

(Iza)
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University of California
Los Angeles
Computer Science Department

CSM51A/EEM16 Midterm Exam
Winter Quarter 2016
February 8th 2016

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

Problem (possible points)	Points
1 (20)	20
2 (20)	20
3 (20)	20
4 (20)	16
5 (20)	20
Total (100)	96

Problem 1 (20 points)

Use only the "E" gate defined below to implement Boolean function:
 $F = w'xy' + wxz + w'x'z + wx'y'z'$
 You may also use constants 0 and 1 as inputs.

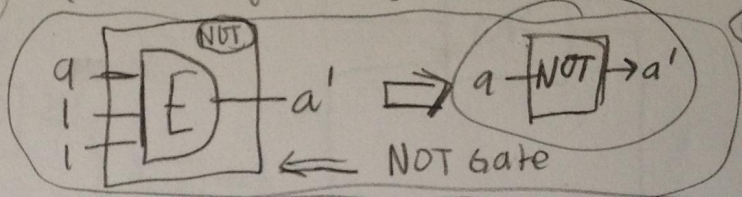
a	b	c	E(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$E = (a+b+c)(a'+b'+c') = ab'+ac'+a'b+bc'+a'c+bc$

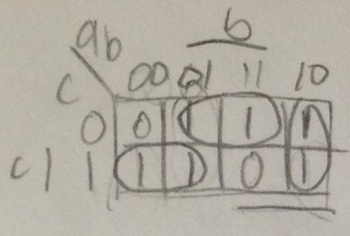
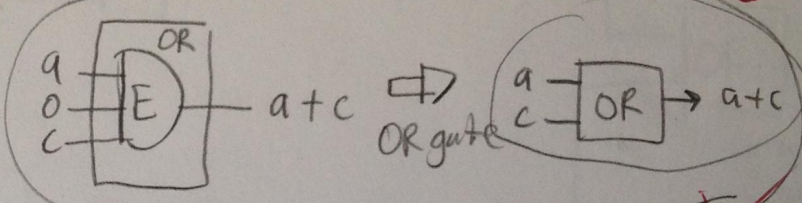
$E(a,a,a) = (a)(a') = 0$

$E(a,1,1) = (a+1+1)(a'+0+0) = (1)(a') = a'$

NOT gate



MODULES



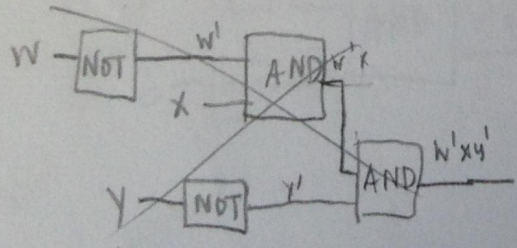
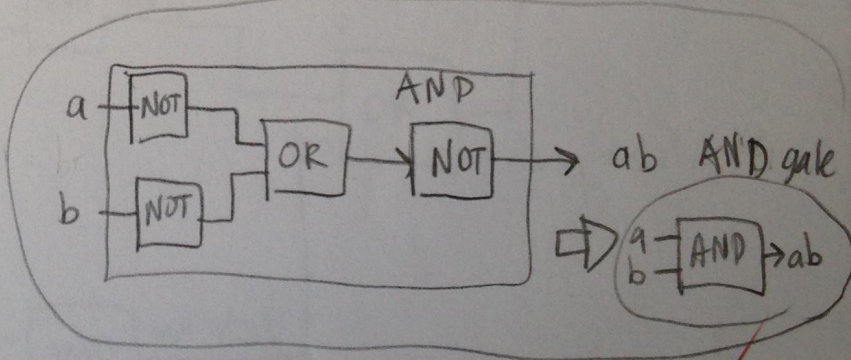
$E = bc' + ab' + a'c$

OR $E(a,0,c) = 0 + a + a'c = a + c$

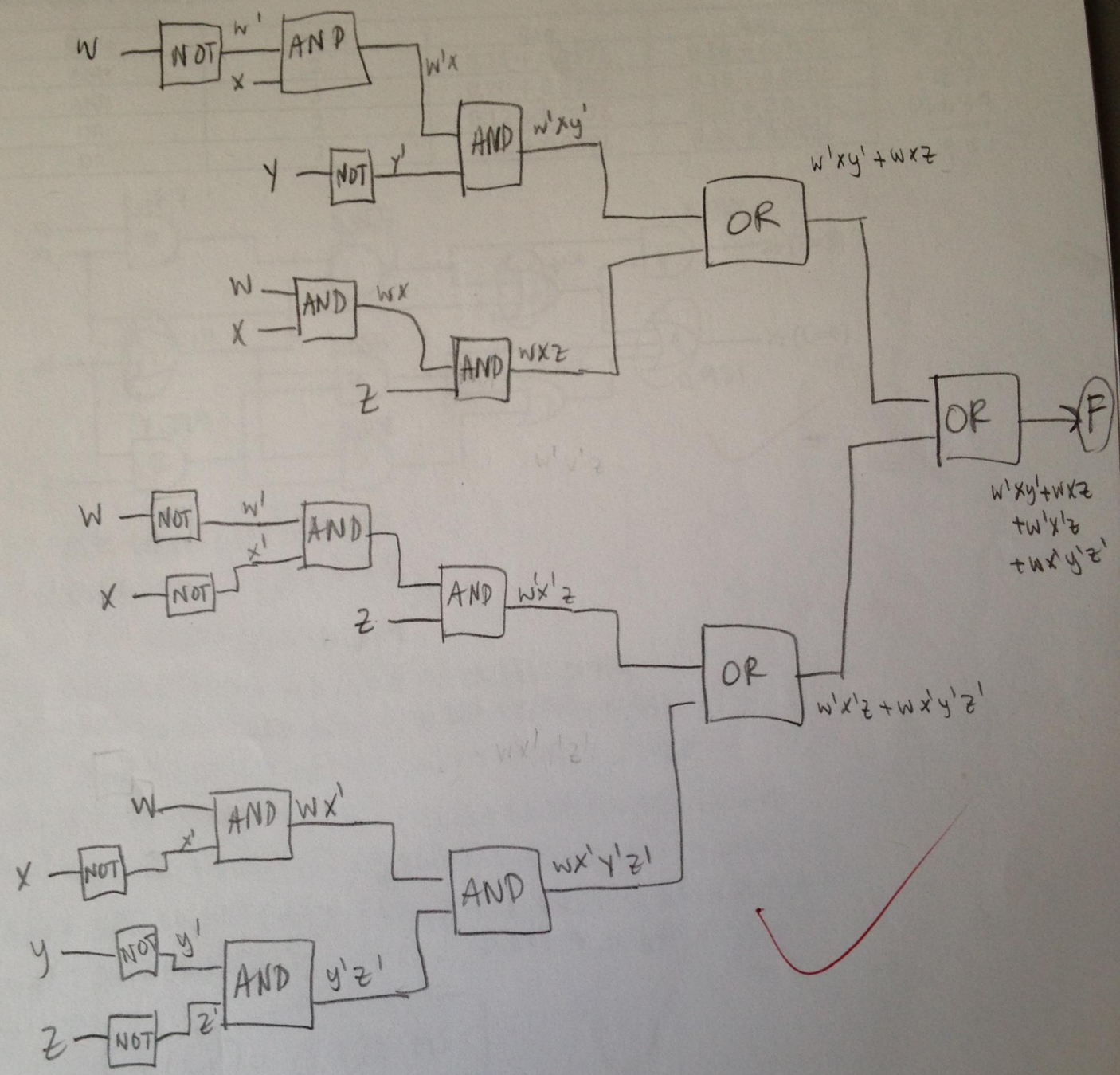
AND $E(c,0,0) = bc + 0 + 0 = bc$

$(a'+b')' = ab$

$NOT(OR(NOT(a), NOT(b))) \Rightarrow AND$



Problem 1) Extra Page



Handwritten calculations for propagation delays:

$$\begin{array}{r} 0.074 \\ 0.150 \\ \hline 0.224 \\ 0.231 \\ \hline 0.455 \end{array}$$

$$\begin{array}{r} 0.037 \\ \times 2 \\ \hline 0.074 \end{array}$$

$$\begin{array}{r} 0.637 \\ \times 3 \\ \hline 0.111 \\ 0.120 \\ \hline 0.231 \end{array}$$

$$\begin{array}{r} 0.150 \\ 0.037 \\ \hline 0.187 \end{array}$$

$$\begin{array}{r} 0.187 \\ 0.231 \\ \hline 0.418 \end{array}$$

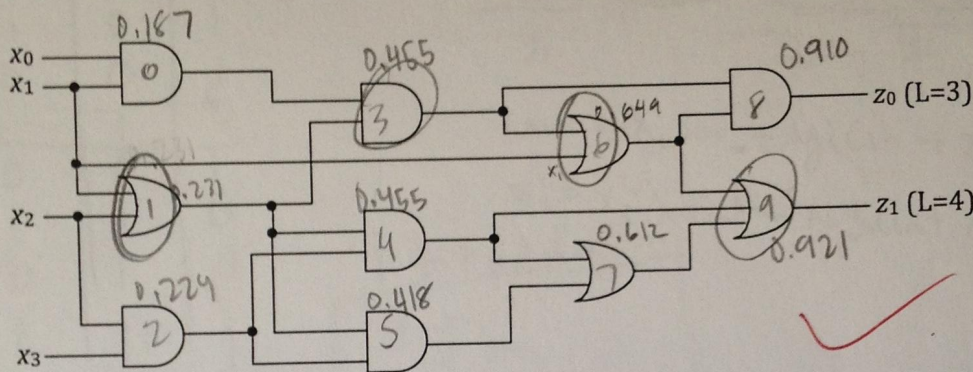
$$\begin{array}{r} 0.074 \\ 0.120 \\ \hline 0.194 \\ 0.455 \\ \hline 0.649 \end{array}$$

$$\begin{array}{r} 0.120 \\ 0.037 \\ \hline 0.157 \\ 0.455 \\ \hline 0.612 \end{array}$$

Problem 2 (20 points)

Given the network below, calculate the critical path delay. Consider L → H delay when calculating the critical path.

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$



AND $d(G_0) = 0.15 + 0.037(1) = 0.187$
 OR $d(G_1) = 0.12 + 0.037(2) = 0.231$
 AND $d(G_2) = 0.15 + 0.037(2) = 0.224$
 AND $d(G_3) = 0.15 + 0.037(2) + d(G_1) = 0.224 + 0.231 = 0.455$
 AND $d(G_4) = 0.15 + 0.037(2) + d(G_1) = 0.224 + 0.231 = 0.455$
 AND $d(G_5) = 0.15 + 0.037(1) + d(G_1) = 0.187 + 0.231 = 0.418$
 OR $d(G_6) = 0.12 + 0.037(2) + d(G_3) = 0.12 + 0.074 + 0.455 = 0.649$
 OR $d(G_7) = 0.12 + 0.037(1) + d(G_4) = 0.157 + 0.455 = 0.612$
 AND $d(G_8) = 0.15 + 0.037(3) + d(G_6) = 0.261 + 0.649 = 0.910$
 OR $d(G_9) = 0.12 + 0.038(4) + d(G_6) = 0.272 + 0.649 = 0.921$

$d(CP) = 0.921 \text{ ns}$

CP = G_1, G_3, G_6, G_9

Handwritten calculations on the right margin:

$$\begin{array}{r} 13 \\ 0.038 \\ \times 4 \\ \hline 0.152 \\ 0.120 \\ \hline 0.272 \\ 0.649 \\ \hline 0.921 \end{array}$$

$10 = (1110)_2$
 adding 4 bit #
 max $1111 = 15$
 $\frac{15}{2} = 7R1$
 $\frac{7}{2} = 3R1$
 $\frac{3}{2} = 1R1$
 $\frac{1}{2} = 0R1$
 $2 \times 4 + 2 + 1 = 30$
 max $15 + 15 = 30$
 $(11110)_2 \Rightarrow 5 \text{ bit}$

Problem 3 (20 points)

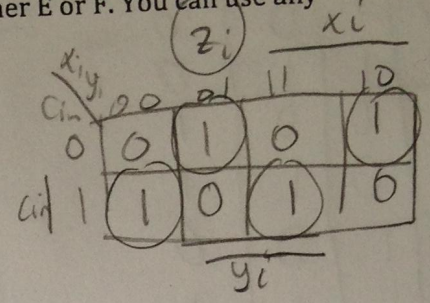
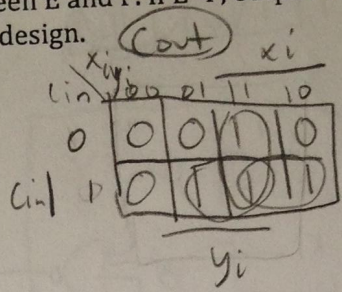
Four 4-bit numbers A, B, C, and D are given as inputs. $E = A + B$, $F = C + D$. Design a system that outputs the larger number between E and F. If $E = F$, output either E or F. You can use any type of gates to implement your design.

1. 1-bit adder
2. 4-bit adder
3. have 2 = 4-bit adders for E, F
4. 1-bit cmp \rightarrow 5-bit cmp
5. 5-bit cmp for E & F \rightarrow selector (mux)

20

① 1-bit adder

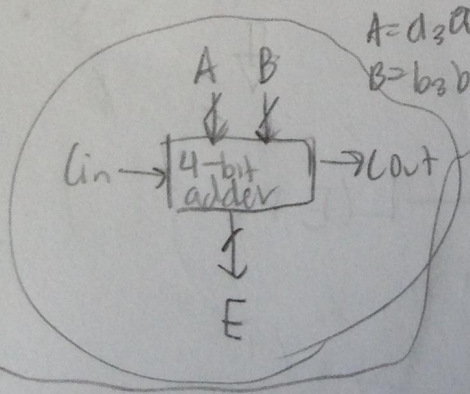
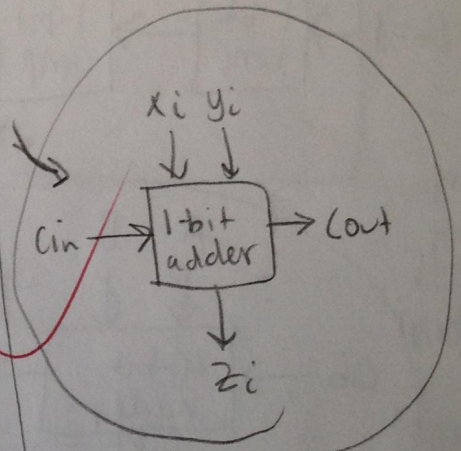
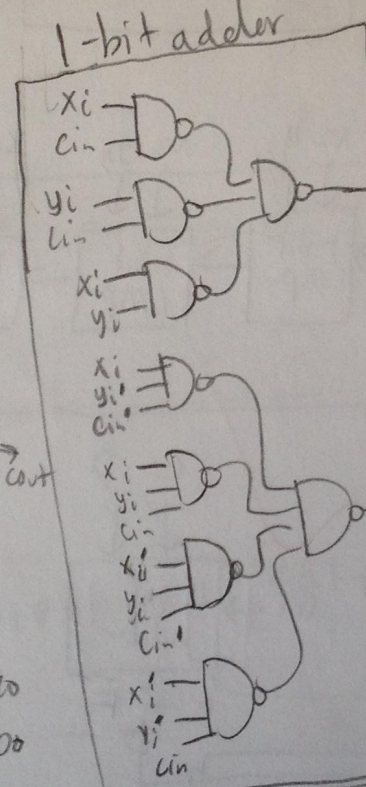
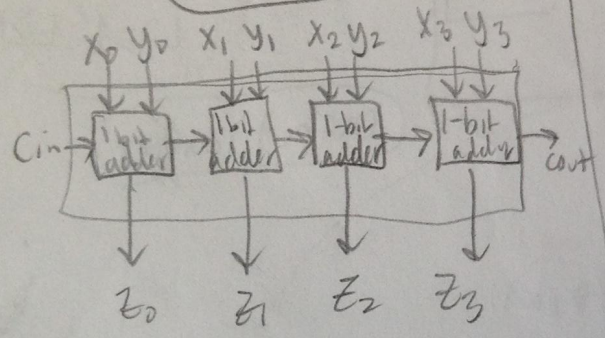
x_i	y_i	c_{in}	Cout	z_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



$Cout = x_i c_{in} + y_i c_{in} + x_i y_i$

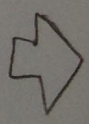
$z_i = x_i y_i' c_{in}' + x_i y_i c_{in} + x_i' y_i c_{in}' + x_i' y_i' c_{in}$

4-bit adder



$A = a_3 a_2 a_1 a_0$
 $B = b_3 b_2 b_1 b_0$

z_i

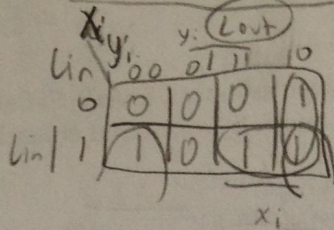


4. 1-bit comparator $X, Y = 4 \text{ bit numbers}$

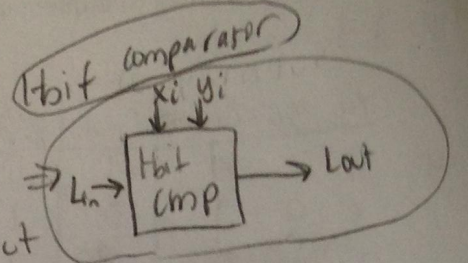
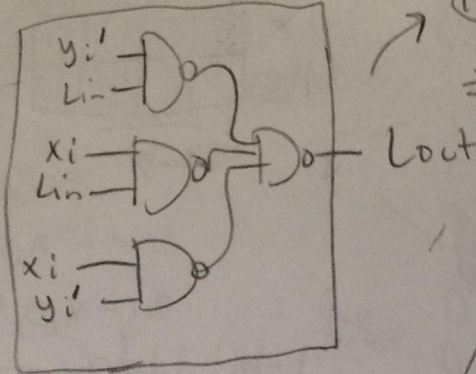
x_i	y_i	L_{in}	L_{out}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$L_{in} = 0 : X < Y$
 $L_{in} = 1 : X \geq Y$

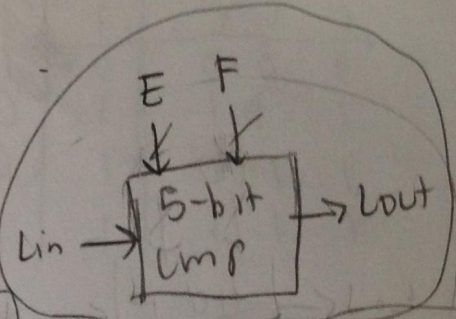
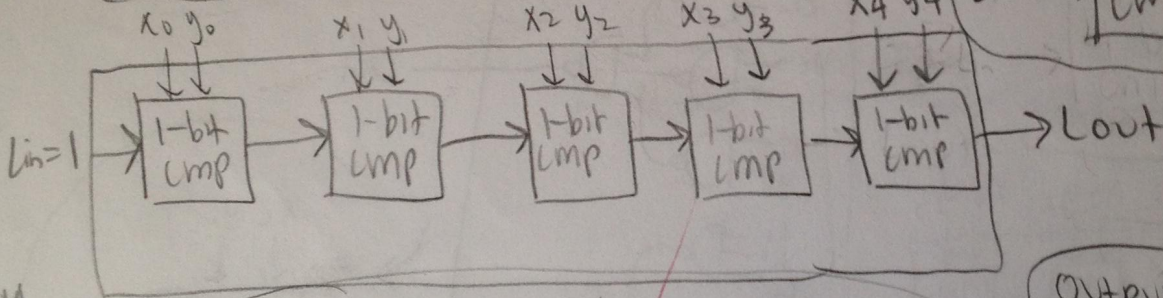
least sig to most sig bit



$$L_{out} = y_i' L_{in} + x_i L_{in} + x_i y_i'$$

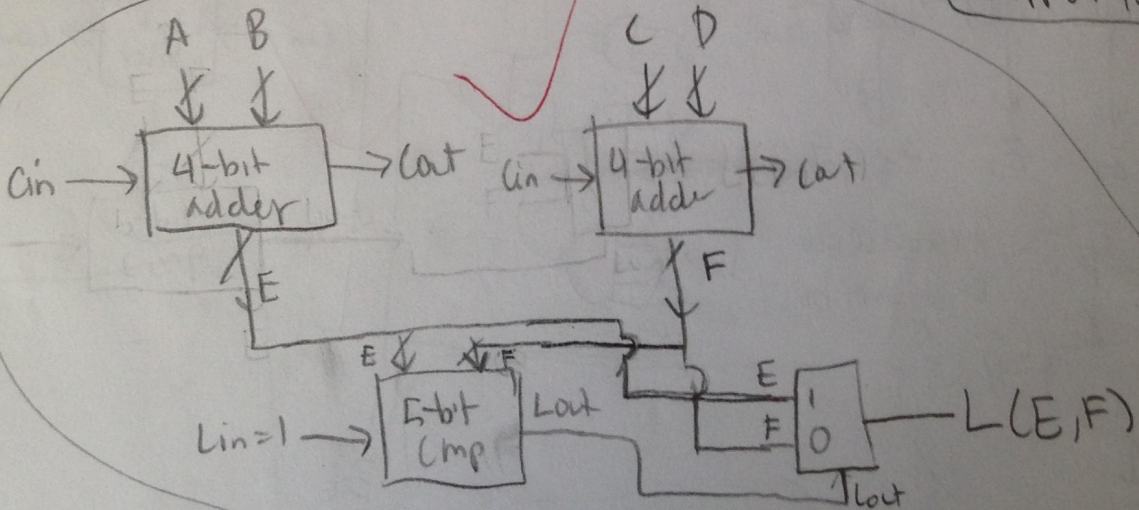
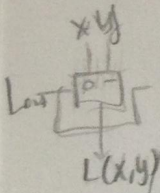


5-bit cmp



$L_{in} = 0 : E < F$
 $L_{in} = 1 : E \geq F$

Output carries



16

easier to not
don't care

M: PI
N: EPI

Problem 4 (20 points)

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4x4 K-map that has the largest value of $P=M-N$ among all the 4x4 K-maps.

= more most #PI & EPI

For example, in the following 4x4 K-map, $M=3, N=2, P=M-N=1$.

	x_0				
	0	0	0	0	
	1	1	0	0	x_2
x_3	1	1	1	0	
	0	0	1	0	
	x_1				

		x_0				
	00	01	11	10		
	00	1	1	0	0	
	01	0	1	0	0	
x_2	11	0	1	1	1	
	10	0	0	0	1	
		x_1				

6 PI
2 EPI
 $P=6-2=4$

0	1	1	1
1	0	0	1
0	0	0	1
1	0	1	0

PI: 1111
EPI: 11
 $P=6-2=4$

0	1	1	1
1	1	0	1
1	0	1	1
1	1	1	0

10 PI
0 EPI
 $P=10-0=10$

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

0	1	1	1
1	1	0	1
1	0	1	1
1	1	1	0

0	1	1	1
1	1	0	1
1	0	1	1
1	1	1	0

10 PI
0 EPI
 $P=10-0$

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

20

$$0 \leq x \leq 8$$

$$y \leq 2 \neq 3$$

$$z \leq 2 \neq 3$$

Problem 5 (20 points)

Use only multiplexers to design a system with input $x \in (0, 1, 2, \dots, 8)$ outputs y and z that implements the following equation

$$(x)_{10} = (yz)_3$$

In the system, x is encoded as $x_3x_2x_1x_0$ in binary. y is encoded as y_1y_0 in binary, and z is encoded as z_1z_0 in binary.

Note that the outputs y and z represent the two digits of a base-3 number.

For example, if $x=7$ ($x_3x_2x_1x_0=0111$), then the system will solve: $(7)_{10} = (21)_3$. Thus $y=2$ ($y_1y_0=10$) and $z=1$ ($z_1z_0=01$).

1. TT ✓
2. K-map ✓
3. expression ✓
4. decomp ✓
5. MUX

x	x_3	x_2	x_1	x_0	y_1	y_0	z_1	z_0	$(yz)_3$
0	0	0	0	0	0	0	0	0	$(00)_3$
1	0	0	0	1	0	0	0	1	$(01)_3$
2	0	0	1	0	0	0	1	0	$(02)_3$
3	0	0	1	1	0	1	0	0	$(10)_3$
4	0	1	0	0	0	1	0	1	$(11)_3$
5	0	1	0	1	0	1	1	0	$(12)_3$
6	0	1	1	0	1	0	0	0	$(20)_3$
7	0	1	1	1	1	0	0	1	$(21)_3$
8	1	0	0	0	1	0	1	0	$(22)_3$

y_1

	x_3	x_2	x_1	x_0
x_3	00	01	11	10
x_2	00	01	11	10
x_1	00	01	11	10
x_0	00	01	11	10

y_0

	x_3	x_2	x_1	x_0
x_3	00	01	11	10
x_2	00	01	11	10
x_1	00	01	11	10
x_0	00	01	11	10

z_1

	x_3	x_2	x_1	x_0
x_3	00	01	11	10
x_2	00	01	11	10
x_1	00	01	11	10
x_0	00	01	11	10

z_0

	x_3	x_2	x_1	x_0
x_3	00	01	11	10
x_2	00	01	11	10
x_1	00	01	11	10
x_0	00	01	11	10

DL

9	1	0	0	1	—	—
10	1	0	0	0	—	—
11	1	0	1	0	—	—
12	1	1	0	0	—	—
13	1	1	0	1	—	—
14	1	1	1	0	—	—
15	1	1	1	1	—	—

$$y_1 = x_3 + x_2x_1$$

$$y_0 = x_2x_1 + x_2'x_1x_0$$

$$z_1 = x_3 + x_2'x_1x_0 + x_2x_1'x_0$$

$$z_0 = x_2x_1'x_0 + x_2'x_1'x_0 + x_2x_1x_0$$

$$y_1 \begin{cases} y_1x_3 = 1 + x_2x_1 = 1 \\ y_1x_3' = x_2x_1 \end{cases}$$

$$y_0 \begin{cases} y_0x_2 = x_1 \\ y_0x_2' = x_1x_0 \end{cases}$$

$$z_1 \begin{cases} z_1x_1 = x_3 + x_2'x_0 \\ z_1x_1x_3 = 1 + x_2'x_0 = 1 \\ z_1x_1x_3' = x_2'x_0 \\ z_1x_1'x_3 = 1 + x_2x_0 \\ z_1x_1'x_3' = x_2x_0 \end{cases}$$

$$z_0 \begin{cases} z_0x_2 = x_1'x_0 + x_1x_0 \\ z_0x_2' = x_1'x_0 \\ z_0x_2x_1 = x_0 \\ z_0x_2x_1' = x_0' \end{cases}$$

Problem 5) Extra Page

$$c = a + b$$

$$a + 0 \text{ - AND}$$

Answer

