Stats
highest - 100
Lowest - 33
avg - ~<80
median - ~<80
Final more difficult.

University of California

Los Angeles

Computer Science Department

CSM51A/EEM16 Midterm Exam

Winter Quarter 2016

February 8th 2016

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

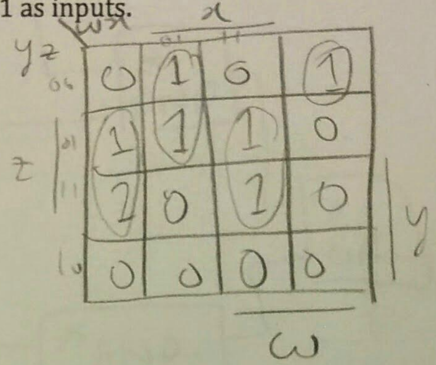
Problem (possible points)	Points
1 (20)	20
2 (20)	20
3 (20)	20
4 (20)	20
5 (20)	20
Total (100)	100

20

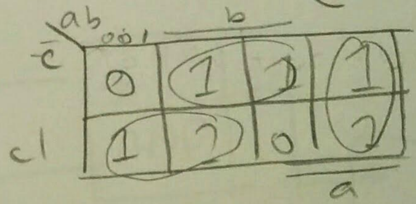
Problem 1 (20 points)

Use only the "E" gate defined below to implement Boolean function:
F=w'xy'+wxz+w'x'z+wx'y'z'
You may also use constants 0 and 1 as inputs.

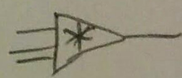
a	b	c	E(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



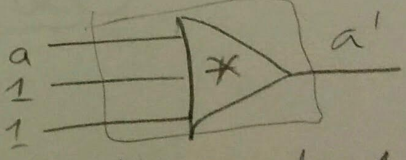
$E(a,b,c) = (a+b+c)\cdot(a'+b'+c')$



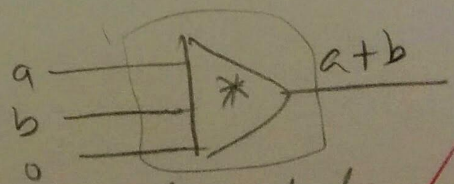
$E(a,b,c) = a'c + bc' + ab'$



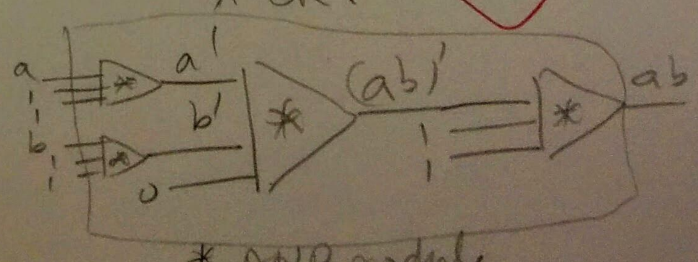
$E(a,1,1) = a'.1 + 1.0 + a.0$
$= a'$ → NOT



$E(a,b,0) = a'.0 + b.1 + ab'$
$= ab' + b = a + b$

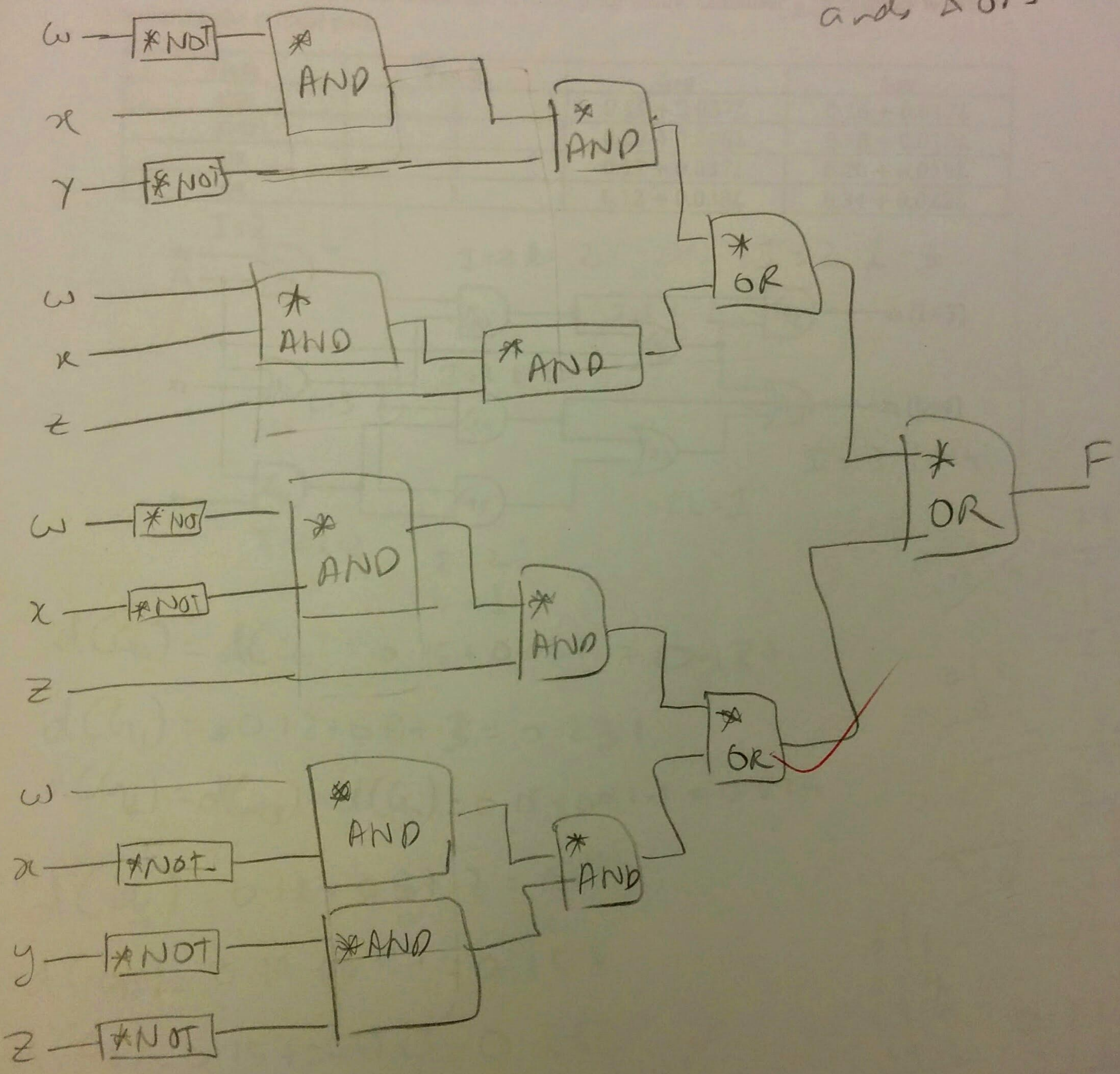


$E(a',b',0) = a.0 + b'.1 + a'b$
$= a' + b'$
$= (ab)'$



Problem 1) Extra Page

Because I have 2 input ands & ors



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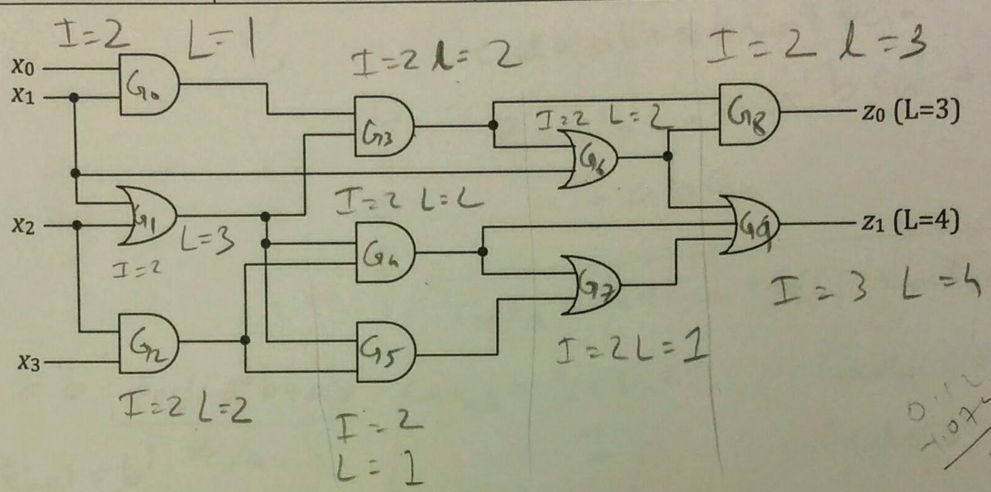
Problem 2 (20 points)

Given the network below, calculate the critical path delay. Consider L → H delay when calculating the critical path.

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$



$d(G_0) = d(G_5) = 0.15 + 0.037 = 0.187$

$d(G_1) = 0.12 + 0.037 \cdot 3 = 0.231$

$d(G_2) = d(G_3) = d(G_4) = 0.15 + 0.037 \times 2 = 0.224$

$d(G_6) = 0.12 + 0.037 \cdot 2 = 0.194$

$d(G_7) = 0.12 + 0.037 = 0.157$

$d(G_8) = 0.15 + 0.037 \times 3 = 0.261$

$d(G_9) = 0.12 + 0.038 \times 4 = 0.272$

Critical path

$G_1 \to G_3 \to G_6 \to G_9$

$T(CP) = 0.231 + 0.224 + 0.194 + 0.272$

$= 0.921$

we can see this looking gate with max delay in every layer

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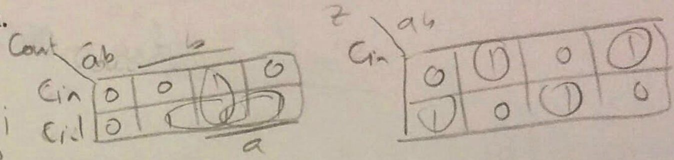
Problem 3 (20 points)

Four 4-bit numbers A,B,C, and D are given as inputs. E=A+B, F=C+D. Design a system that outputs the larger number between E and F. If E=F, output either E or F. You can use any type of gates to implement your design.

First 1 bit adder

a	b	Cin	Cout	z_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



$Cout = ab + aC_{in} + bc_{in}$

$Z = a'b'C_{in} + a'bC_{in}' + ab'C_{in}' + abC_{in}$

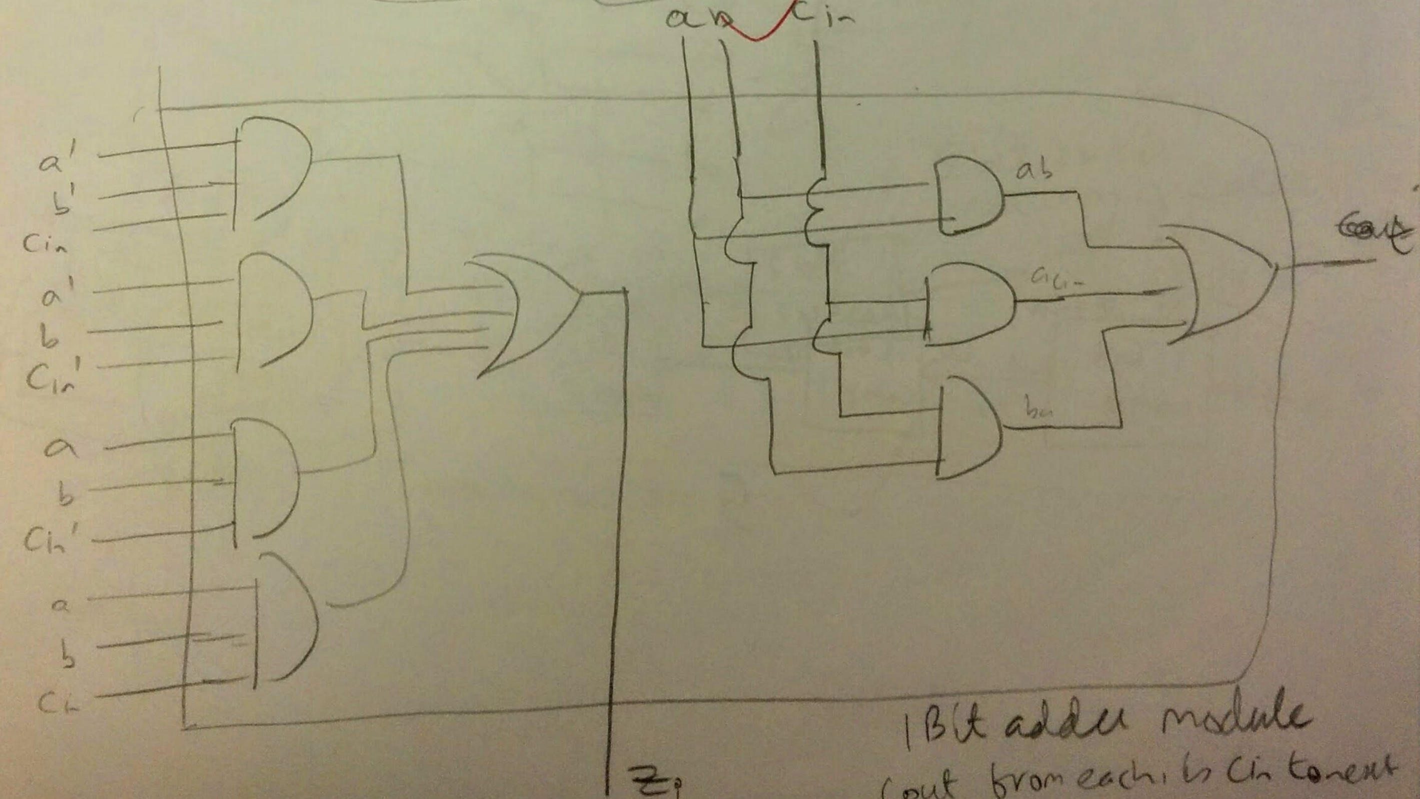
$Cout_a = b + C_{in} + bC_{in}$ $\quad Cout_{a'} = bC_{in}$

$Cout_{ab} = 1 + C_{in} + C_{in} = 1$ $\quad Cout_{a'b} = C_{in}$

$Cout_{ab'} = C_{in}$ $\quad Cout_{a'b'} = 0$

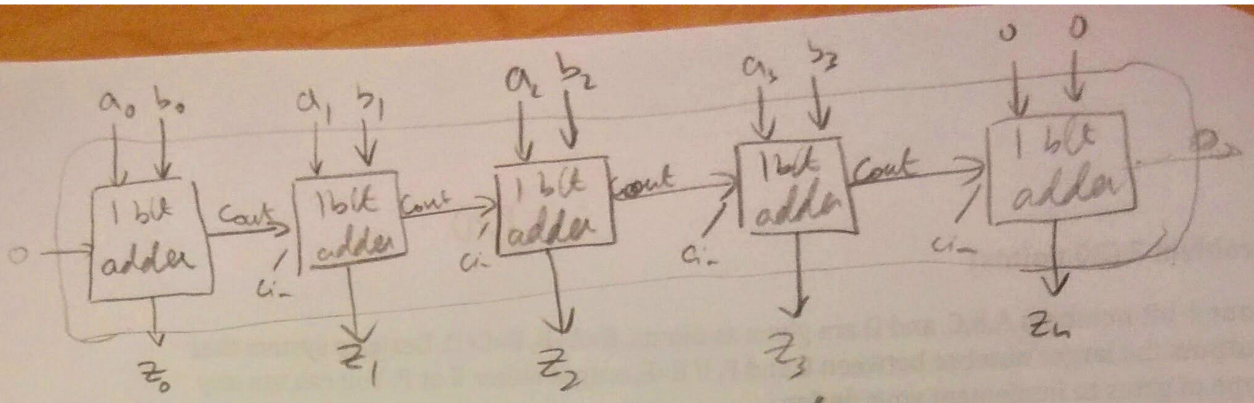
$z_{C_{in}} = a'b' + ab$ | $z_{C_{in}'} = a'b + ab'$

$z_{C_{in}a} = b$ | $z_{C_{in}a'} = b'$ | $z_{C_{in}'a} = b'$ | $z_{C_{in}'a'} = b$



1 Bit adder module

Cout from each is Cin to next



4-bit adder module

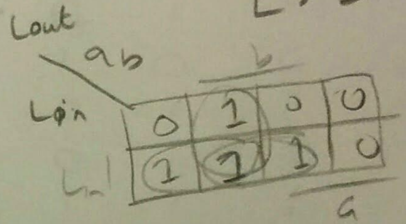
Comparator

a	b	L_{in}	L_{out}
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

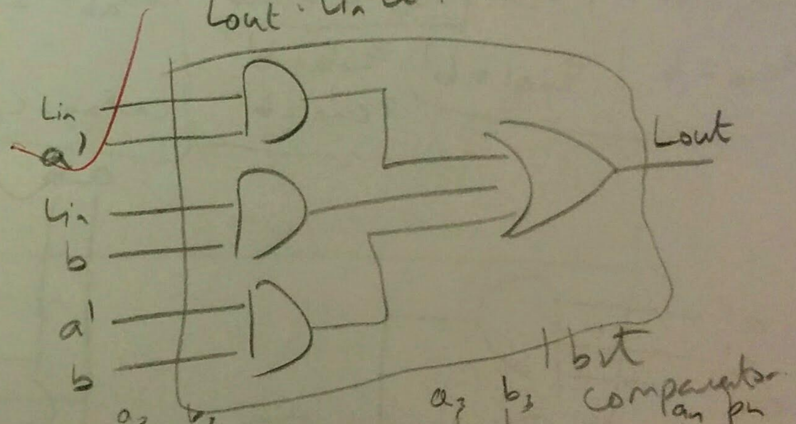
A, B

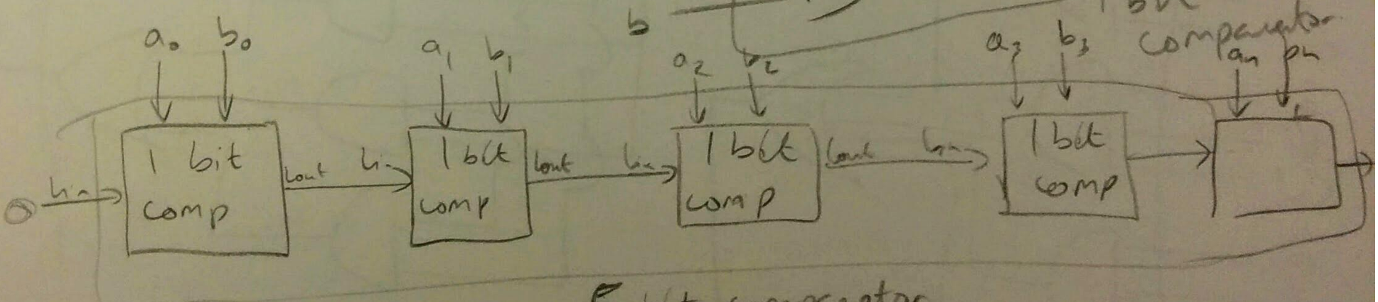
L → 0 → A > B

L → 1 → A ≤ B



$L_{out} = L_{in} a' + L_{in} b + a'b$





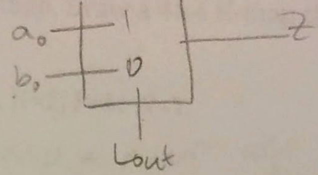
5 bit comparator

Problem 3) Extra Page

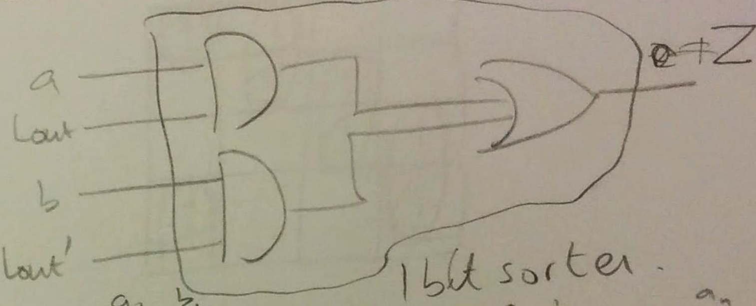
E) If a > b

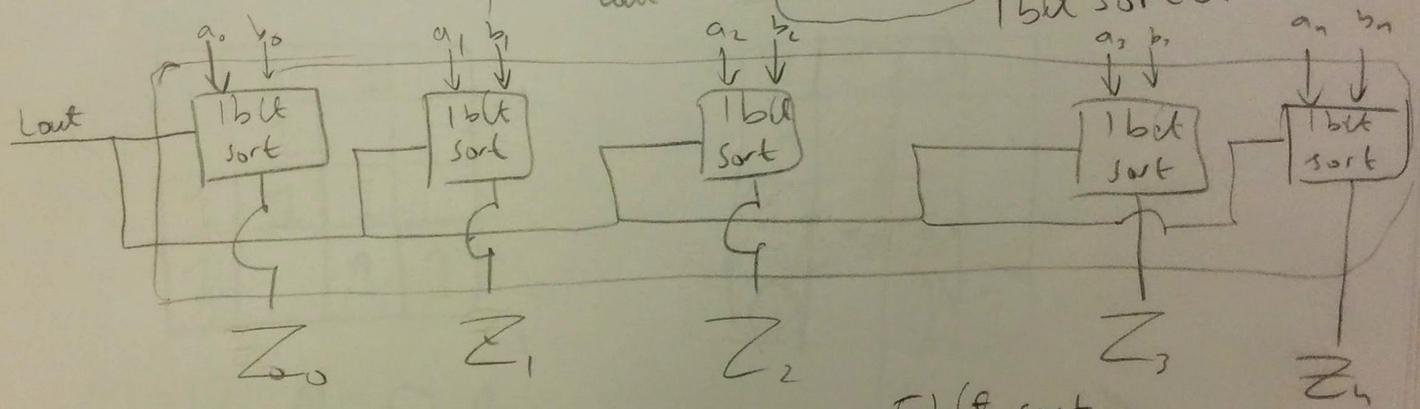
Sorter

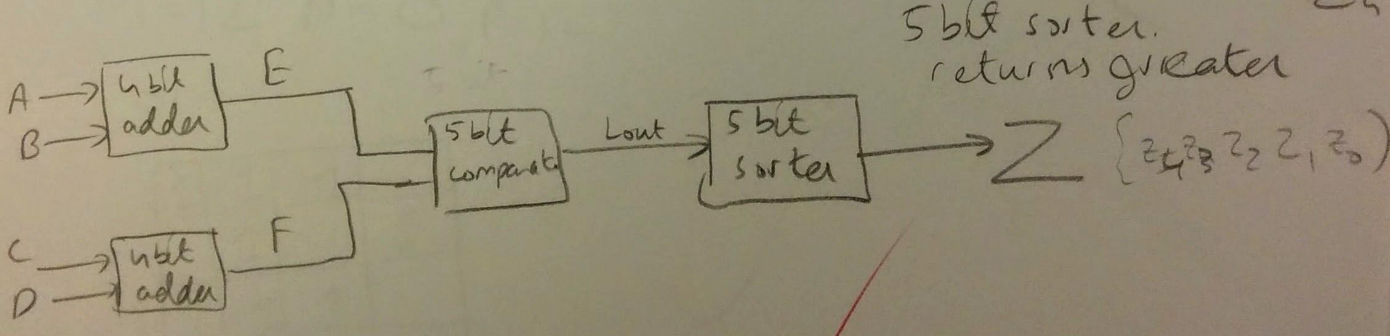
a	b	Lout	Z
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



$Z = Lout\, a + Lout'\, b$







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Problem 4 (20 points)

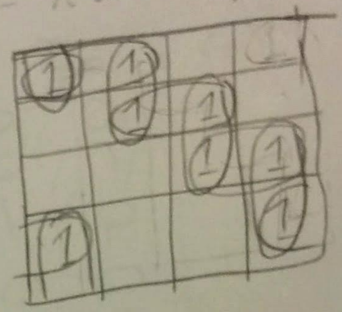
For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of P=M-N among all the 4×4 K-maps.

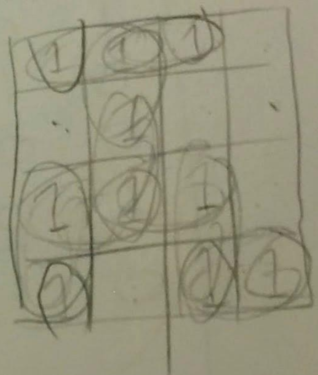
For example, in the following 4×4 K-map, M=3, N=2, P=M-N=1.

X_0 (top), X_2 (right), X_3 (left), X_1 (bottom)

0	0	0	0
1	1	0	0
1	1	1	0
0	0	1	0

max num of prime
m- number of ep





1	1	1	
	1		
1	1	1	
1		1	1

M=12 N=0 P=12

or

1	1	1	
	1		1
1	1	1	
1		1	1

M=13 N=1

P=12

✓

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Problem 5 (20 points)

Use only multiplexers to design a system with input $x \in (0,1,2,\ldots 8)$, outputs y and z that implements the following equation

$$(x)_{10} = (yz)_3$$

In the system, x is encoded as $x_3x_2x_1x_0$ in binary. y is encoded as y_1y_0 in binary, and z is encoded as z_1z_0 in binary.

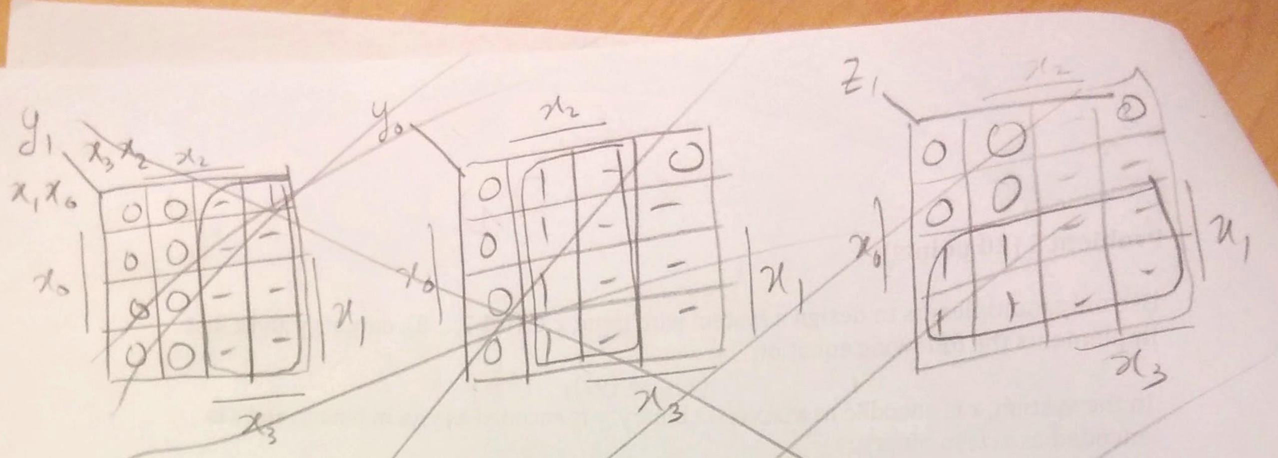
Note that the outputs y and z represent the two digits of a base-3 number.

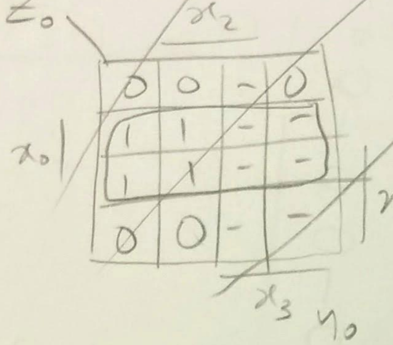
For example, if x=7 ($x_3x_2x_1x_0$=0111), then the system will solve: $(7)_{10} = (21)_3$. Thus $y = 2$ (y_1y_0=10) and $z = 1$ (z_1z_0=01).

x_3	x_2	x_1	x_0	y_1	y_0	z_1	z_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	1	0	0	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	1	0
1	0	0	1	–	–	–	–
1	0	1	0	–	–	–	–
1	0	1	1	–	–	–	–
1	1	0	0	–	–	–	–
1	1	0	1	–	–	–	–
1	1	1	0	–	–	–	–
1	1	1	1	–	–	–	–

ref 8 3×2+2 6+2=8

0 0
0 1
0 2
1 0
1 1
1 2
2 0
2 1
2 2
3×2+2
8



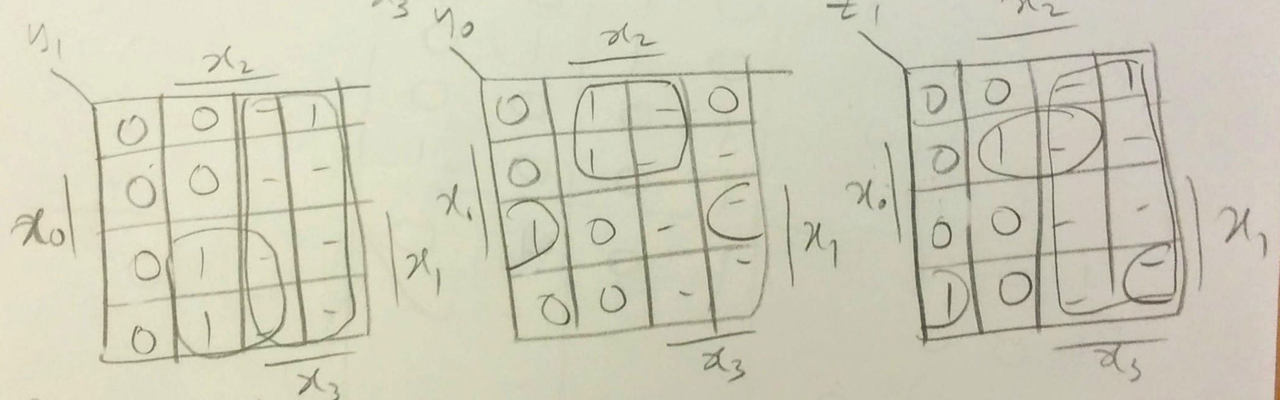


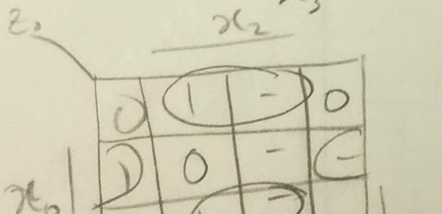
$y_1 = x_3$

$y_0 = x_2$

$z_1 = x_1$

$z_0 = x_0$





Problem 5) Extra Page

$y_1 = x_3 + x_2 x_1$ $\quad y_0 = x_2 x_1' + x_2' x_1 x_0$

$z_1 = x_3 + x_2 x_1' x_0$ $\quad z_0 = x_2 x_1' x_0' + x_2' x_1 x_0 + x_2 x_1 x_0$

$y_1 x_3 = 1 + x_2 x_1 = 1$

$y_1 x_3' = x_2 x_1$

$y_1 x_3' x_2 = x_1$

$y_1 x_3' x_2' = 0$

$y_0 x_2 = x_1'$

$y_0 x_2' = x_1 x_0$

$y_0 x_2' x_1 = x_0$

$y_0 x_2' x_1' = 0$

$z_1 x_3 = 1$

$z_1 x_3' = x_2 x_1' x_0$

$z_1 x_3' x_2 = x_1' x_0$

$z_1 x_3' x_2' = 0$

$z x_3' x_2 x_0 = x_1'$

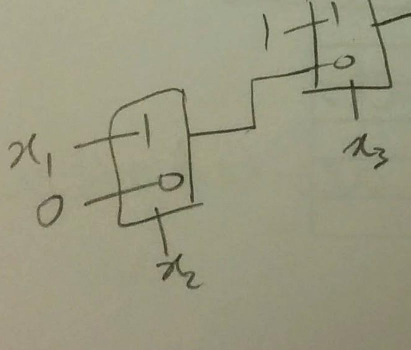
$z x_3' x_2 x_0' = 0$

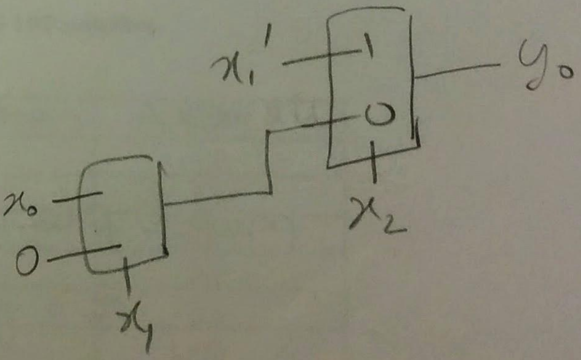
$z_0 x_1 = x_2' x_0$ $\quad z_0 x_1 x_2 = x_0$

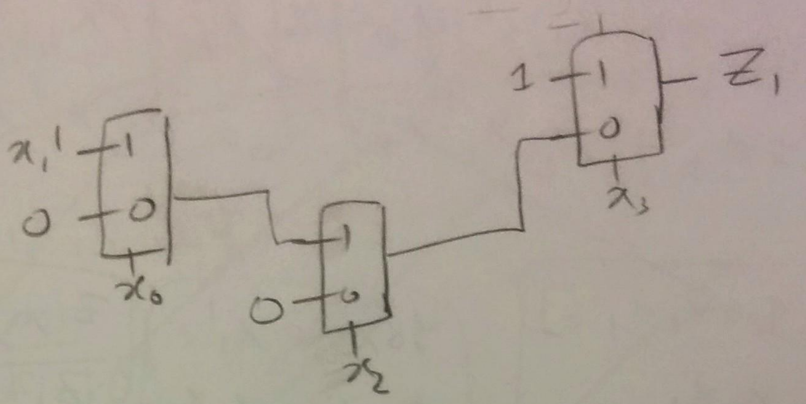
$z_0 x_1' = x_2 x_0' + x_2' x_0$ $\quad z_0 x_1 x_2' = 0$

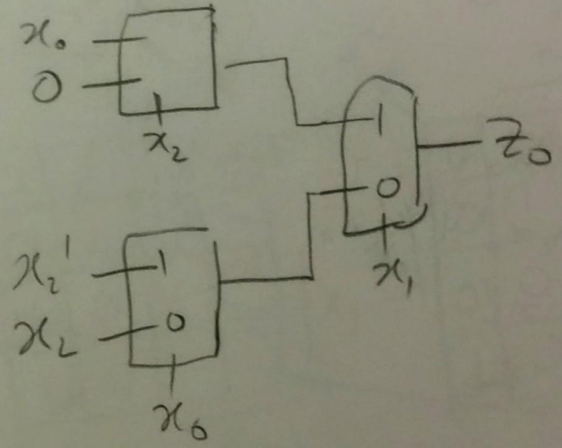
$z_0 x_1' x_0 = x_2'$

$z_0 x_1' x_0' = x_2$









We can box all these multiplexer sets to form desired output