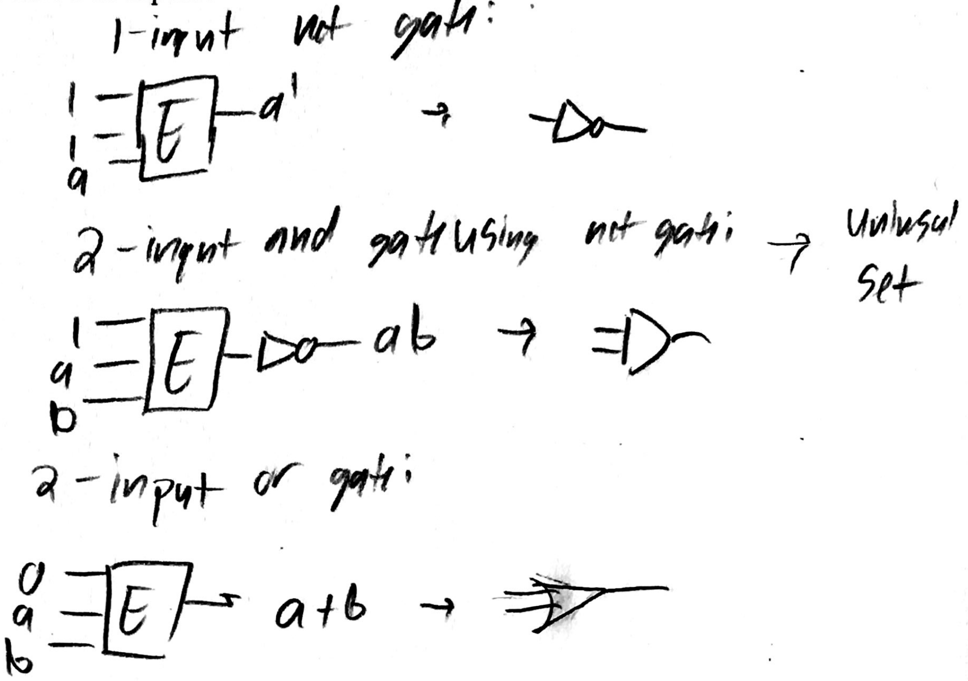
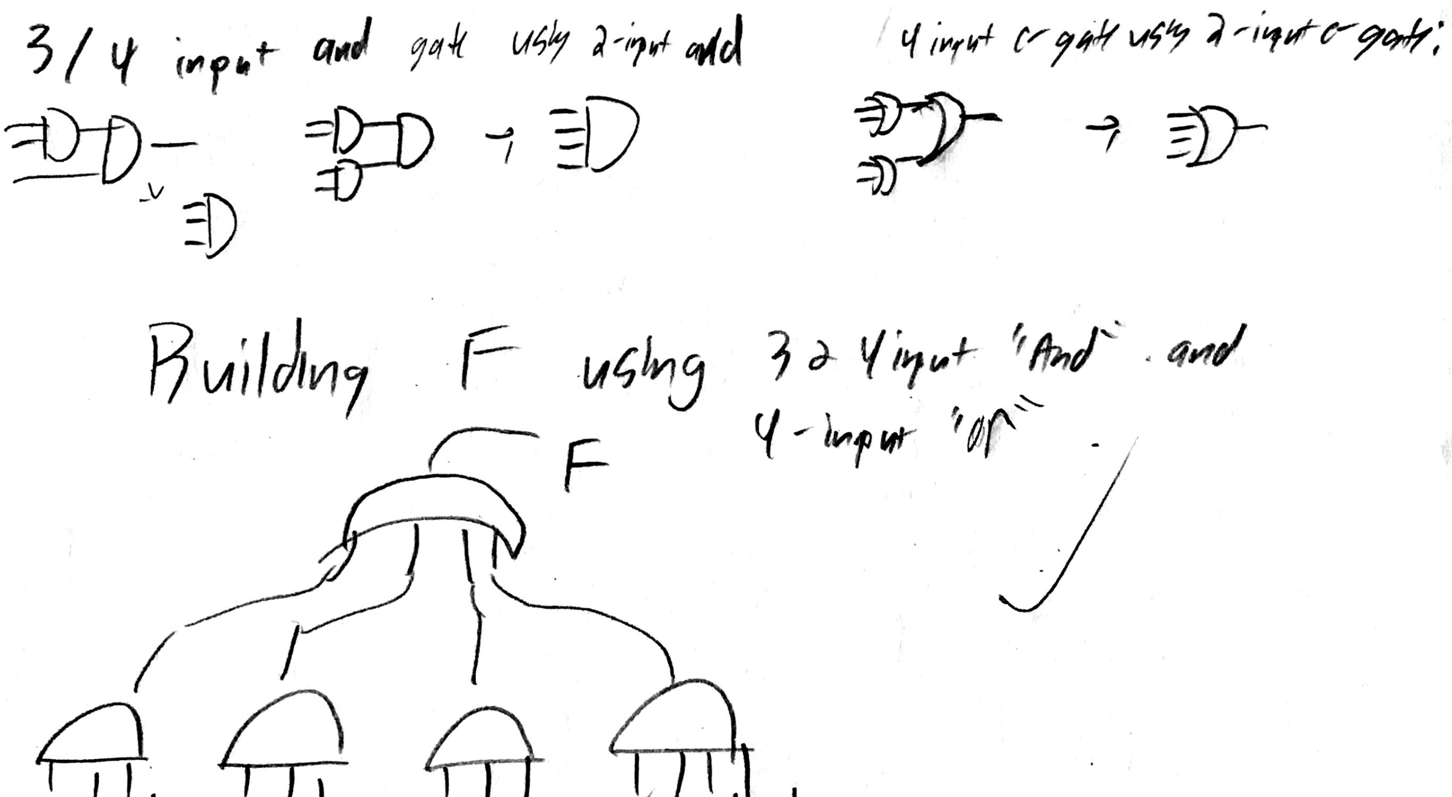
20

**Problem 1 (20 points)**

Use only the "E" gate defined below to implement Boolean function:
F=w'xy'+wxz+w'x'z+wx'y'z'
You may also use constants 0 and 1 as inputs.

| a | b | c | E(a,b,c) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |



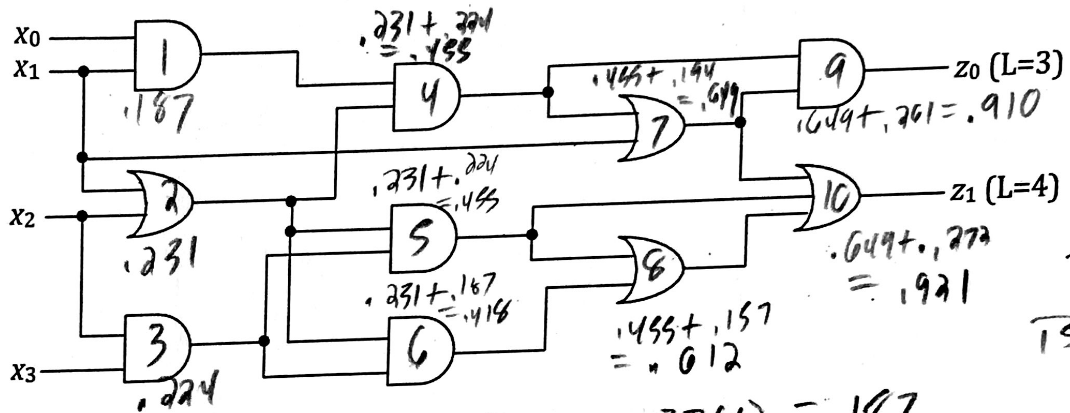
.194
.455
.649

.649
.272
.921

## Problem 2 (20 points)

Given the network below, calculate the critical path delay. Consider L → H delay when calculating the critical path.

| Gate | Fan-in | $t_{pLH}$ | $t_{pHL}$ |
|---|---|---|---|
| AND | 2 | $0.15 + 0.037L$ | $0.16 + 0.017L$ |
| AND | 3 | $0.20 + 0.038L$ | $0.18 + 0.018L$ |
| OR | 2 | $0.12 + 0.037L$ | $0.20 + 0.019L$ |
| OR | 3 | $0.12 + 0.038L$ | $0.34 + 0.022L$ |




1. .15 + .037(1) = .187
2. .12 + .037(3) = .231
3. .15 + .037(2) = .224
4. .15 + .037(2) = .224
5. .15 + .037(2) = .224
6. .15 + .037(1) = .187
7. .12 .037(2) = .194
8. .12 + .037(1) = .157
9. .15 + .037(3) = .261
10. .12 + .038(4) = .272

$Z_0$ critical path length: .910 (2 4 7 9)

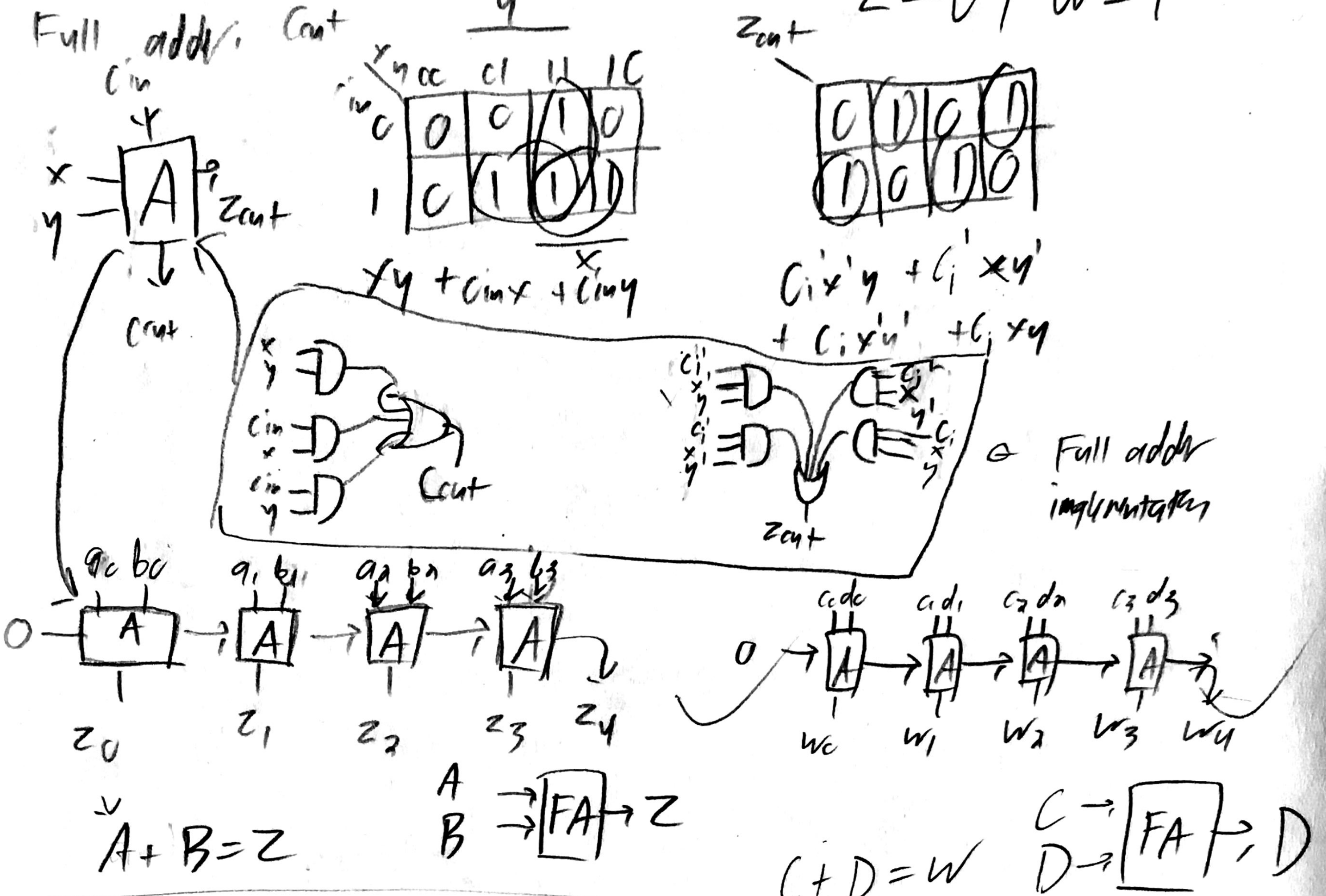
$Z_1$ critical path length: .921 (2 4 7 10)

16

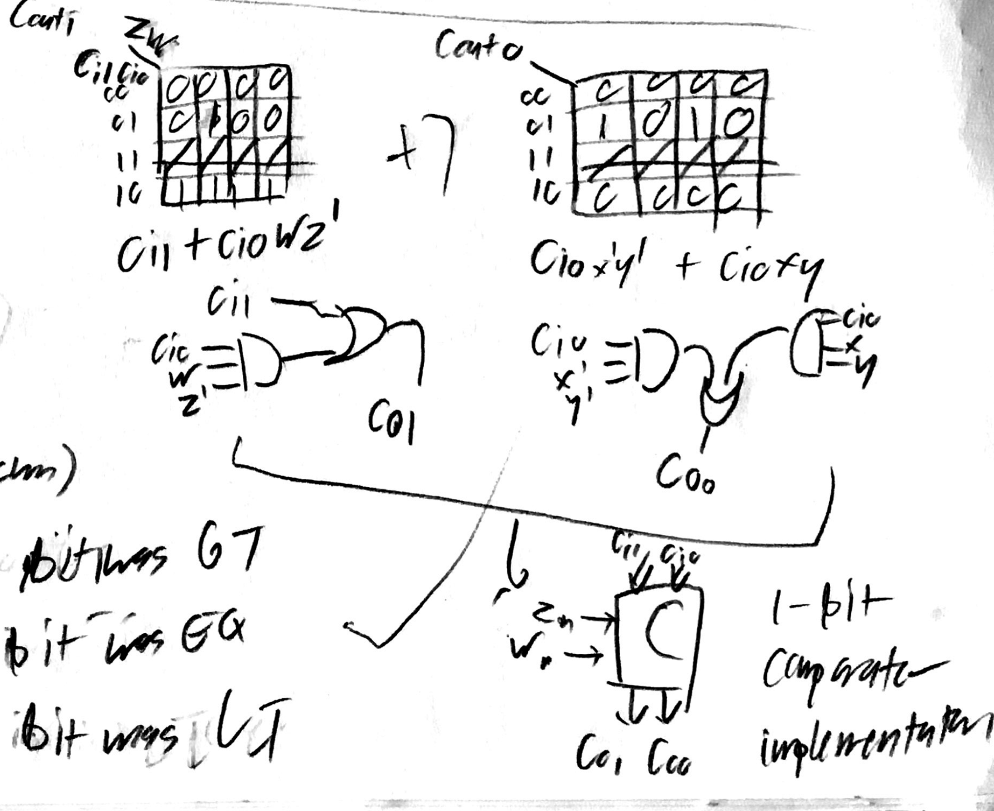
**Problem 3 (20 points)**

Four 4-bit numbers A,B,C, and D are given as inputs. E=A+B, F=C+D. Design a system that outputs the larger number between E and F. If E=F, output either E or F. You can use any type of gates to implement your design.

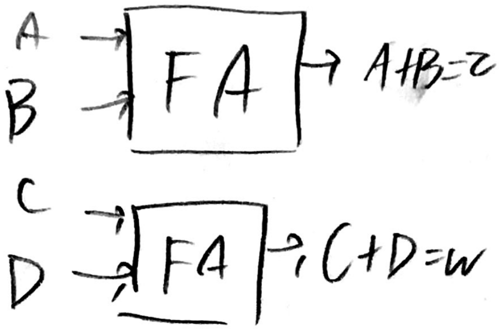
$Z=E, W=F$

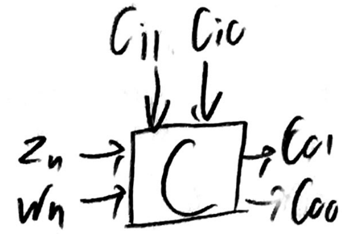



5-bit number comparator
input: $Z_n \in \{0,1\}$
$W_n \in \{0,1\}$
$C_{in} \in \{0,1,2\}$
(GT, EQ, LT)
Output: $C_{out} \in \{0,1,2\}$
($Z_n > W_n$, $Z_n = W_n$, $Z_n < W_n$)
0 if $Z_n > W_n$ OR previous bit was GT
1 if $Z_n = W_n$ And previous bit was EQ
2 if $Z_n < W_n$ OR previous bit was LT



## Problem 3) Extra Page



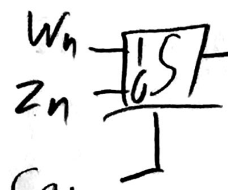




$C_{out}$ = 0 if $Z_n > W_n$
1 if $Z_n = W_n$
2 if $Z_n < W_n$

→ output $Z_n$ if $C_{out}$ = 0 or 1
→ output $W_n$ if $C_{out}$ = 2

| $C_{o1}$ | $C_{o0}$ | |
|---|---|---|
| 0 | 0 | $Z_n$ |
| 0 | 1 | $Z_n$ |
| 1 | 0 | $W_n$ |

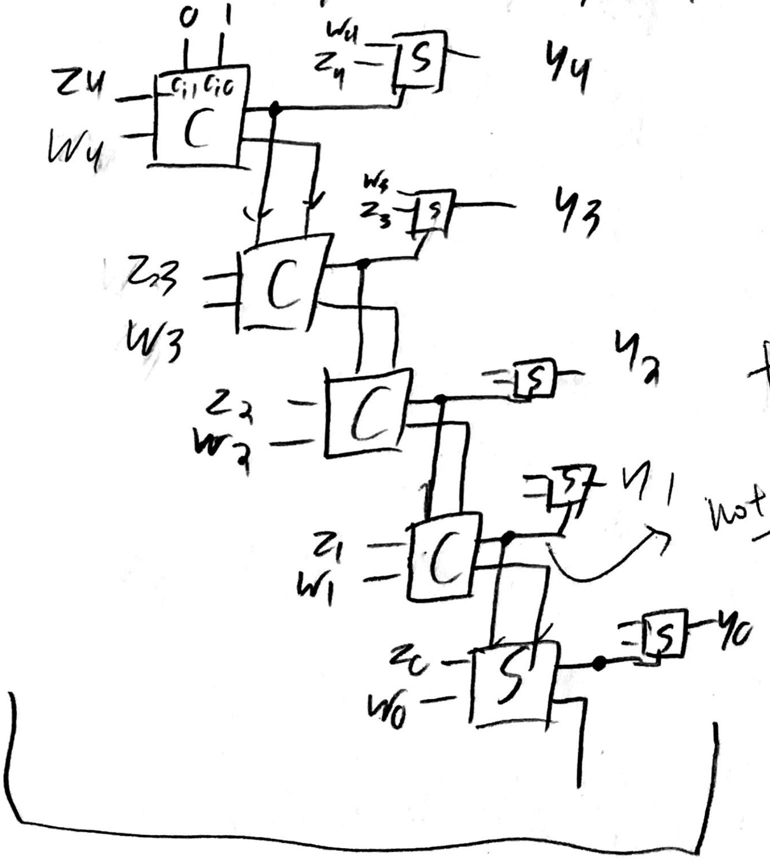
+7.

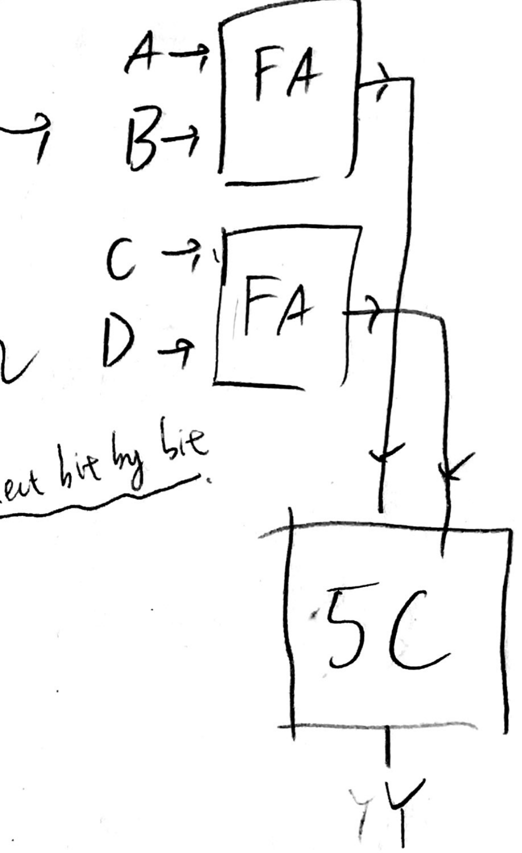



$W_n > Z_n$ ? $W_n$ : $Z_n$

1-bit GT output multiplexer

5-bit comparator and outputter → outputs Y = Max(Z,W)




5C


Y = max(Z,W) = Max(A+B, C+D).

20

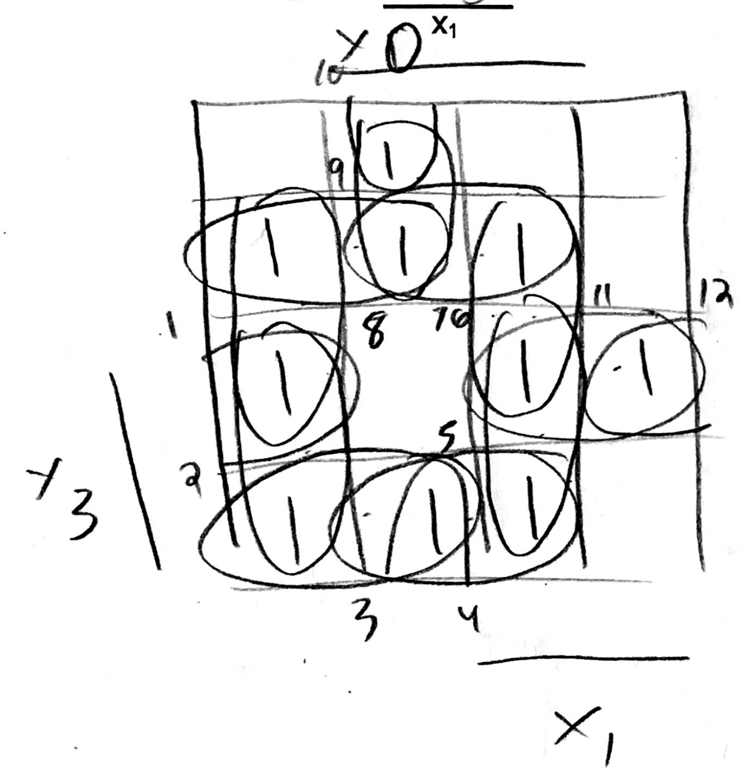
**Problem 4 (20 points)**

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of P=M-N among all the 4×4 K-maps.

For example, in the following 4×4 K-map, M=3, N=2, P=M-N=1.

| | | $X_0$ | $X_0$ | | |
|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | |
| | 1 | 1 | 0 | 0 | $X_2$ |
| $X_3$ | 1 | 1 | 1 | 0 | $X_2$ |
| $X_3$ | 0 | 0 | 1 | 0 | |
| | | | $X_1$ | $X_1$ | |

→ Most # of Prime Implicants
→ Least # of Epis



m = 12 prime implicants
n = 0 essential prime implicants

P = M - n = 12

$x_3 x_1' x_0' + x_3 x_1 x_0 + x_2 x_1' x_0' + x_2 x_1 x_0 + x_1 x_3 x_2 + x_3' x_0 x_1' + x_0' x_3 x_2 + x_2' x_0 x_1'$

$+ x_1' x_3 x_2' + x_1' x_3' x_2 + x_0 x_3 x_2' + x_0 x_3' x_2$

**Problem 5 (20 points)**

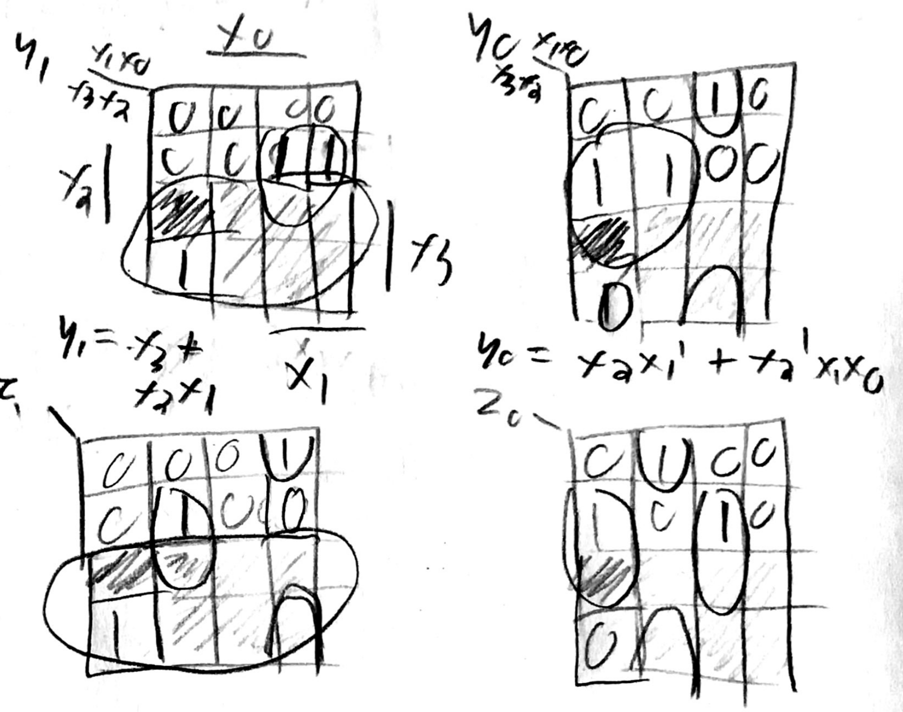
20

Use only multiplexers to design a system with input $x \in (0,1,2,\ldots 8)$, outputs y and z that implements the following equation

$$(x)_{10} = (yz)_3$$

In the system, x is encoded as $x_3x_2x_1x_0$ in binary. y is encoded as $y_1y_0$ in binary, and z is encoded as $z_1z_0$ in binary.
Note that the outputs y and z represent the two digits of a base-3 number.
For example, if x=7 ($x_3x_2x_1x_0$=0111), then the system will solve: $(7)_{10} = (21)_3$. Thus $y = 2$ ($y_1y_0$=10) and $z = 1$ ($z_1z_0$=01).

| $x_3$ | $x_2$ | $x_1$ | $x_0$ | $y_1$ | $y_0$ | $z_1$ | $z_0$ | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 00 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 01 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 02 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 10 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 11 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 12 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 20 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 21 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 22 |



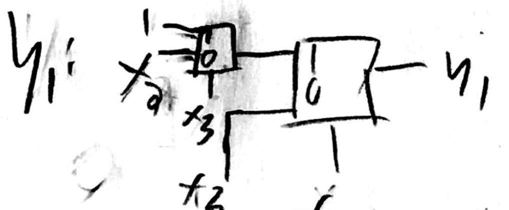
$y_1 = x_3 + x_2x_1$

$y_0 = x_2x_1' + x_2'x_1x_0$

$Z_1 = x_3 + x_2x_0x_1' + x_2'x_0'x_1$

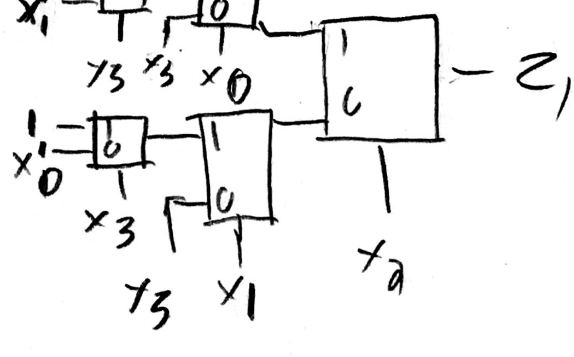
$z_0 = x_2x_1'x_0' + x_2x_1x_0 + x_2'x_1'x_0$

$y_1$:



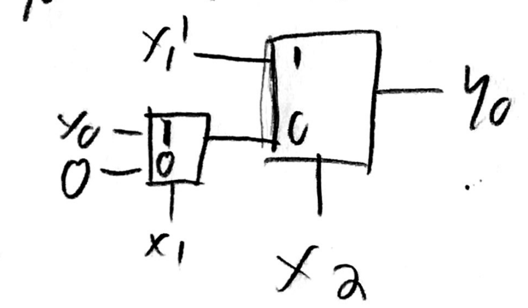
$Z_1$: $F_{x_2} = x_0x_1' + x_3$

$F_{x_2'} = x_3 + x_0'x_1$



$y_0$: $F_{x_2} = x_1'$

$F_{x_2'} = x_1x_0$

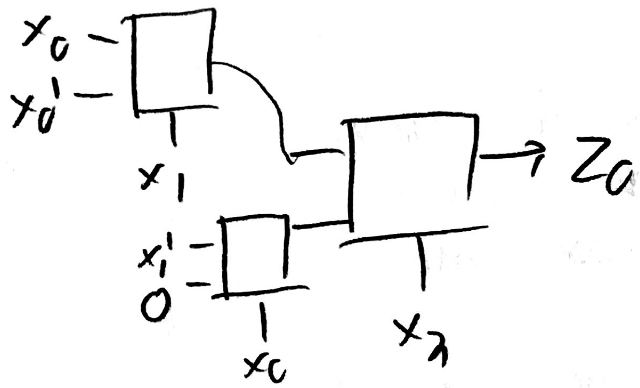


**Problem 5) Extra Page**

$Z_0: x_2 x_1' x_0' + x_2 x_1 x_0 + x_2' x_1' x_0$

$f_{x_2} = x_1' x_0' + x_1 x_0$

$f_{x_2'} = x_1' x_0$



Name ____________________

Student ID # ____________________

University of California

Los Angeles

Computer Science Department

CSM51A/EEM16 Midterm Exam

Winter Quarter 2016

February 8th 2016

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

**Time allowed 100 minutes.**

| Problem (possible points) | Points |
|---|---|
| 1 (20) | 20 |
| 2 (20) | 20 |
| 3 (20) | 16 |
| 4 (20) | 20 |
| 5 (20) | 20 |
| Total (100) | 96 |