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CSM51A/EEM16 Midterm Exam

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This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

Problem (possible points)	Points
1 (20)	20
2 (20)	20
3 (20)	16
4 (20)	16
5 (20)	20
Total (100)	92

20

Problem 1 (20 points)

Use only the “E” gate defined below to implement Boolean function:

$$F = w'xy' + wxz + w'x'z + wx'y'z'$$

You may also use constants 0 and 1 as inputs.

a	b	c	E(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$E = (a+b+c)(a'+b'+c')$$

$$E = ab' + ac' + a'b + bc' + ac + b'c$$

$b' + \emptyset$

$$(ab)' = (a.b)' + c'$$

$$F = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4$$

$$w^{1,1}(x+x^{\dagger}z^{\dagger})$$

$$= w'xy' + w'y'z' + wxz + w'xz$$

$$\text{D}_0 \rightarrow \text{D}^-$$

$$F(a, b) = \frac{a + b}{a^2 + b^2} \rightarrow OR$$

$$E(a', b', 1) = \overline{ab + \bar{c}} \quad \text{NAND}$$

$$\frac{a+bc}{a+b} + \frac{c}{(a+b)} \text{ AND } \frac{c}{a+b}$$

A hand-drawn logic diagram for a 3-input AND gate. The inputs are labeled A, B, and C. Each input connects to one of two AND gates. Input A connects to the first AND gate's top input and the second AND gate's bottom input. Input B connects to the first AND gate's bottom input and the second AND gate's top input. Input C connects to both inputs of the second AND gate. The outputs of the first AND gate are labeled $a' + b' + c'$ and $(a'b)c$. The output of the second AND gate is labeled abc . The final output is labeled abc , representing the product of all three inputs.

A circuit diagram showing a NOT gate. It consists of a rectangular box labeled 'E' with two input terminals on the left and one output terminal on the right. The top input terminal is labeled 'a' and the bottom input terminal is labeled 'b'. The output terminal is labeled 'a''. Above the box, the text 'NOT gate' is written under a horizontal line.

3 input NAND

$$\begin{aligned} b' &= ((a'+c') + b) \\ &= (a''+c'') + b \\ &= (a'+c') b \\ &= ac'b \end{aligned}$$

$$E((a'+b')^1, c, 1) = (a'+b'+c')^1 = abc$$

OR gate: Input OR:

$$\begin{array}{c} a \\ b \\ \hline \end{array} \rightarrow \begin{array}{c} D \\ D \\ \hline D = \end{array} = \Rightarrow a+b$$

$$a+b \xrightarrow{b} ab$$

$$\frac{(a^1+b^1)^{1/c}}{(a^1+b^1)^c} = a^{1/c} b^{1/c}$$

3x =

3 x 11

$\tilde{x} \in \exists D$

$\gamma^w = D$

$$(a^1 + b^1)^1$$

~~implemented using~~

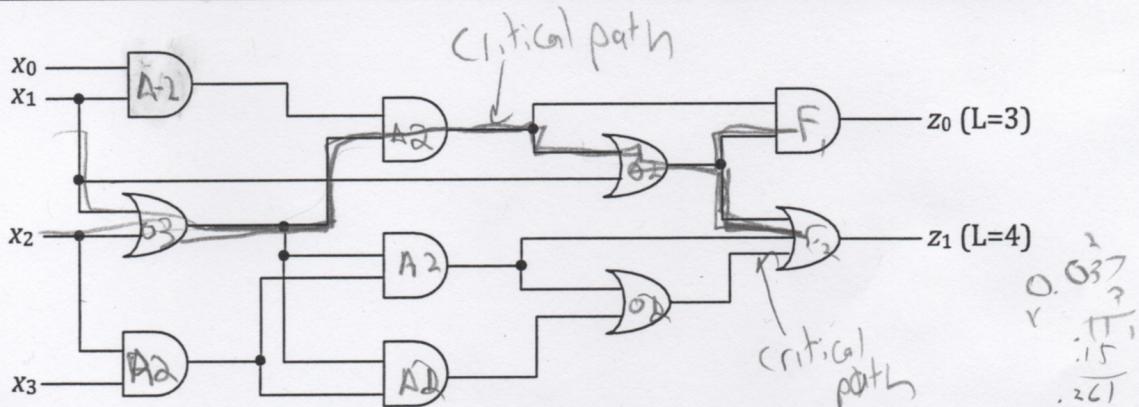
The NOT, AND, and OR gates
in boxes above, which use the
E gate.

20

Problem 2 (20 points)

Given the network below, calculate the critical path delay. Consider L → H delay when calculating the critical path.

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$



z_0 : critical path:

$$\text{either } A_2 \rightarrow A_2 \rightarrow O_2 \rightarrow f,$$

$$\text{OR } O_3 \rightarrow A_2 \rightarrow O_2 \rightarrow F,$$

$O_3 - 3 \text{ outputs} \rightarrow A_2 - 1 \text{ output}, \text{ so}$

$$t_{z_0} = 0.12 + 0.037(3) + 0.15 + 0.037(2) + 0.12 + 0.037(2) + f,$$

$$= 0.649 + 0.261$$

$$= 0.910$$

$$f = 0.15 + 0.037(3)$$

t_{z_1} critical path: up L: $A_2 \rightarrow A_2 \rightarrow O_2 \rightarrow F_2$ $O_3 \rightarrow A_2$

or: $O_3 \rightarrow A_2 \rightarrow O_2 \rightarrow F_2 \rightarrow \text{critical}$

$$t_{z_1} = 0.649 + 0.12 + 0.038(4)$$

$$= 0.921$$

$$0.649 + 0.12 + 0.038(4) = 0.921$$

16

Problem 3 (20 points)

Four 4-bit numbers A, B, C, and D are given as inputs. E = A + B, F = C + D. Design a system that outputs the larger number between E and F. If E = F, output either E or F. You can use any type of gates to implement your design.

$$a_3 a_2 a_1 a_0$$

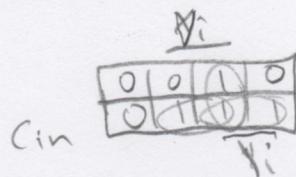
$$b_3 b_2 b_1 b_0$$

$$c_3 c_2 c_1 c_0 \quad d_3 d_2 d_1 d_0$$

Address output is 5 bit $Cout\ 2_3\ 2_2\ 2_1\ 2_0$

Cin	X _i	Y _i	Z _i	Cout
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Cout :

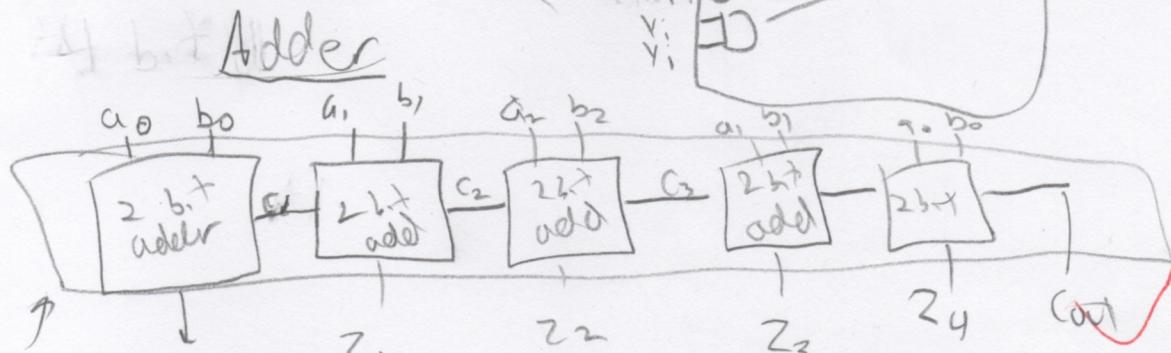
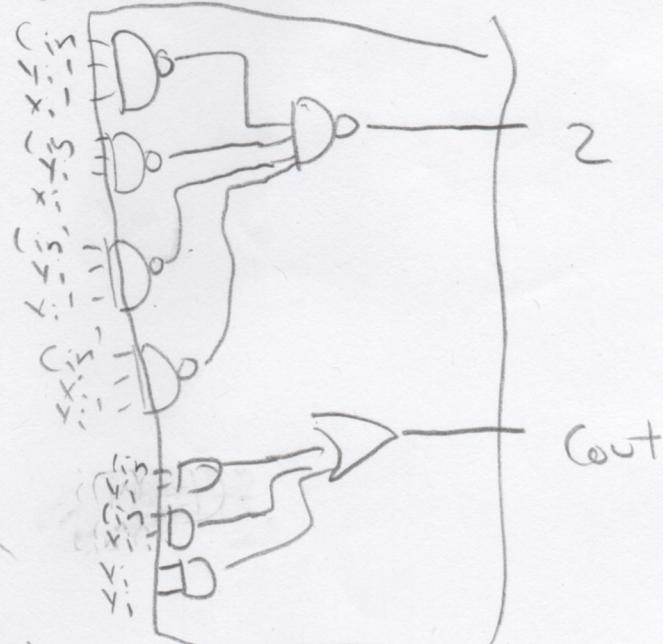


$$Cout = Cin Y_i + Cin X_i + X_i Y_i$$

$$\begin{array}{l} Z_i \\ \hline \text{Cin} \quad | \quad \begin{array}{|c|c|c|c|} \hline & 0 & 1 & 0 & 1 \\ \hline 1 & 1 & 0 & 1 & 0 \\ \hline \end{array} \\ \hline \quad \quad \quad X_i \end{array}$$

$$Z = Cin Y_i' X_i' + Cin Y_i X_i + (in' Y_i X_i' + Cin' X_i Y_i)$$

2 bit adder

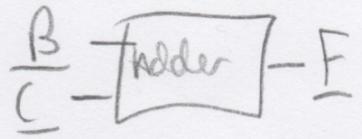


Let $x_{h.s.}$ be represented as

$$\frac{A}{B} = \boxed{\text{Adder}} = Z$$

where A, B, Z are 4 bit numbers

Problem 3) Extra Page



to compare $E \geq F$

indicates $x > y$ \rightarrow 2 bit sorter will put digit of that is higher at output.

$x > y$	$Cin \times Y$	Cin
0	000	0
0	001	0
0	010	1
0	011	0
1	100	0
1	101	0
1	110	1
1	111	0

OR
0 if equal
1 otherwise

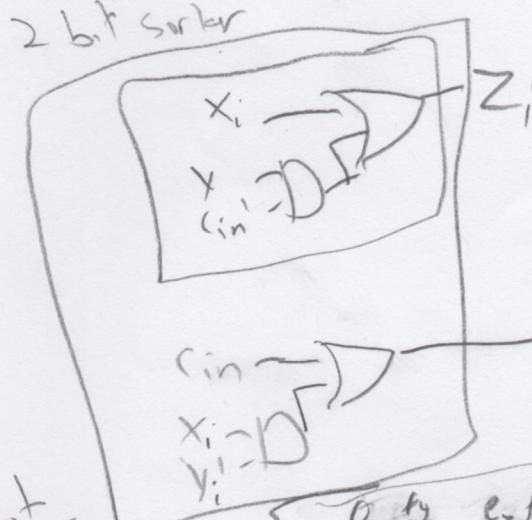
Must sort from highest bit to lowest bit

$E: 01010$
 $F: 10110$ \rightarrow output
1011

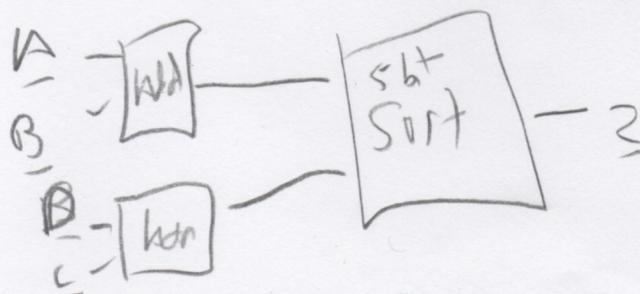
$$Z = X + Y \text{ (in)} = X + Y \text{ (in)}$$

$$C_{out} = \begin{array}{|c|c|c|c|} \hline 0 & 1 & 1 & 1 \\ \hline 0 & 0 & 1 & 1 \\ \hline \end{array}$$

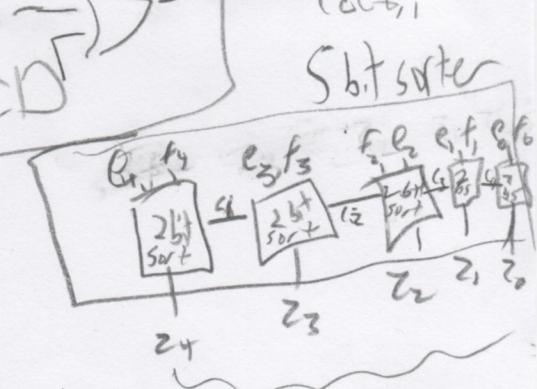
$$Y = \begin{array}{|c|c|c|c|} \hline 0 & 0 & 1 & 1 \\ \hline \end{array}$$



Full sorter



5 bit sorter



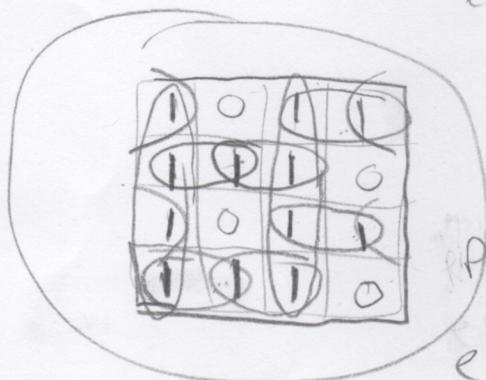
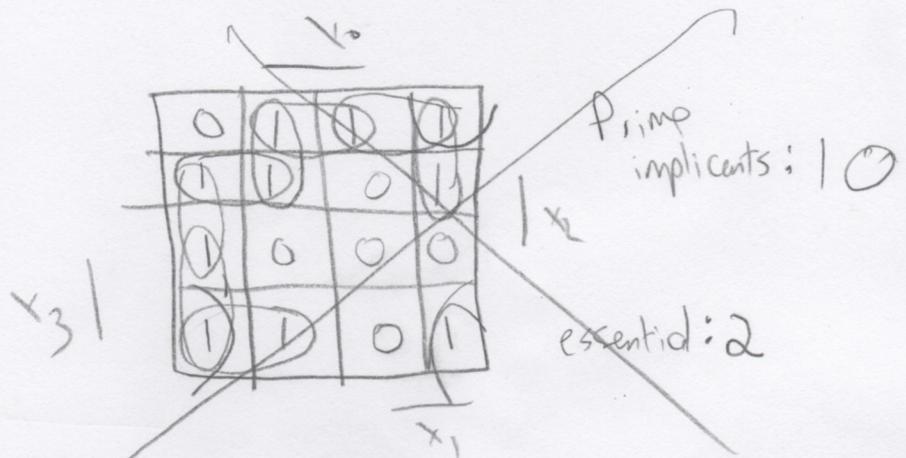
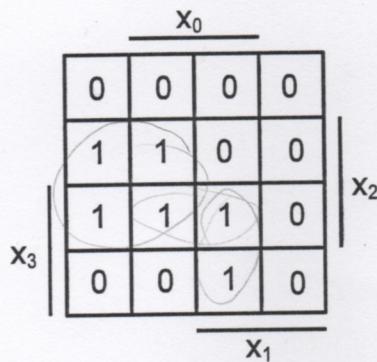
Output Z is the greater of E or F

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Problem 4 (20 points)

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of $P = M - N$ among all the 4×4 K-maps.

For example, in the following 4×4 K-map, $M=3$, $N=2$, $P=M-N=1$.



minimize
ess. prime

maximize
prime imp \rightarrow use 2×4 , 1×2 prime implicants

$M - N = 8$

prime imp: 10
essential prime
imp: 0

$$\therefore p = 10$$

20

Problem 5 (20 points)

Use only multiplexers to design a system with input $x \in \{0, 1, 2, \dots, 8\}$, outputs y and z that implements the following equation

$$(x)_{10} = (yz)_3$$

In the system, x is encoded as $x_3x_2x_1x_0$ in binary. y is encoded as y_1y_0 in binary, and z is encoded as z_1z_0 in binary.

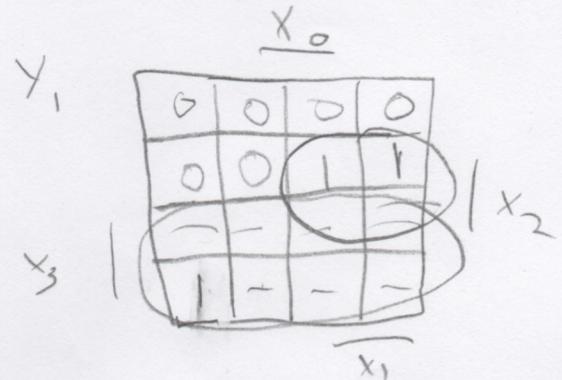
Note that the outputs y and z represent the two digits of a base-3 number.

For example, if $x=7$ ($x_3x_2x_1x_0=0111$), then the system will solve: $(7)_{10} = (21)_3$. Thus $y = 2$ ($y_1y_0=10$) and $z = 1$ ($z_1z_0=01$).

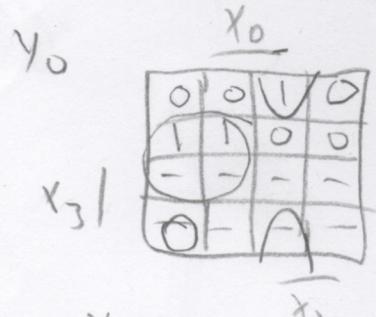
x_3	x_2	x_1	x_0	y_1, y_0, z_1, z_0
0	0	0	0	0000
0	0	0	1	0001
0	0	1	0	0010
0	0	1	1	0011
0	1	0	0	0100
0	1	0	1	0101
0	1	1	0	0110
0	1	1	1	0111
1	0	0	0	1000
1	0	0	1	1001
1	0	1	0	1010
1	0	1	1	1011
1	1	0	0	1100
1	1	0	1	1101
1	1	1	0	1110
1	1	1	1	1111
2	0	0	0	2000
2	0	0	1	2001
2	0	1	0	2010
2	0	1	1	2011
2	1	0	0	2100
2	1	0	1	2101
2	1	1	0	2110
2	1	1	1	2111
3	0	0	0	3000
3	0	0	1	3001
3	0	1	0	3010
3	0	1	1	3011
3	1	0	0	3100
3	1	0	1	3101
3	1	1	0	3110
3	1	1	1	3111
4	0	0	0	4000
4	0	0	1	4001
4	0	1	0	4010
4	0	1	1	4011
4	1	0	0	4100
4	1	0	1	4101
4	1	1	0	4110
4	1	1	1	4111
5	0	0	0	5000
5	0	0	1	5001
5	0	1	0	5010
5	0	1	1	5011
5	1	0	0	5100
5	1	0	1	5101
5	1	1	0	5110
5	1	1	1	5111
6	0	0	0	6000
6	0	0	1	6001
6	0	1	0	6010
6	0	1	1	6011
6	1	0	0	6100
6	1	0	1	6101
6	1	1	0	6110
6	1	1	1	6111
7	0	0	0	7000
7	0	0	1	7001
7	0	1	0	7010
7	0	1	1	7011
7	1	0	0	7100
7	1	0	1	7101
7	1	1	0	7110
7	1	1	1	7111
8	0	0	0	8000
8	0	0	1	8001
8	0	1	0	8010
8	0	1	1	8011
8	1	0	0	8100
8	1	0	1	8101
8	1	1	0	8110
8	1	1	1	8111

→ don't care

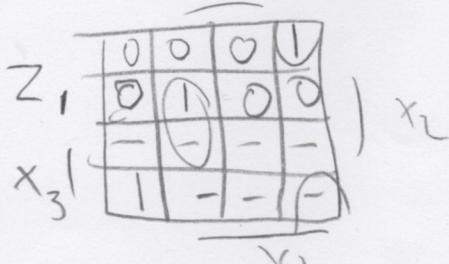
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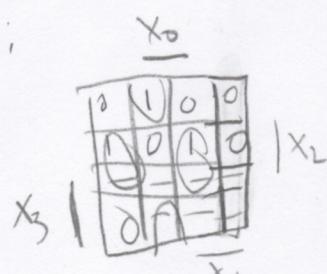
$$y_1 = x_2'x_3 + x_3'$$



$$y_0 = x_2'x_1' + x_2'x_0x_1$$



$$z_1 = x_3 + x_2x_0x_1' + x_2'x_0x_1$$

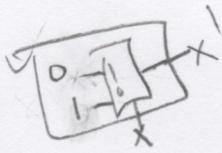


$$z_0 = x_2x_1x_0 + x_2x_1'x_0' + x_2'x_0x_1'$$

Problem 5) Extra Page

$$Y_1 = x_2 x_1 + x_3$$

NOT Gate:



$$f_{x_3} = 1$$

$$f_{x_3'} = x_2 x_1$$

$$f_{x_3' x_2} = x_1$$

$$f_{x_3' x_2'} = 0$$

$$Y_0 = x_2 x_1' + x_2' x_1 x_0$$

$$f_{x_2} = x_1'$$

$$f_{x_2'} = x_1 x_0$$

$$f_{x_1' x_0} = x_0$$

$$f_{x_1' x_0'} = 0$$

$$Z_1 = x_3 + x_2 x_0 x_1' + x_2' x_1 x_0'$$

$$f_{x_3} = 1$$

$$f_{x_3'} = x_2 x_0 x_1' + x_2' x_1 x_0'$$

$$f_{x_3' x_2} = x_0 x_1'$$

$$f_{x_3' x_2 x_1} = 0$$

$$f_{x_3' x_2'} = x_1 x_0$$

$$f_{x_3' x_2 x_1'} = x_0$$

$$f_{x_3' x_2' x_1} = x_0$$

$$f_{x_3' x_2' x_1'} = 0$$

$$Z_0 = x_2 x_1 x_0 + x_2 x_1' x_0' + x_2' x_0 x_1'$$

$$f_{x_2} = x_1 x_0 + x_1' x_0' \quad f_{x_2 x_0} = x_1$$

$$f_{x_2'} = x_0 x_1'$$

$$f_{x_2' x_0} = x_1'$$

$$f_{x_2' x_0'} = 0$$

