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University of California

Los Angeles

Computer Science Department

CSM51A/EEM16 Midterm Exam #1

Winter Quarter 2019

February 6th 2019

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

Problem (Possible Points)	Points
1(20)	20
2(20)	20
3(20)	15
4(20)	14
5(20)	20
Total (100)	89

Problem 1 (20 points):

Given the network below, find the critical path and calcuate critical path delay. Assume the values of (x_3, x_2, x_1, x_0) are initially $(1,1,1,1)$ and they change to $(0,0,1,0)$ in the next clock cycle. Now, choose a gate on the citical path which maximally decreases overall delay when the gate decreases its delay by 20%. Finally, find the critical path in the new network and its length.

Part 1

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$

Low = 1
High = 0

Part 2: Gate I ✓
has the largest time delay both on the whole network
6 on the critical path, so decreasing it's delay by 20% maximally decreases overall delay.
(Continue on Back)

$B = 0$ $G = 0$, $J = 0$

$X_0 \to A \to P$
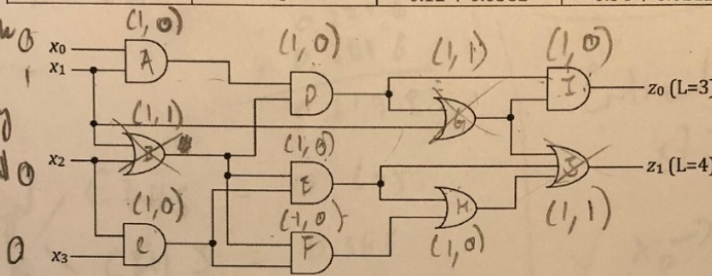
✓ critical path is
$X_0 \to A \to D \to G \to J \to Z_0$
or
$X_0 \to A \to D \to I \to Z_0$
both have time delay
0.672

A - Fanin 2, L=1, AND
$0.15 + 0.037 = 0.187$

C = Fanin 2, L=2, AND
$0.15 + 2(0.037) = 0.224$

$\begin{array}{r} 0.187 \\ + 0.037 \\ \hline 0.224 \end{array}$

D = Fanin 2, L=2, And
$= 0.224$

E = And, Fanin 2, L=2
$= 0.224$

F = And, Fanin 2, L=1
$= 0.187$

H = OR, Fanin 2, L=1
$0.12 + (0.037) = 0.157$

I = AND, Fanin 2, L=3
$0.15 + 3(0.037) = 0.261$

$\begin{array}{r} .187 \\ .224 \\ .261 \\ \hline .672 \end{array}$

Same as
ADGI

ADI / CEH
J / CEH

$\boxed{\text{ADI} = 0.187 + 0.224 + 0.261 = 0.672}$

$\begin{array}{r} 0.448 \\ + 0.157 \\ \hline 0.605 \end{array}$

CEHJ = 0.224 + 0.224 + 0.157 + 0 = 0.605

$\begin{array}{r} 0.037 \\ \times 3 \\ \hline 0.11 \\ + 0.15 \\ \hline 0.261 \end{array}$

$\begin{array}{r} 0.224 \\ 0.187 \\ 0.157 \\ \hline 0.568 \end{array}$

CFHJ = 0.224 + 0.187 + 0.157 + 0 = 0.568

Possible Paths

A P H I   4 > 3
A D G J
A D G J
C E H J   4 > 3
C E H J
C F H J   J = 0

$$0.261 \times 0.8 = .2088$$

Continued From Front

20% decrease of $I \Rightarrow 0.8I = 0.8(0.261)$
$= 0.2088$ s

New Critical Path calculations

$AD G3 \& ADJ = 0.1870$
$+ 0.224,0$
$+ 0.2088$
$\overline{ 0.6198}$

unchanged / $C EHJ = 0.605$
from part #1 \ $CFH J = 0.568$

Critical path is still AM

$\checkmark$ $x_0 \to A \to D \to G \to J \to z_0$
or
$x_0 \to A \to D \to J \to z_0$

but time delay is now

0.6198

$$AND = (x' + y')'$$

$$x_6 x_5'(x_2 + x_3' x_2') = x_6 x_5'(x_2 + x_3')$$
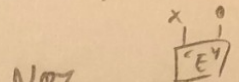
Problem 2 (20 points):

You are given the following Boolean function.

$$F(x_6, x_5, x_4, x_3, x_2, x_1, x_0) = x_6 x_5 x_4 x_3 + x_6 x_5' x_2 + x_6 x_5' x_3' x_2' + x_6' x_1 x_0 + x_6' x_5 x_0'$$

Given the universal operation E as specified in the table, implement F using only the gates specfied by E.

X	Y	E (X, Y)
0	0	1
0	1	1
1	0	0
1	1	1

$$E(x, y) = x' + y$$

NOT

$$E(x, 0) = (x)' + (0) = x'$$

OR

$$E(E(x,0), y) = ((x)' + (0))' + y$$
$$= x + y$$

AND

$$E(E(x, E(y,0)), 0)$$
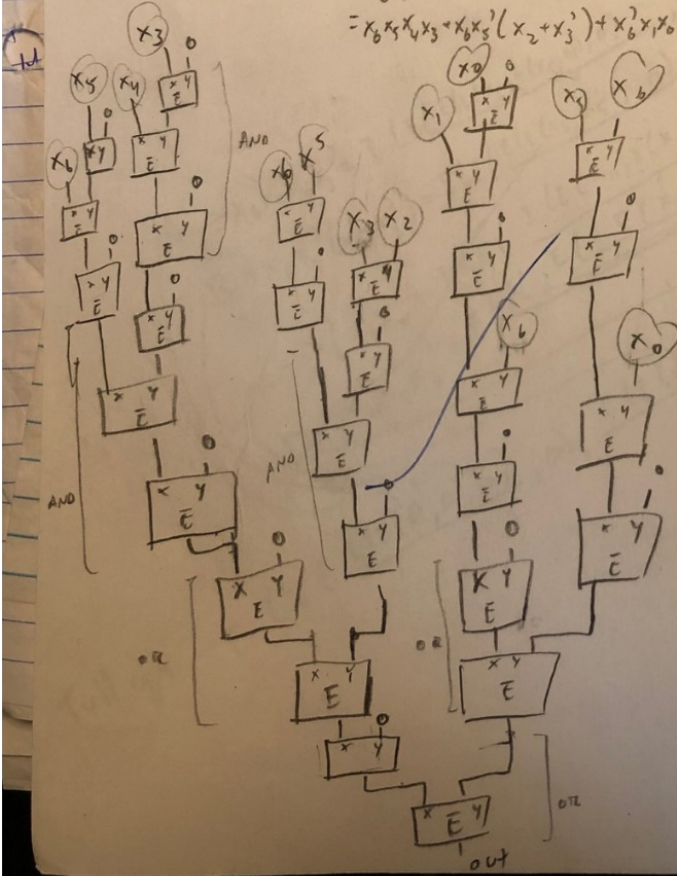$$= ((x' + ((y)' + (0)))' + (0)$$
$$= (x' + y')'$$
$$= xy \quad \text{De morgan's}$$

$$xy' = E(E(x,y),0)$$
$$= (x' + y)' + 0$$
$$= xy'$$

eqn on back

$$\overline{E} (E(xy', E(z, 0), 0)$$
$$((xy')' + z')'$$
$$xy' z$$

$$F(x_6, x_5, x_4, x_3, x_2, x_1, x_0) = x_6 x_5 x_4 x_3 + x_6 x_5' x_2 + x_6 x_5' x_3' x_2' + x_6' x_1 x_0 + x_6' x_5 x_0'$$
$$= x_6 x_5 x_4 x_3 + x_6 x_5'(x_2 + x_3' x_2') + x_6' x_1 x_0 + x_6' x_5 x_0'$$
$$= x_6 x_5 x_4 x_3 + x_6 x_5'(x_2 + x_3') + x_6' x_1 x_0 + x_6' x_5 x_0'$$

Problem 2 (Extra Page):

$A \rightarrow X_6 X_5 X_4 X_3 = E\left(E\left(E\left(E(X_6, E(X_5,0)),0\right), E\left(\bar{E}\left(E(X_4, E(X_3,0))P\right),0\right)\right),0\right)$

$B \rightarrow X_6 X_5 (X_2 + X_3) = E\left(E\left(E(E(X_6, X_5)),0\right), E\left(\frac{E(X_3, X_2)}{},0\right)\right),0\right)$

$C \rightarrow X_1^2 X_1 X_0 = E\left(E\left(E\left(E(X_1, E(X_0,0)),0\right), X_6\right),0\right)$

$D \rightarrow X_6 X_5 X_0 = E\left(E\left(E\left(\frac{E(X_5, X_6)}{},0\right), X_0\right),0\right)$

Full eqn: $E\left(E\left(\frac{E(E(A,0)B)}{},0\right), E\left(E(C,0),D\right)\right)$

Problem 3 (20 points):

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of P=M-N among all the 4×4 K-maps.

For example, in the following 4×4 K-map, M=3, N=2, P=M-N=1.

X_0

0	0	0	0
1	1	0	0
1	1	1	0
0	0	1	0

X_2

X_3

X_1

13 PI

$P = 14 - 0 = 14$

13 PI, Oessutial

$P = 13 - 0 = 13$

actuo

actually

(11).

14

Problem 4 (20 points):

Given an input stream X, we want to recognize interchangably patterns A and B. We recognize A first, then B, followed by A again, then B again and so on.
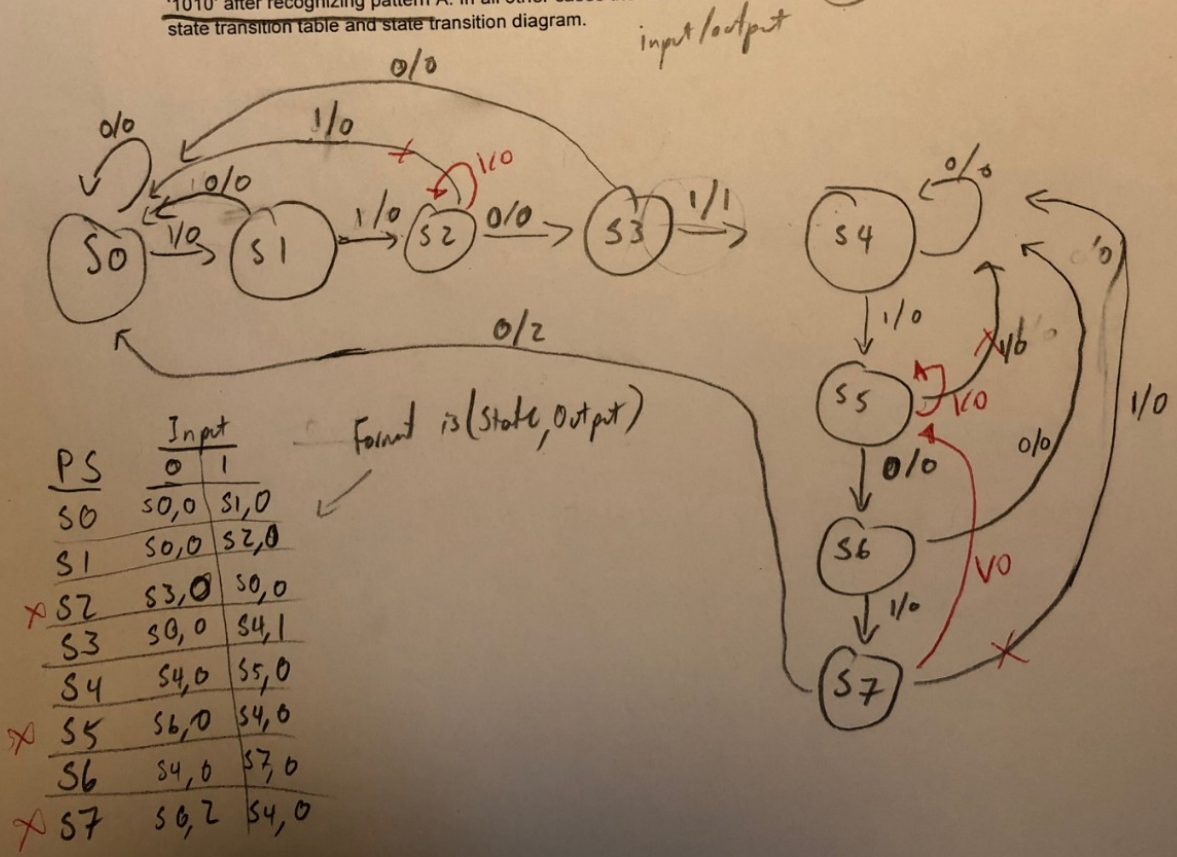
For example,

1. Assume X= 01011010010101, A=101 and B=001.

   We will first recognize A, then look for B. Please note that we ignore the second '101' (A) in X and we only search for B once we have found A. After finding B, we again search for A.

2. Assume that we have X= 1011, A = 101 and B=011

   We recognize A, but we do not recognize B as we only start looking for B once we have detected A. In other words, A and B do not overlap.

Now, you are given any input stream X. Design a finite state machine such that the system outputs 1 when it recognizes pattern A= '1101' and outputs 2 when it recognizes pattern B= '1010' after recognizing pattern A. In all other cases the machine should output 0. Show the state transition table and state transition diagram.

input /output

Format is (State, Output)

PS	Input	
	0	1
S0	S0,0	S1,0
S1	S0,0	S2,0
✗ S2	S3,0	S0,0
S3	S0,0	S4,1
S4	S4,0	S5,0
✗ S5	S6,0	S4,0
S6	S4,0	S7,0
✗ S7	S0,2	S4,0

Problem 5 (20 points):

Perform the following conversions:

 a) $(B2451)_{16} \to (x)_8$
 b) $(354)_7 \to (y)_5$

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

a)
B   2   4   5   1
1011 0010 0100 0101 0001

010 110 010   010 001 010 001
 2   6   2   2   1   2   1

$(B2451)_{16} \to (2622121)_8$

b) $(354)_7 = 3(7^2) + 5(7^1) + 4(7^0)$
         147   35   4

↓

$(186)_{10}$

$\begin{array}{r} 49 \\ \times\ 3 \\ \hline 147 \end{array}$

$\begin{array}{r} 147 \\ 35 \\ +\ 4 \\ \hline 186 \end{array}$

$\begin{array}{r} 147 \\ +\ 39 \\ \hline 186 \end{array}$

37 R1
5)186
 15
 36
 35
  1

7 R2
5)37
 35
  2

1 R2
5)7
 5
 2
 or 1
5)1

5)186
5) 37 1
5) 7 2
5) 1 2
    0 1

5)186
 15
 36
 35
  1

$(1221)_5$

Checking

010 110 010 010 001 010 001
1011 0010 0100 0101 0001
 B   2   4   5   1

$(1221)_5 = 1 \times 5^3 + 2 \times 5^2 + 2 \times 5^1 + 1 \times 5^0$
         $+\ 125 + 50 + 10 + 1$
               186

26 R4
7)186
 14
 46
 43
  4

3 R5   or R3
7)26   7)3
 21
  5

$(354)_7$