (0,1)
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$



t_{LH} =
A: $0.15 + 0.037 \cdot 1 = 0.187$
B: $t_{LH} = 0.15 + 0.037 \cdot 2 = 0.224$
C: $t_{LL} = 0$
D: $0.15 + 0.037 \cdot 3 = 0.261$
E: $t_{LL} = 0$
F: $t_{LH} = 0.15 + 0.037 \cdot 2 = 0.224$
0 delay G: $t_{LH} = 0.12 + 0.038 \cdot 4 = 0.272$
H: $t_{LH} = 0.15 + 0.037 \cdot 2 = 0.224$
I: $t_{LH} = 0.15 + 0.037 = 0.187$
J: $t_{LH} = 0.12 + 0.037 = 0.157$

Before changing delay:

Delay (A, B, D) = 0.187 + 0.224 + 0.261 X
same (Delay (A, B, C, D)
Delay (A, B, C, G) = 0.187 + 0.224 + 0.272 = 0.683
Delay (E, B, D) = 0.224 + 0.261 X
Same (Delay (E, B, C, D)
Delay (E, B, C G) = 0.224 + 0.272 X
Delay (E, F, G) = 0.224 + 0.272 X
Delay (E F J G) = 0.224 + 0.157 + 0.272 = 0.653
Delay (E, I, J G) = 0.187 + 0.157 + 0.272 X
Delay (H, F, J, G) = 0.224 + 0.224 + 0.157 + 0.272 = **0.877** ns.
Delay (H, F G) = 0.224 + 0.224 + 0.272 X
Delay (H, I, J, G) = 0.224 + 0.187 + 0.157 + 0.272 X

Critical path : HFJG, with time delay 0.877 ns

Problem 1 (Extra Page):

Decrease G Gate by 20%

Delay(G) = 0.272 · (1-20%) = 0.272 · 0.8 = 0.2176 ns

The other gates are the same.

same (Delay (A,B, D) = 0.187 + 0.224 + 0.261 = 0.672 X

Delay (A, B, C, D)

Delay (A, B, C G) = 0.187 + 0.224 + 0.2176 X

same (Delay (E, B, D) = 0.224 + 0.261 X

Delay (E, B, C, D)

Delay (E, B, C G) = 0.224 + 0.2176 X

Delay (E, F, G) = 0.224 + 0.2176 X

Delay (E, F, J G) = 0.224 + 0.157 + 0.2176 X

Delay (E J, J G) = 0.187 + 0.157 + 0.2176 X

Delay (H, F, J, G) = 0.224 + 0.224 + 0.157 + 0.2176 = **0.8226** ns.

Delay (H, F, G) = 0.224 + 0.224 + 0.2176 X

Delay (H J, J, G) = 0.224 + 0.187 + 0.157 + 0.2176 X

The Critical path in the new network is : H F J G with 0.8226 ns time delay

Problem 2 (20 points):

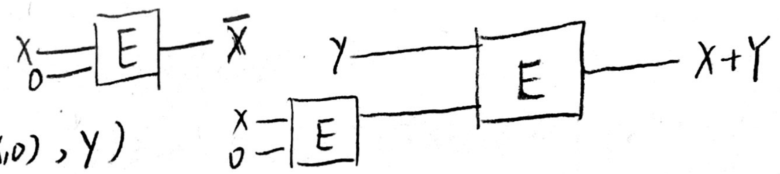
You are given the following Boolean function.

$$F(x_6, x_5, x_4, x_3, x_2, x_1, x_0) = x_6x_5x_4x_3 + x_6x_5'x_2 + x_6x_5'x_3'x_2' + x_6'x_1x_0 + x_6'x_5x_0'$$

Given the universal operation E as specified in the table, implement F using only the gates specfied by E.

X	Y	E (X, Y)
0	0	1
0	1	1
1	0	0
1	1	1

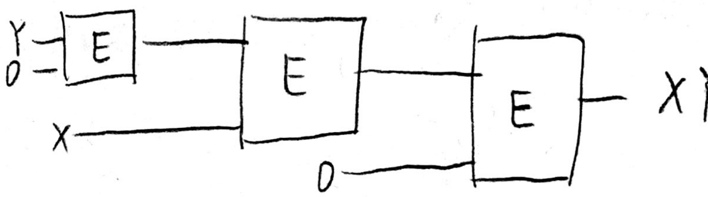
$E(X,Y) = X' + Y = (XY')'$

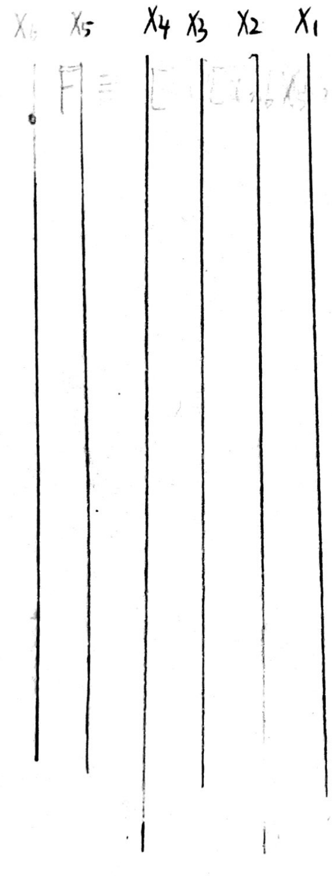


$\bar{X}$ NOT : $E(X,0)$

$X+Y$ OR : $E(X',Y) = E(E(X,0),Y)$

XY AND : $E(E(X,Y'),0) = E(E(X,E(Y,0)),0)$





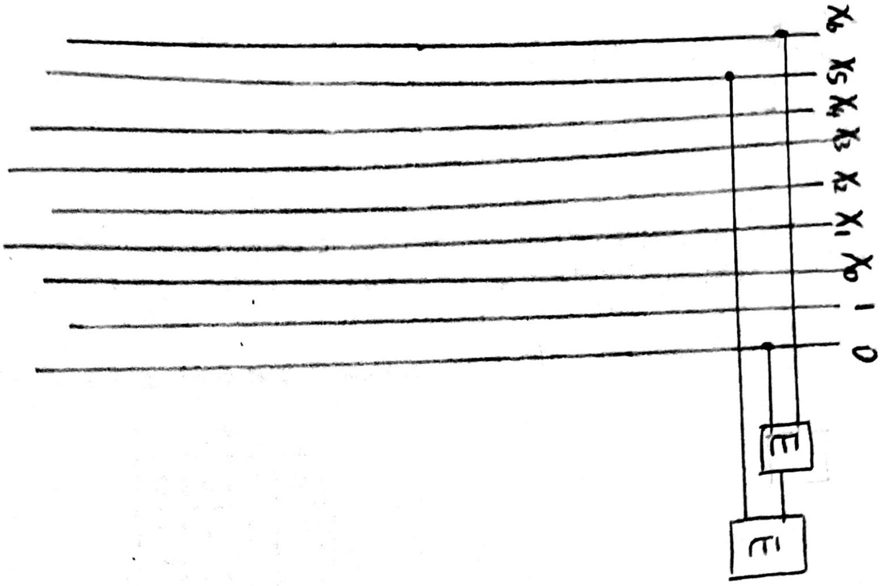
$F = X_6X_5X_4X_3 + X_6X_5'X_2 + X_6X_5'X_3'X_2' + X_6'X_1X_0 + X_6'X_5X_0'$

$(X_3+X_2)'$

$= E(E(X_6X_5, E(X_4X_3,0)),0) + E(E(X_6X_2, E(X_5',0)),0)$
$+ E(E(X_6X_5', E(X_3'X_2',0)),0) + E(E(X_6', E(X_1X_0,0)),0)$
$+ E(E(X_6', E(X_5X_0',0)),0)$

$= E(E(E(E(E(X_6, E(X_5,0)),0)), E(E(E(X_4, E(X_3,0))),0)),0)),0)$
$+ E(E(E(E(E(X_6, E(X_2,0))),0)), E(E(X_5,0),0))),0)$
$+ E(E(E(E(E(X_6, E(E(X_5,0),0))),0)), E(E(X_3+X_2,0)),0))),0)$
$+ E(E(E(X_6,0)), E(E(E(X_1, E(X_0,0)),0))),0)$
$+ E(E(E(E(X_6,0)), E(E(E(X_5, E(E(X_6,0),0))),0)),0))),0)$

Problem 2 (Extra Page):



$$= E(E(E(E(E(x_6, E(x_5,0)),0), E(E(E(x_4, E(x_3,0)),0),0),0),0)$$

$$+ E(E(E(E(E(x_6, E(x_2,0)),0), E(E(x_5,0),0)),0)$$

$$+ E(E(E(E(E(x_6, E(E(x_5,0),0)),0), E(E(E(E(x_3,0), x_2),0),0)),0)$$

$$+ E(E(E(E(x_6,0), E(E(E(x_1, E(x_0,0)),0))),0)$$

$$+ E(E(E(E(x_6,0), E(E(E(x_5, E(E(x_0,0),0)),0),0)),0)$$

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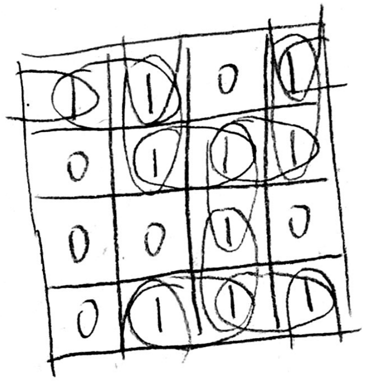
Problem 3 (20 points):

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of P=M-N among all the 4×4 K-maps.

For example, in the following 4×4 K-map, M=3, N=2, P=M-N=1.

x_0 (columns 2–3), x_1 (columns 3–4), x_2 (rows 2–3), x_3 (rows 3–4)

0	0	0	0
1	1	0	0
1	1	1	0
0	0	1	0



M=12

N=0

P= M-N =12

(same)

1	1	0	1
0	1	1	1
0	0	1	0
0	1	1	1

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Problem 4 (20 points):

Given an input stream X, we want to recognize interchangably patterns A and B. We recognize A first, then B, followed by A again, then B again and so on.

For example,

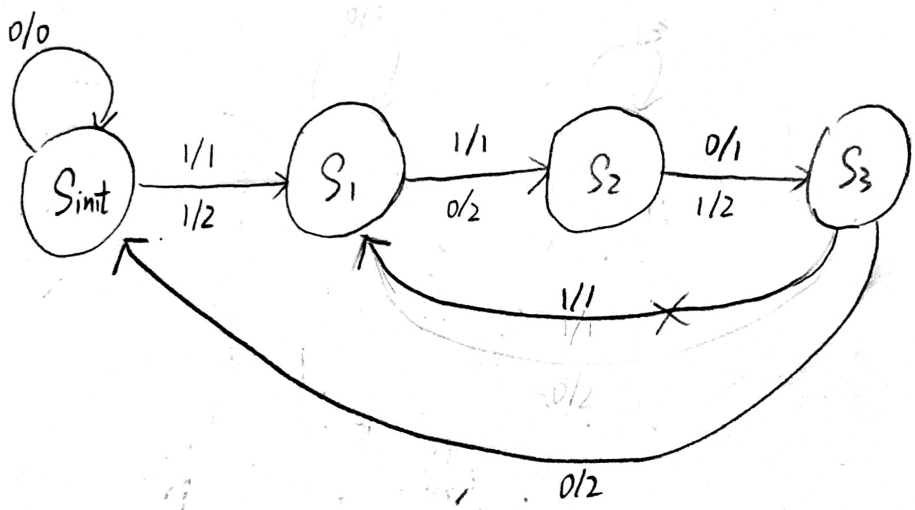
1. Assume X= 01011010010101, A=101 and B=001.

   We will first recognize A, then look for B. Please note that we ignore the second '101' (A) in X and we only search for B once we have found A. After finding B, we again search for A.

2. Assume that we have X= 1011, A = 101 and B=011

   We recognize A, but we do not recognize B as we only start looking for B once we have detected A. In other words, A and B do not overlap.

Now, you are given any input stream X. Design a finite state machine such that the system outputs 1 when it recognizes pattern A= '1101' and outputs 2 when it recognizes pattern B= '1010' after recognizing pattern A. In all other cases the machine should output 0. Show the state transition table and state transition diagram.



Needs 8 states.

11011 outputs 1 twice

State \ Input	0	1
S_{init}	0 S_{init}	1 S_1 2 S_1
S_1	2 S_2	1 S_1
S_2	1 S_3	2 S_3
S_3	2 S_{init}	1 S_1

Problem 5 (20 points):

Perform the following conversions:

a) $(B2451)_{16} \rightarrow (x)_8$
b) $(354)_7 \rightarrow (y)_5$

8 4 2 1

a) $(B2451)_{16}$

$= 0\,10\,11\;00\,1\,0\;01\,00\;01\,01\;0\,001$

$= 2622121_8$

$X = 2622121$

b) $(354)_7 = (3 \cdot 7^2 + 5 \cdot 7^1 + 4 \cdot 7^0)_{10}$

$= (147 + 35 + 4)_{10}$

$= 186_{10}$

$= 1221_5$

$Y = 1221$

186 / 5	1
37 / 5	2
7 / 5	2
1 / 5	1
0	

Problem (Possible Points)	**Points**
1(20)	15
2(20)	16+4.
3(20)	20·
4(20)	6
5(20)	20
Total (100)	77