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Computer Science Department

CSM51A/EEM16 Midterm Exam

Winter Quarter 2016

February 8th 2016

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

Problem (possible points)	Points
1 (20)	20
2 (20)	20
3 (20)	16
4 (20)	16
5 (20)	20
Total (100)	92

20

Problem 1 (20 points)

Use only the "E" gate defined below to implement Boolean function:
F=w'xy'+wxz+w'x'z+wx'y'z'
You may also use constants 0 and 1 as inputs.

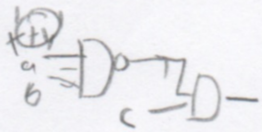
a	b	c	E(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$E = (a+b+c)(a'+b'+c')$

$E = ab' + ac' + a'b + bc' + a'c + b'c$

$b' + 0$

$(ab)' \quad (ab)'' + c''$



$F = w'xy' + wxz + w'x'z + wx'y'z'$

$w'y'(x + x'z')$

$= w'xy' + w'y'z' + wxz + w'x'z$

c

	0	1	1	1
a	1	1	0	1

b

$(ab)'$ → NAND gate

NAND $a'+b'$

$E(a,b,c)$

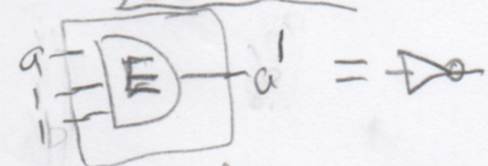
$ab' + a'c + bc'$

$E(a,1,1) = 0 + a' + 0 = a'$

$E(a',b',1) = a'b + a + 0 = a+b$ → OR

NAND

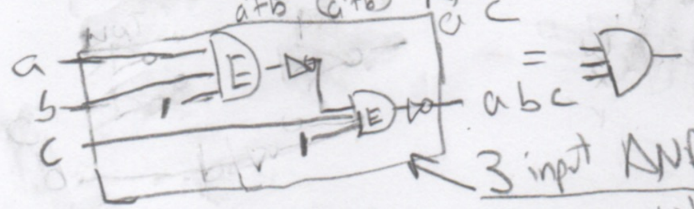
NOT Gate



AND

3 input NAND

$a'+b'$ $(a'+b')' + c'$ AND



3 input AND GATE

$, 0, b') = ((a'+c') + b')$

$= (a'+c')' b$

$= acb$

$E((a'+b')', c, 1)$

$= (a'+b'+c')'$

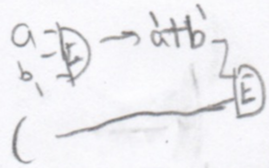
$= abc$

OR gate:

2 input OR:

a+c

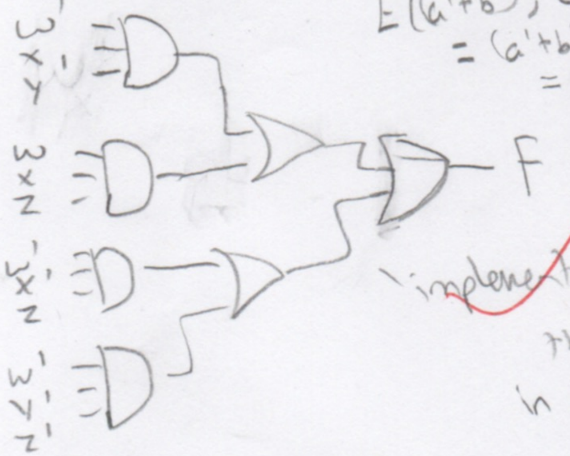
a+b



$(a'+b')' + c')'$

$(a'+b')'c$

abc



implemented using the NOT, AND, and OR gates in boxes above, which use the E gate.

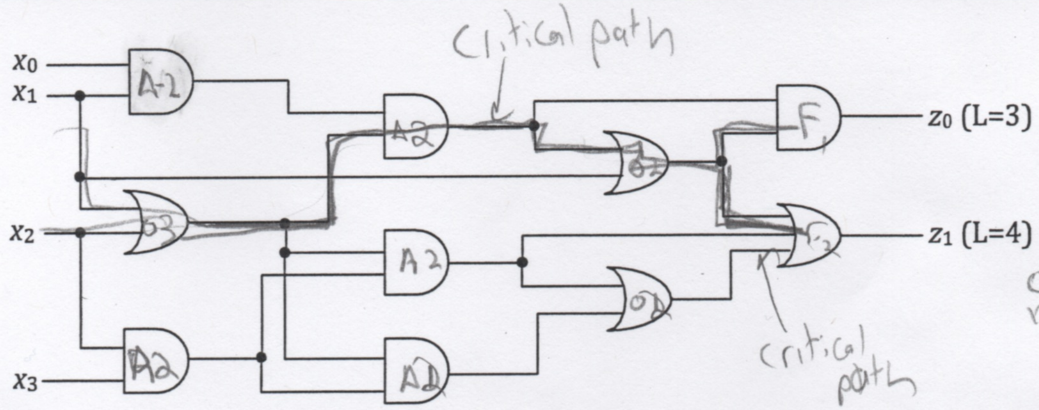
$(a'+b')'$

20

Problem 2 (20 points)

Given the network below, calculate the critical path delay. Consider L → H delay when calculating the critical path.

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$



0.037 × .11 .15 .261

z_0 : critical path :

either A2 → A2 → O2 → F1

OR A3 → A2 → O2 → F1

O3 - 3 outputs > A2 - 1 output, so

.24 .15 .39

0.037 × 7 .259 + .390 .649

.649 .261 .910

$t_{z_0} = 0.12 + 0.037(3) + 0.15 + 0.037(2) + 0.12 + 0.037(2) + F_1$

$= 0.649 + 0.261$

$= 0.910$

$F_1 = 0.15 + 0.037(3)$

t_{z_1} critical path : L: 2 2 2

A2 → A2 → O2 → F2

O3 > A2

or : O3 → A2 → O2 → F2 → critical

$t_{z_1} = 0.649 + 0.12 + 0.038(4)$

$= 0.921$

0.038 × 4 0.152 +0.12 .272

0.649 +0.272 0.921

16

Problem 3 (20 points)

Four 4-bit numbers A,B,C, and D are given as inputs. E=A+B, F=C+D. Design a system that outputs the larger number between E and F. If E=F, output either E or F. You can use any type of gates to implement your design.

$a_3a_2a_1a_0$ $b_3b_2b_1b_0$ $c_3c_2c_1c_0$ $d_3d_2d_1d_0$

Adder output is 5 bit $C_{out}Z_3Z_2Z_1Z_0$

C_{in}	X_i	Y_i	X_{out}	Z_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

C_{out}:

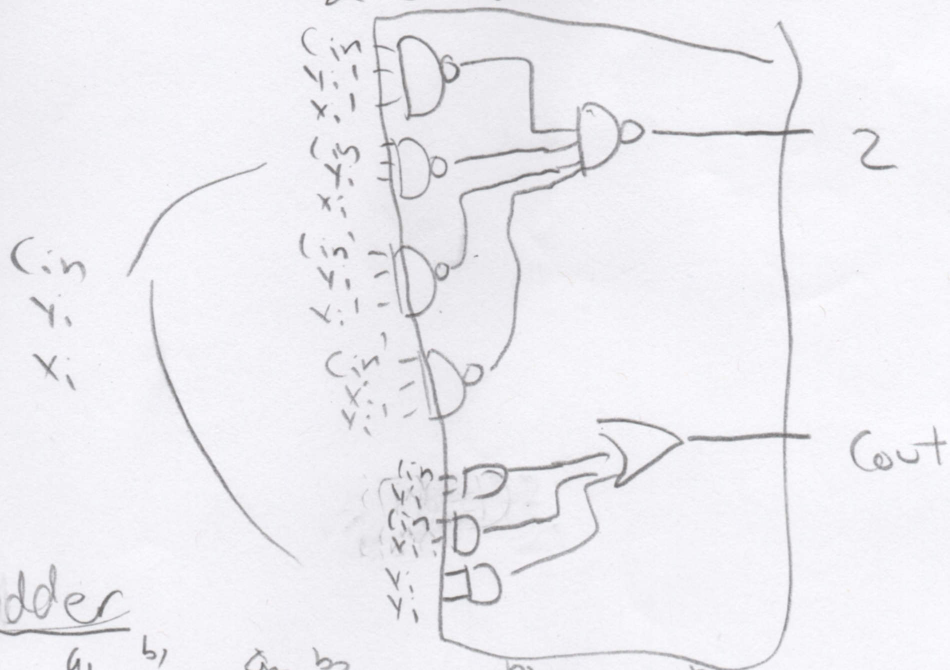
			X_i	
	0	0	1	0
C_{in}	0	1	1	1
			Y_i	

$C_{out} = C_{in}Y_i + C_{in}X_i + X_iY_i$

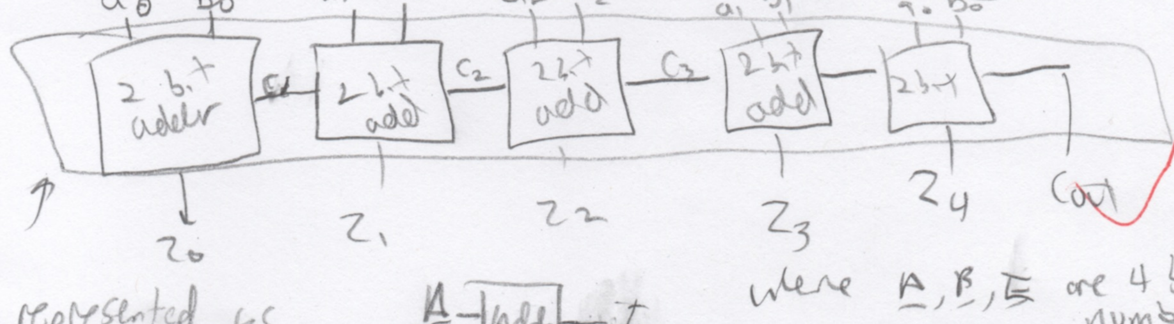
		Y_i		
Z_i	0	1	0	1
C_{in} 1	1	0	1	0
			X_i	

$Z = C_{in}Y_i'X_i' + C_{in}Y_iX_i + C_{in}'Y_iX_i' + C_{in}'X_iY_i'$

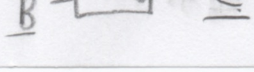
2 bit adder



Adder

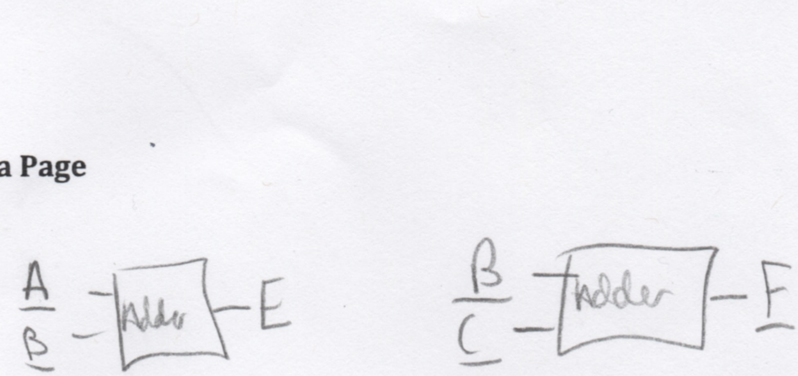


let this be represented as



where A, B, E are 4 bit numbers

Problem 3) Extra Page



to compare E & F

E : 01010
F : 10110 → output 1011

2 bit sorter

indicates X>Y or 0 if equal

Cin	X	Y	Cout	Z
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

Z ← will output ~~digit~~ of 1 that is higher

1 ~~both~~

1 because Cin

if Cin

Cin $\begin{cases} 1 & \text{if } \underline{X} > \underline{Y} \\ 0 & \text{otherwise} \end{cases}$

Must sort from highest bit to lowest bit

bit is set, then we know 4 bit $\underline{X}$ is greater than 4 bit $\underline{Y}$

not output the larger between each bit.

Z:

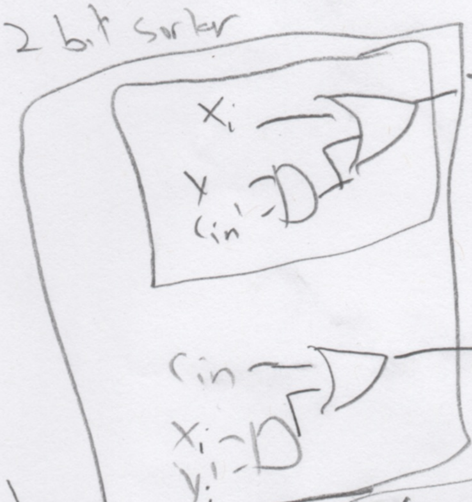
	0	1	1	1
Cin	0	0	1	1

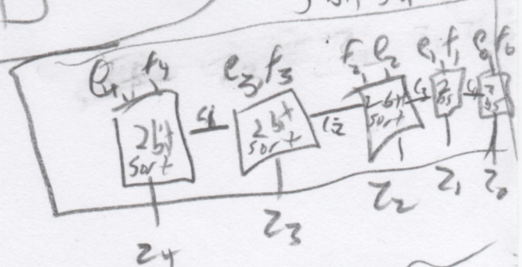
$= X + YCin'$

Cout:

	0	0	0	1
Cin	1	1	1	1

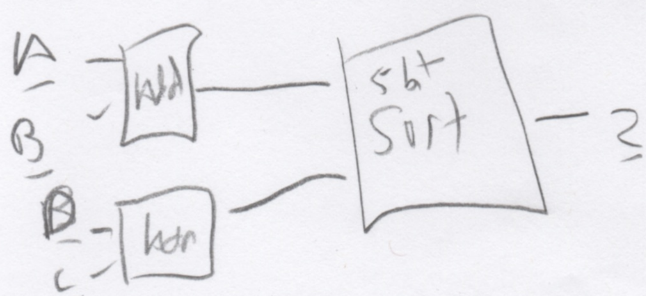
$Cin + XY'$





full sorter

5 bit sorter:



output $\underline{Z}$ is the greater of $\underline{E}$ or $\underline{F}$

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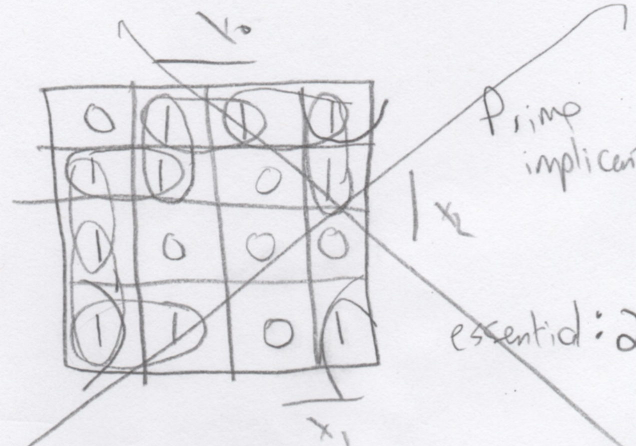
Problem 4 (20 points)

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of P=M-N among all the 4×4 K-maps.

For example, in the following 4×4 K-map, M=3, N=2, P=M-N=1.

		x_0	x_0		
	0	0	0	0	
	1	1	0	0	x_2
x_3	1	1	1	0	x_2
x_3	0	0	1	0	
			x_1	x_1	



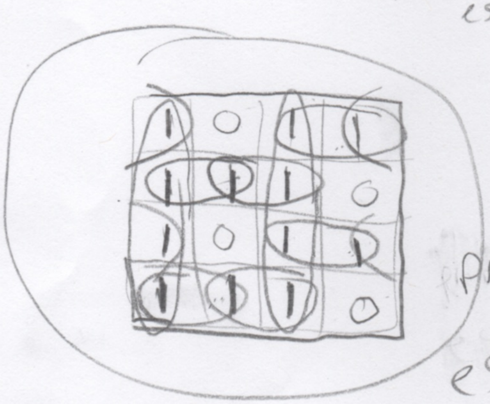
Prime implicants: 10

essential: 2

minimize ess. prime

maximize prime imp → use 2x2, 1x2 prime implicants

M-N=8



prime imp: 10

essential prime imp: 0

p=10

20

Problem 5 (20 points)

Use only multiplexers to design a system with input $x \in (0,1,2, \ldots 8)$, outputs y and z that implements the following equation

$$(x)_{10} = (yz)_3$$

In the system, x is encoded as $x_3x_2x_1x_0$ in binary. y is encoded as y_1y_0 in binary, and z is encoded as z_1z_0 in binary.
Note that the outputs y and z represent the two digits of a base-3 number.
For example, if x=7 ($x_3x_2x_1x_0$=0111), then the system will solve: $(7)_{10} = (21)_3$. Thus $y = 2$ (y_1y_0=10) and $z = 1$ (z_1z_0=01).

3	10	x_3	x_2	x_1	x_0	y_1	y_0	z_1	z_0
00	0	0	0	0	0	0	0	0	0
01	1	0	0	0	1	0	0	0	1
02	2	0	0	1	0	0	0	1	0
10	3	0	0	1	1	0	1	0	0
11	4	0	1	0	0	0	1	0	1
12	5	0	1	0	1	0	1	1	0
20	6	0	1	1	0	1	0	0	0
21	7	0	1	1	1	1	0	0	1
22	8	1	0	0	0	1	0	1	0
		1	0			—			
		1	0			—			
		1	0			—			
		1	0			—			
		1	1			—			
		1	1			—			
		1	1			—			

→ don't care

Y_1

x_0

0	0	0	0
0	0	1	1
–	–	–	–
1	–	–	–

x_2, x_3, x_1

$Y_1 = x_2 x_1 + X_3$

Y_0

x_0

0	0	1	0
1	1	0	0
–	–	–	–
0	–	–	–

x_2, x_3, x_1

$Y_0 = x_2 x_1' + x_2' x_1 x_0$

Z_1

x_0

0	0	0	1
0	1	0	0
–	–	–	–
1	–	–	–

x_2, x_3, x_1

$Z_1 = x_3 + x_2 x_0 x_1' + x_2' x_1 x_0'$

Z_0:

x_0

0	1	0	0
1	0	1	0
–	–	–	–
0	–	–	–

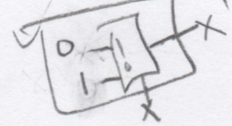
x_2, x_3, x_1

$Z_0 = x_2 x_1 x_0 + x_2 x_1' x_0' + x_2' x_0 x_1'$

Problem 5) Extra Page

$Y_1 = x_2 x_1 + x_3$

NOT gate:



$f_{x_3} = 1$

$f_{x_3'} = x_2 x_1$

$f_{x_3' x_2} = x_1$

$f_{x_3' x_2'} = 0$

$Y_0 = x_2 x_1' + x_2' x_1 x_0$

$f_{x_2} = x_1'$

$f_{x_2'} = x_1 x_0$

$f_{x_2' x_1} = x_0$

$f_{x_2' x_1'} = 0$

$Z_1 = x_3 + x_2 x_0 x_1' + x_2' x_1 x_0'$

$f_{x_3} = 1$

$f_{x_3'} = x_2 x_0 x_1' + x_2' x_1 x_0'$

$f_{x_3' x_2} = x_0 x_1'$ $\quad f_{x_3' x_2 x_1} = 0$

$f_{x_3' x_2'} = x_1 x_0'$ $\quad f_{x_3' x_2 x_1'} = x_0$

$f_{x_3' x_2' x_1} = x_0'$

$f_{x_3' x_2' x_1'} = 0$

$Z_0 = x_2 x_1 x_0 + x_2 x_1' x_0' + x_2' x_0 x_1'$

$f_{x_2} = x_1 x_0 + x_1' x_0'$ $\quad f_{x_2 x_0} = x_1$

$f_{x_2 x_0'} = x_1'$

$f_{x_2'} = x_0 x_1'$

$f_{x_2' x_0} = x_1'$

$f_{x_2' x_0'} = 0$