

Name Jorge Fuentes
First Last
Student ID # XXXXXXXXXX

University of California
Los Angeles
Computer Science Department

CSM51A/EEM16 Midterm Exam
Winter Quarter 2016
February 8th 2016

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

Problem (possible points)	Points
1 (20)	20
2 (20)	20
3 (20)	20
4 (20)	16
5 (20)	20
Total (100)	96

20

Problem 1 (20 points)

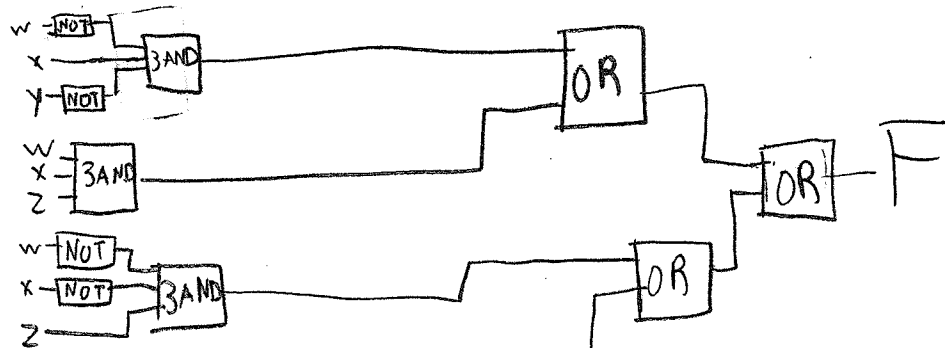
Use only the "E" gate defined below to implement Boolean function:

$$F = w'xy' + wxz + w'x'z + wx'y'z'$$

You may also use constants 0 and 1 as inputs.

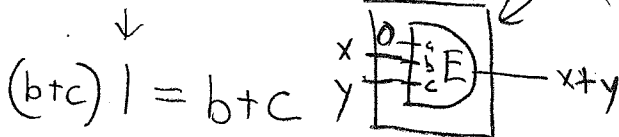
a	b	c	E(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$F = w'xy' + wxz + w'x'z + wx'y'z'$$

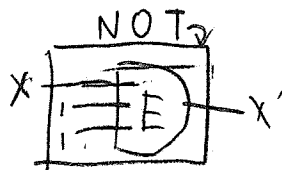


$$E = (a+b+c)(a'+b'+c')$$

$a=0 \quad b=b \quad c=c$ (2-input OR)



OR



$b=1 \quad c=1$

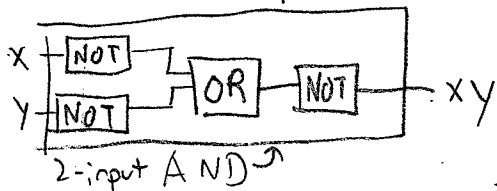
$$(a+1+1)(a'+0+0) = a'$$

↑

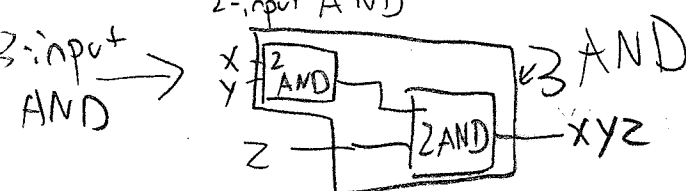
$a=0 \quad b=1 \quad c=1$ NOT

$$x+y = (x'y)'$$

$$x'+y' = (xy)'$$



2-input AND



11 11
10 00
01 00
00 00

Problem 1) Extra Page

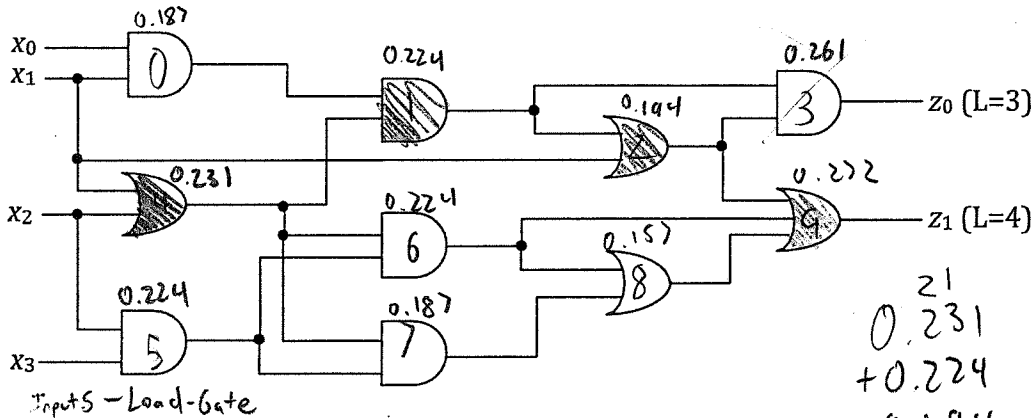
2

Problem 2 (20 points)

Given the network below, calculate the critical path delay. Consider L → H delay when calculating the critical path.

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$

L → H
for AND & OR
means everything
L → H



- 0: 2-1-AND = $0.15 + 0.037 = 0.187$
- 1: 2-2-AND = $0.15 + 0.074 = 0.224$
- 2: 2-2-OR = $0.12 + 0.074 = 0.194$
- 3: 2-3-AND = $0.15 + 0.111 = 0.261$
- 4: 2-3-OR = $0.12 + 0.111 = 0.231$
- 5: 2-2-AND = 0.224
- 6: 2-2-AND = 0.224
- 7: 2-1-AND = 0.187
- 8: 2-1-OR = $0.12 + 0.037 = 0.157$
- 9: 3-4-OR = $0.12 + 0.152 = 0.272$

$$\begin{array}{r}
 21 \\
 0.231 \\
 + 0.224 \\
 0.194 \\
 + 0.272 \\
 \hline
 0.921
 \end{array}$$

$$\begin{array}{r}
 11 \\
 .455 \\
 + .466 \\
 \hline
 .921
 \end{array}$$

4-1-2-9

$$\begin{array}{r}
 13 \\
 0.038 \\
 \times 4 \\
 \hline
 .152
 \end{array}$$

$$\begin{array}{r} 1 \\ 11 \\ +11 \\ \hline 111 \end{array}$$
 20

Problem 3 (20 points)

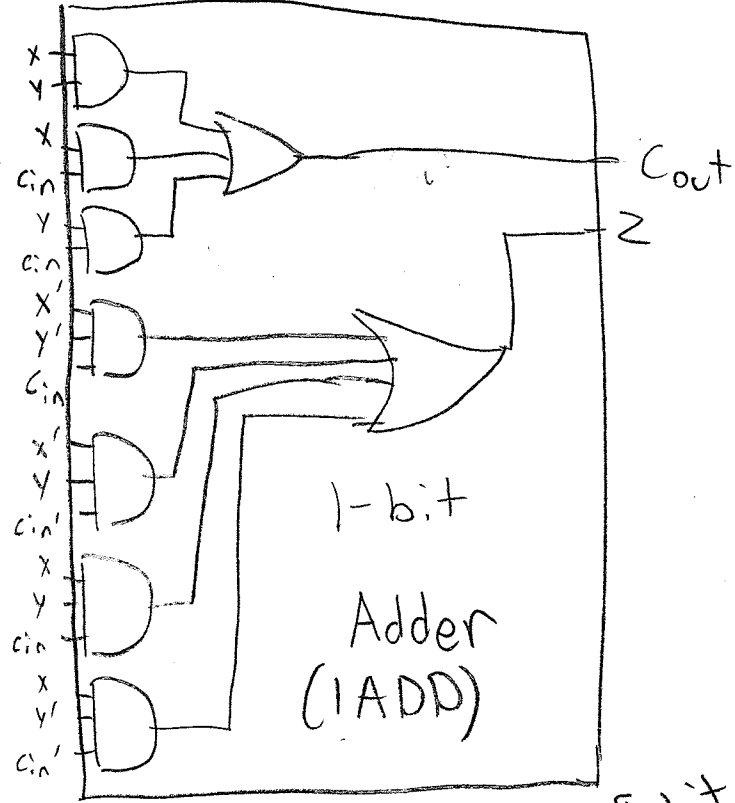
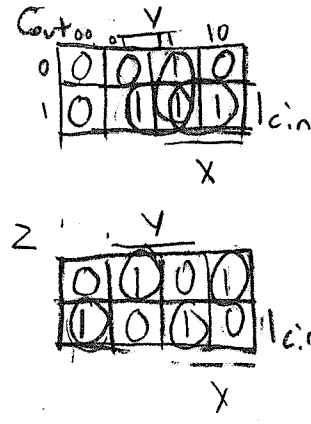
Four 4-bit numbers A, B, C, and D are given as inputs. $E=A+B$, $F=C+D$. Design a system that outputs the larger number between E and F. If $E=F$, output either E or F. You can use any type of gates to implement your design.

1) 1-bit adder with carry over

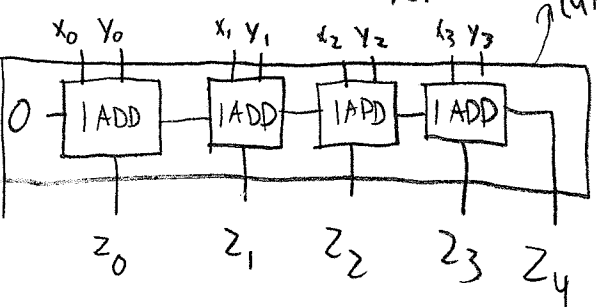
$$z = x'y'c_{in} + x'y c_{in}' + xy c_{in} + xy'c_{in}'$$

$$C_{out} = xy + x c_{in} + y c_{in}$$

x	y	c _{in}	C _{out}	z
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



2) 4-bit adder

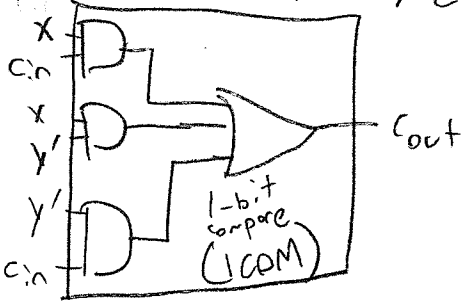


3) 1-bit comparator with carry over

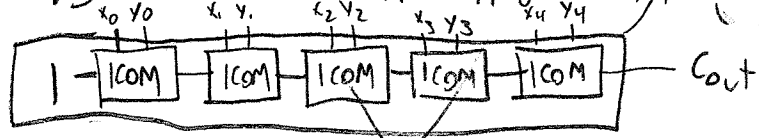
$x \geq y \rightarrow 1$
 $x < y \rightarrow 0$

x	y	c _{in}	C _{out}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

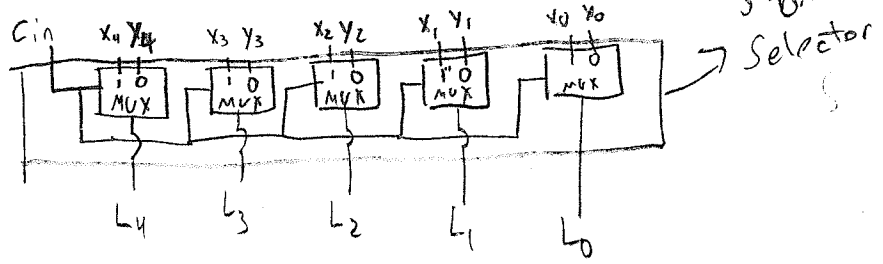
$$C_{out} = x c_{in} + x y' + y' c_{in}$$



4) 5-bit compare



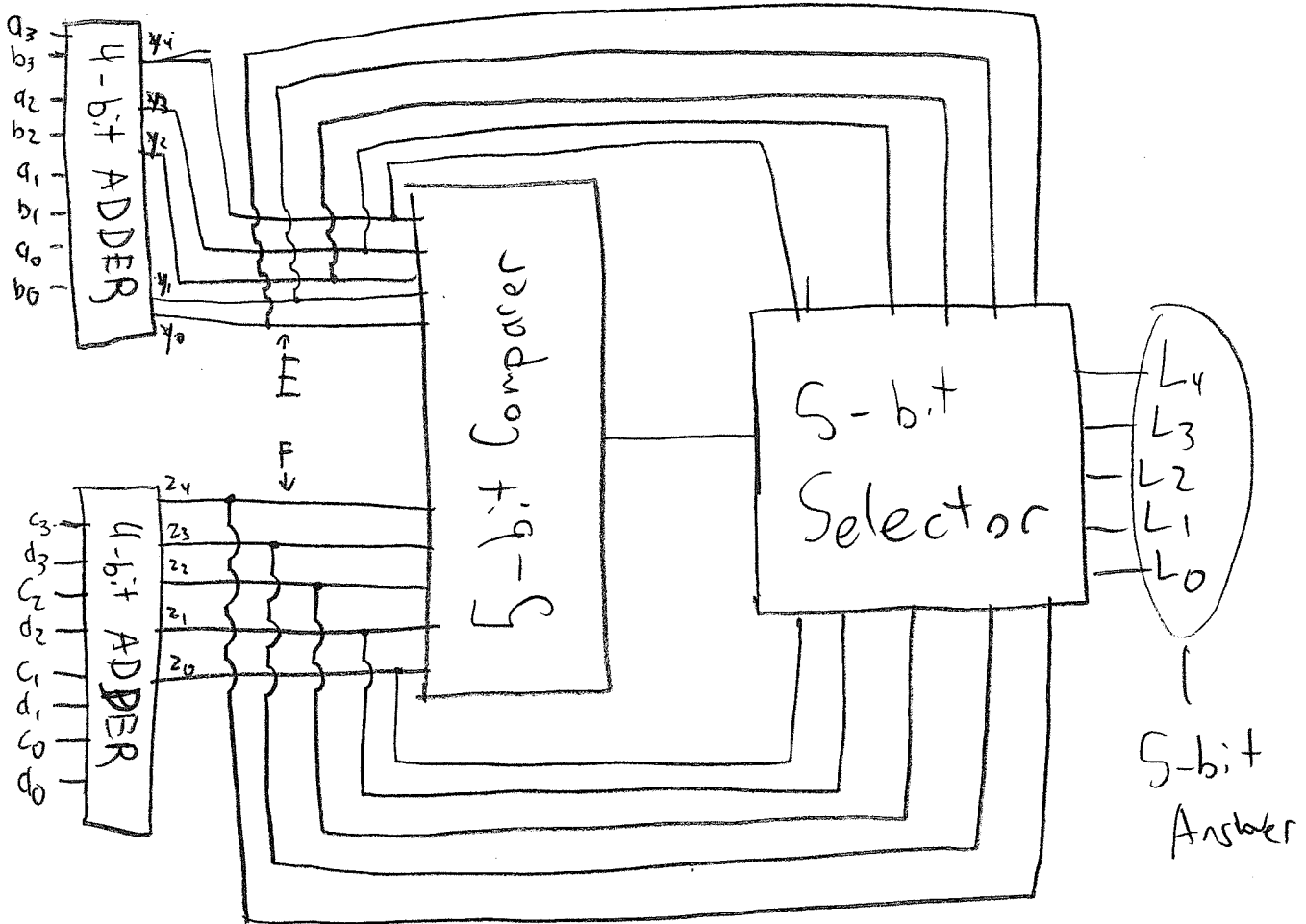
5) 5-bit Selector



Problem 3) Extra Page

b) Finish the Job

A $a_3 a_2 a_1 a_0$
B $b_3 b_2 b_1 b_0$
C $c_3 c_2 c_1 c_0$
D $d_3 d_2 d_1 d_0$



16

Problem 4 (20 points)

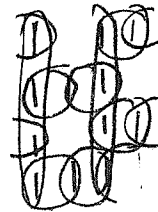
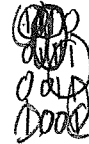
For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of $P=M-N$ among all the 4×4 K-maps.

For example, in the following 4×4 K-map, $M=3$, $N=2$, $P=M-N=1$.

	x_0				
	0	0	0	0	
	1	1	0	0	x_2
x_3	1	1	1	0	
	0	0	1	0	
	x_1				

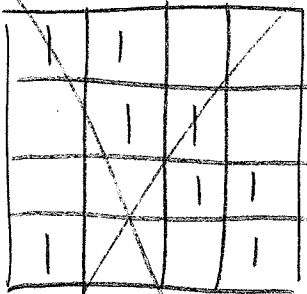


$P=8$

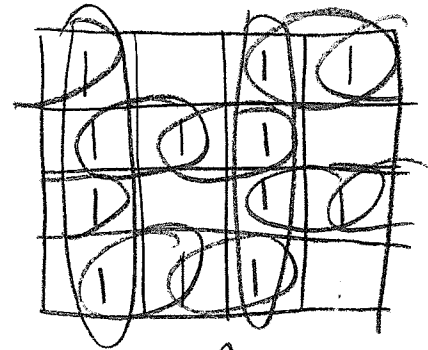
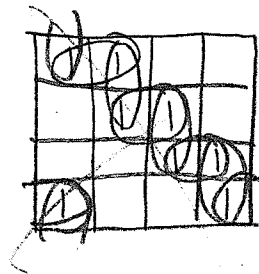


10

$M=8$
 $N=0$



$P=8$



$M=10$
 $N=0$

1	0	1	1
1	1	1	0
1	0	1	1
1	1	1	0

$\leftarrow P = 10$
* Winner

Problem 5 (20 points)

4-b: + 20
 ↑
 0000
 ↑

Use only multiplexers to design a system with input $x \in \{0,1,2, \dots, 8\}$, outputs y and z that implements the following equation

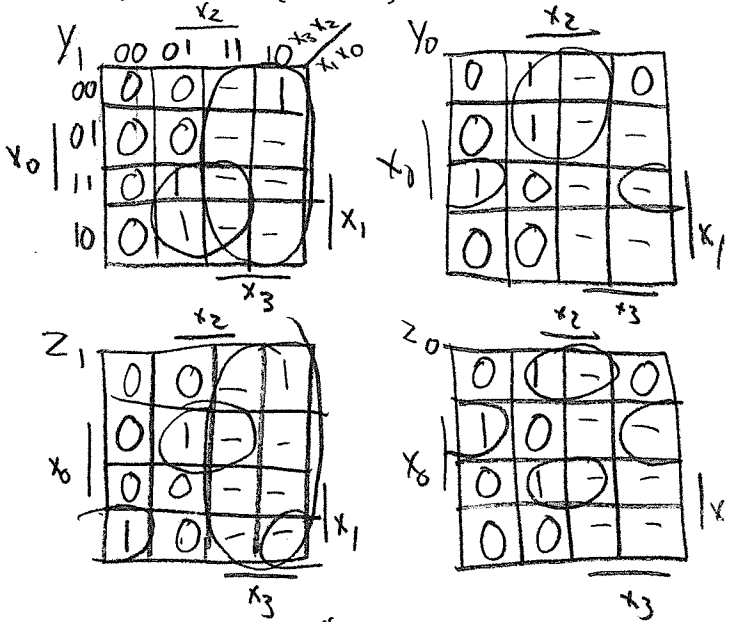
$$(x)_{10} = (yz)_3$$

In the system, x is encoded as $x_3x_2x_1x_0$ in binary. y is encoded as y_1y_0 in binary, and z is encoded as z_1z_0 in binary.

Note that the outputs y and z represent the two digits of a base-3 number.

For example, if $x=7$ ($x_3x_2x_1x_0=0111$), then the system will solve: $(7)_{10} = (21)_3$. Thus $y = 2$ ($y_1y_0=10$) and $z = 1$ ($z_1z_0=01$).

0	→	00	0000	→	0000
1	→	01	0001	→	0001
2	→	02	0010	→	0010
3	→	10	0011	→	0100
4	→	11	0100	→	0101
5	→	12	0101	→	0110
6	→	20	0110	→	1000
7	→	21	0111	→	1001
8	→	22	1000	→	1010
		$Y Z$	$x_3 x_2 x_1 x_0$		

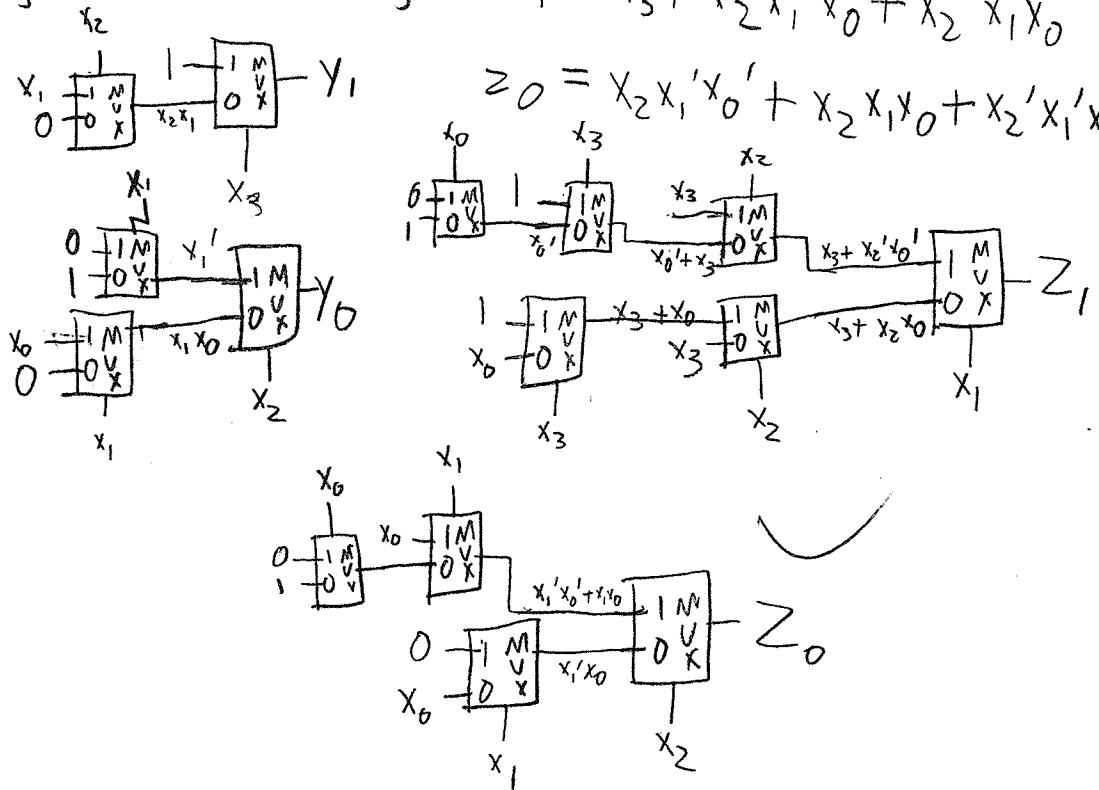


$$Y_1 = X_3 + X_2 X_1$$

$$Y_0 = X_2 X_1' + X_2' X_1 X_0$$

$$Z_1 = X_3 + X_2 X_1' X_0 + X_2' X_1 X_0'$$

$$Z_0 = X_2 X_1' X_0' + X_2 X_1 X_0 + X_2' X_1' X_0$$



Problem 5) Extra Page