

CS m51A: Logic Design of Digital Systems
UCLA Computer Science Department
Winter 2010
Midterm 1

Time: 100 minutes

Note: Closed book, closed notes, no electronic computing or communications devices.

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Question	Points	Grades
1	15	15
2	15	15
3	15	14
4	20	20
5	20	18
6	15	15
Total	100	97

"The best way to predict the future is to invent it."
-Alan Kay

Question 1: Number System Conversion

Find the value of x in the following equations. Show your work for credit.

$$\begin{array}{r} 45056 \\ 3072 \\ 224 \\ \hline 48352 \end{array}$$

a) $BCE0_{16} = x_4$
 11 12 14

1011110011100000 → BCE₀₁₆

$x_4 = 23303200_4$ ✓

$$11 \times 16^3 + 12 \times 16^2 + 14 \times 16 + 0 \times 16^0$$

$$45056 + 3072 + 224 + 0$$

$$= 48352$$

$$\begin{array}{r} 12088 \\ 4 \overline{)48352} \\ \underline{4} \\ 8 \\ \underline{8} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

$$\begin{array}{r} 11 \\ 4 \overline{)47} \\ \underline{4} \\ 7 \\ \underline{4} \\ 3 \\ \underline{3} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 4 \overline{)11} \\ \underline{8} \\ 3 \end{array}$$

$$\begin{array}{r} 3022 \\ 4 \overline{)12088} \\ \underline{12} \\ 8 \\ \underline{8} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

$$\begin{array}{r} 198 \\ 4 \overline{)755} \\ \underline{4} \\ 35 \\ \underline{32} \\ 35 \\ \underline{32} \\ 3 \end{array}$$

$$\begin{array}{r} 47 \\ 4 \overline{)188} \\ \underline{16} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

$$\begin{array}{r} 256 \\ 12 \\ \hline 512 \\ 512 \\ \hline 3072 \end{array}$$

$$\begin{array}{r} 14 \\ 16 \\ \hline 224 \\ 224 \\ \hline 45056 \end{array}$$

$$\begin{array}{r} 256 \\ 16 \\ \hline 1536 \\ 256 \\ \hline 4096 \\ 4096 \\ \hline 16384 \\ 16384 \\ \hline 48352 \end{array}$$

b) $251_8 = x_6$

251_8

$$2 \times 8^2 + 5 \times 8^1 + 1 \times 8^0$$

$$128 + 40 + 1$$

$$= 169$$

$$\begin{array}{r} 28 \\ 6 \overline{)169} \\ \underline{12} \\ 49 \\ \underline{48} \\ 1 \end{array}$$

$$\begin{array}{r} 4 \\ 6 \overline{)28} \\ \underline{24} \\ 4 \end{array}$$

$$\begin{array}{r} 0 \\ 6 \overline{)1} \\ \underline{0} \\ 1 \end{array}$$

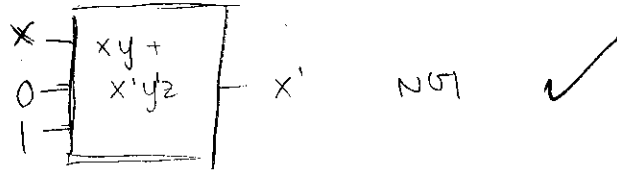
$x_6 = 441_6$ ✓

Question 2: Universal Set

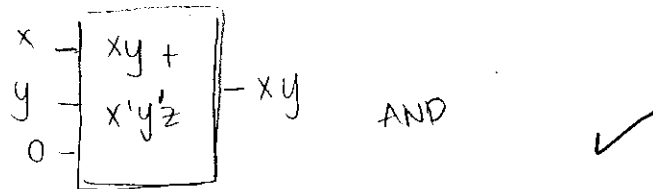
Prove that the operation represented by the switching expression $xy + x'y'z$ is universal. You can use the constants 1 and 0.

$$(a+b)' = a'b'$$

$xy + x'y'z$ is universal



$$x \cdot 0 + x' \cdot 1 \cdot 1 = 0 + x' = x'$$



$$x \cdot y + x' \cdot y' \cdot 0 = xy$$

since $\{AND, NOT\}$, which is a universal set, can be formed from $xy + x'y'z$, then the switching expression $xy + x'y'z$ must be universal as well.

Question 3: Boolean Algebra

a) Simplify the following switching expression, using Boolean algebra. Show your work for credit.

$$f = ((a'+b)'(c+ab'+ad')'+d')'$$

$$f = ((a'+b)'(c+ab'+ad')'+d')'$$

$$= ((ab')(c+ab'+ad')'+d')$$

$$c+a(b'+d)$$

$$= ((ab') [c + a(b'+d)'] + d')$$

$$[(ab)[c \cdot [a(b'+d)']] + d']$$

$$= [(ab)[c \cdot [a' + (b'+d)']]] + d']$$

$$= [(ab)[c \cdot (a' + bd)]] + d']$$

$$= [(ab)[a'c' + bc'd] + d']$$

$$= [aa'b'c' + ab'bc'd + d']$$

$$a \cdot a' = 0$$

$$b' \cdot b = 0$$

$$= 0 \cdot b'c' + a \cdot 0 \cdot c'd + d'$$

$$= 0 + 0 + d'$$

$$\boxed{f = d'}$$

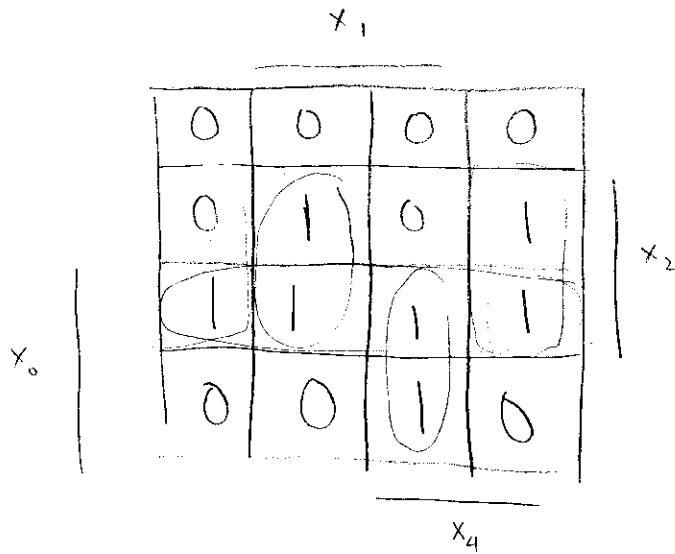
Complement Missed



Question 4:

Give an example of a function of four variables with four prime implicants and seven minterms. For credit, provide the K-map, showing the prime implicants.

$$\Sigma m: (3, 6, 10, 11, 13, 14, 15)$$



$$z = x_0 x_2 + x_1 x_2 x_4' + x_0 x_1 x_4 + x_1' x_2 x_4$$

minterms:

0110	6
0011	3
1010	10
1110	14
1111	15
1011	11
1101	13

Question 5: NOR-NOR network

Give a minimal NOR-NOR network that implements the subtraction of x from y ,

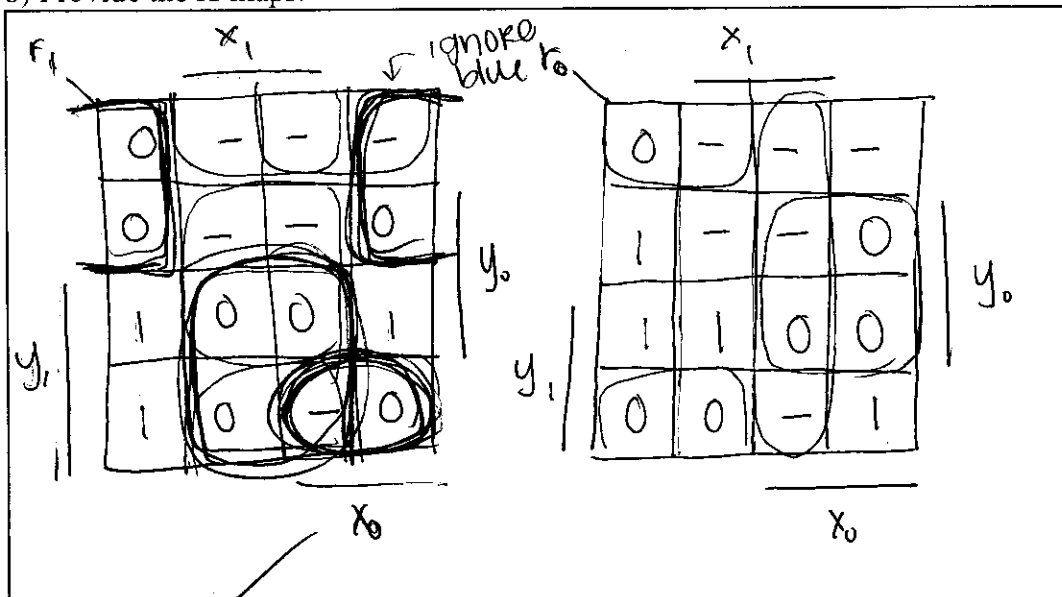
$$y(y_1, y_0) - x(x_1, x_0) = r(r_1, r_0).$$

For example if $x=10$ and $y=11$, then $r=01$. Assume that an input where x is greater than y is never given.

a) Provide the truth table.

y_1, y_0	x_1, x_0	r_1, r_0	
00	00	00	0-0=0
01	00	01	1-0=1
01	01	00	1-1=0
10	00	10	2-0=2
10	01	01	2-1=1
10	10	00	2-2=0
11	00	11	3-0=3
11	01	10	3-1=2
11	10	01	3-2=1
11	11	00	3-3=0

b) Provide the K-maps.

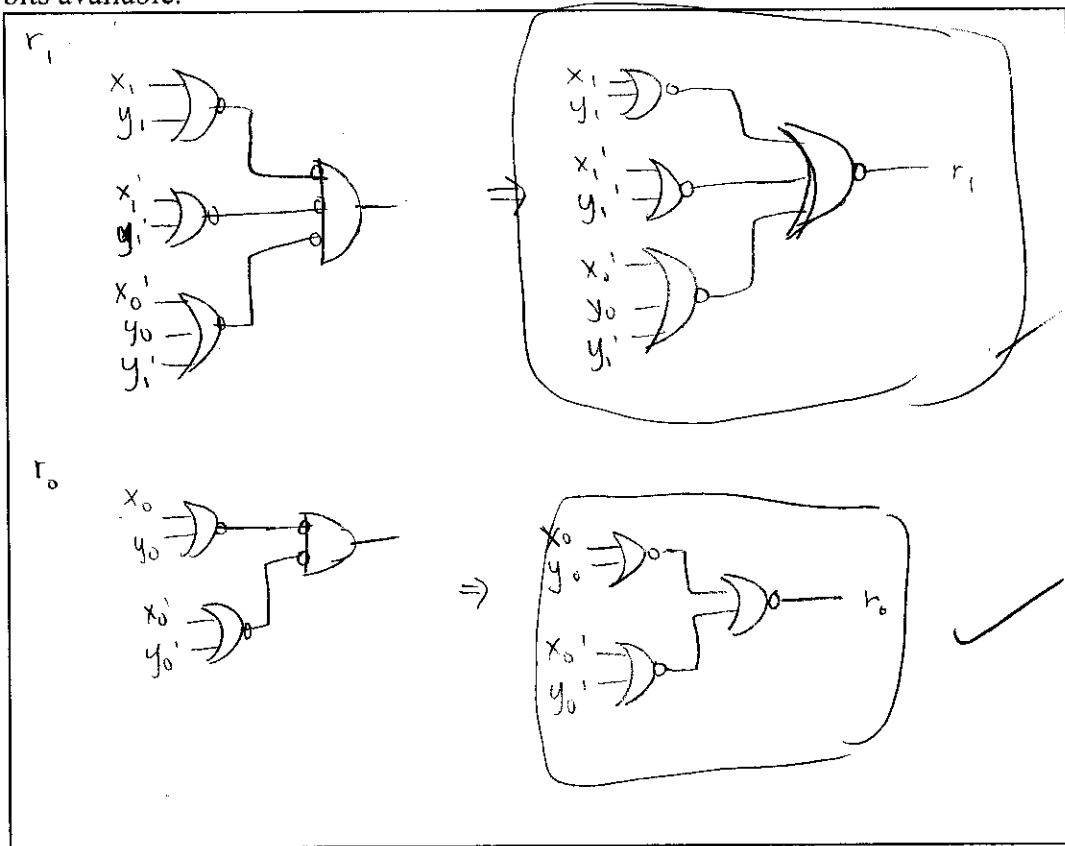


c) Provide the switching expressions.

$r_1 = (y_1 + x_1)(x_1' + y_1')(y_1' + x_0' + y_0)$
 $r_1 = (x_1 + y_1)(x_1' + y_1')(x_0' + y_0 + y_1')$ ✗ (-2)
 $r_0 = (y_0 + x_0)(x_0' + y_0')$
 $r_0 = (x_0 + y_0)(x_0' + y_0')$ ✓

d) Provide a minimal NOR-NOR network. Assume you have the negation of the input bits available.

$d'b'c'$
 $(a+b+c)'$



Question 6:

Give a minimal NAND-NAND implementation for the following function

$$f = a'b'c + b'c'd' + a'b'c'd + a'bc$$

For credit, show your work. Assume you have the negation of the input bits available.

The handwritten solution is contained within a rectangular border and includes the following elements:

- Top Left:** A partial NAND-NAND circuit diagram. It shows three NAND gates. The first gate has inputs a' and c . The second gate has inputs a' and b' . The third gate has inputs b' , c' , and d' . The outputs of these three gates are connected to a fourth NAND gate, which produces the final output f .
- Top Right:** A Karnaugh map for variables a , b , c , and d . The map is a 4x4 grid. The top row is labeled $(a+b+c+d)'$ and the right side is labeled d . The bottom row is labeled a and the right side is labeled c . There are three groups of 1s circled: a vertical group of two 1s in the first column (top two rows), a horizontal group of two 1s in the second row (left two columns), and a vertical group of two 1s in the third column (top two rows).
- Center:** The word "minimal" is written above a rectangular box containing the expression $a'c + a'b' + b'c'd'$.
- Bottom Left:** A complete NAND-NAND circuit diagram enclosed in a rounded rectangle. It shows three NAND gates with inputs a' , c ; a' , b' ; and b' , c' , d' . Their outputs are connected to a final NAND gate that outputs f .
- Bottom Right:** A wavy line indicating the end of the solution.