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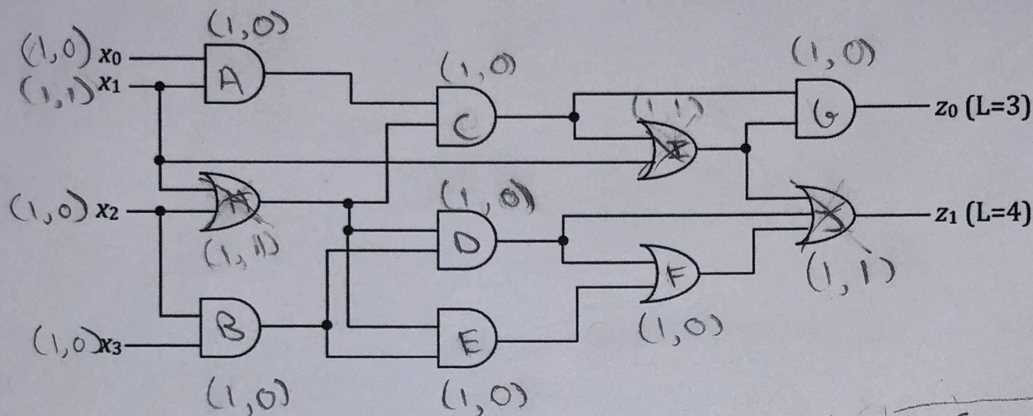
Problem 1 (20 points):

Given the network below, find the critical path and calcuate critical path delay. Assume the values of (x_3, x_2, x_1, x_0) are initially (1,1,1,1) and they change to (0,0,1,0) in the next clock cycle. Now, choose a gate on the citical path which maximally decreases overall delay when the gate decreases its delay by 20%. Finally, find the critical path in the new network and its length.

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$

(first, second)



#s in parens are multiplied by 0.037

OG: $\boxed{0.15 + (1) \quad 0.15 + (2) \quad 0.15 + (3)}$

OG switched: $\boxed{0.15 + (1) \quad 0.15 + (2) \quad 0.12 + (2.4)}$ ← still longest

vs.

competitor: $\boxed{0.15 + (2) \quad 0.15 + (2) \quad 0.12 + (1)}$

	initial delay	new delay
A	0.15 + 0.037	0.15 + 0.037
B	0.15 + (2)0.037	0.15 + (2)0.037
C	0.15 + (2)0.037	0.15 + (2)0.037
D	0.15 + (2)0.037	0.15 + (2)0.037
E	0.15 + 0.037	0.15 + 0.037
F	0.12 + 0.037	0.12 + 0.037
G	0.15 + (3)0.037	$\frac{4}{5}$(0.15 + (3)0.037) ✓

initial critical path :	(x_0, A, C, G, z_0), length = 3 ✓
new critical path :	(x_0, A, C, G, z_0), length = 3 ✓

← The critical path does NOT change (but its delay does)

Problem 2 (20 points):

You are given the following Boolean function.

$$F(x_6, x_5, x_4, x_3, x_2, x_1, x_0) = \underbrace{x_6x_5x_4x_3}_{(1)} + \underbrace{x_6x_5'x_2}_{(2)} + \underbrace{x_6x_5'x_3'x_2'}_{(3)} + \underbrace{x_6'x_1x_0}_{(4)} + \underbrace{x_6'x_5x_0'}_{(5)}$$

Given the universal operation E as specified in the table, implement F using only the gates specfied by E.

$NAND(x,y) = OR(NOT(x), NOT(y))$

$NOT(x) = E(x,0)$

$OR(x,y) = E(NOT(x), y)$

$= E[E(x,0), y]$

X	Y	E (X, Y)
0	0	1
0	1	1
1	0	0
1	1	1

NAND(x,y)	OR(x,y)
1	0
1	1
1	1
0	1

$xy = (x'+y')'$

$AND = NOT[OR[NOT(x), NOT(y)]]$

$= NOT\{OR[E(x,0), E(y,0)]\}$

$= NOT\{E[E(E(x,0),0], E(y,0)]\}$

$= NOT[E[x, E(y,0)]]$

$= E[E[x, E(y,0)], 0]$

$\boxed{*: x = E[E(x,0),0]}$

$\uparrow$

$x = NOT[NOT(x)]$

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(1) $x_6x_5x_4x_3 = (x_6x_5)(x_4x_3) = \underbrace{E[E[x_6, E(x_5,0)],0]}_{L1}\ \underbrace{E[E[x_4, E(x_3,0)],0]}_{R1}$

$= \underbrace{E[E[L1, E(R1,0)],0]}_{G1}$

(2) $x_6x_5'x_2 = (x_6x_5')(x_2) = \underbrace{E[E[x_6, x_5],0]}_{L2}\ \underbrace{x_2}_{R2}$ *

$= \underbrace{E[E[L2, E(R2,0)],0]}_{G2}$

(3) $x_6x_5'x_3'x_2' = (x_6x_5')(x_3'x_2') = \underbrace{E[E[x_6, x_5],0]}_{L3}\ \underbrace{E[E[E(x_3,0), x_2],0]}_{R3}$ * *

$= \underbrace{E[E[L3, E(R3,0)],0]}_{G3}$

→ NEXT PAGE

Problem 2 (Extra Page):

(4) $x_6' x_1 x_0 = (x_6' x_1)(x_0) = \underbrace{E[E[E(x_6,0),E(x_1,0)],0]}_{L4}\ \underbrace{x_0}_{R4}$

$= \underbrace{E[E[L4, E(R4,0)],0]}_{G4}$

(5) $x_6' x_5 x_0' = (x_6' x_5)(x_0') = \underbrace{E[E[E(x_6,0),E(x_5,x_0)],0]}_{L5}\ \underbrace{E(x_0,0)}_{R5}$

$= \underbrace{E[E[L5, E(R5,0)],0]}_{G5}$

$F = G1 + G2 + G3 + G4 + G5$

$G1 + G2$: $E[E(G1,0),G2]$

$(G1 + G2 + G3)$

$G3 + (G1 + G2)$: $\underbrace{E[E(G_3,0), E[E(G1,0),G2]]}_{Z1}$

$G4 + G5$: $\underbrace{E[E(G4,0),G5]}_{Z2}$ $\leftarrow (G4 + G5)$

$(G1 + G2 + G3 + G4 + G5)$

$Z1 + Z2 = E[E(Z1,0),Z2]$

Create ~~diagram~~ diagram?

Z2 and Z1 are defined by G1 → G5, which are in turn defined by L1 → L5 and R1 → R5. these abstractions were performed to increase readability.

$$\boxed{F = E[E(Z1,0),Z2]}$$

where $F = F(x_6, x_5, x_4, x_3, x_2, x_1, x_0)$

Problem 3 (20 points):

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of P=M-N among all the 4×4 K-maps.

For example, in the following 4×4 K-map, M=3, N=2, P=M-N=1.

		X_0	X_0		
	0	0	0	0	
	1	1	0	0	X_2
X_3	1	1	1	0	X_2
X_3	0	0	1	0	
			X_1	X_1	

		X_0	X_0		
	1	0	1	1	
	1	1	1	0	X_2
X_3	0	1	1	1	X_2
X_3	1	1	0	1	
			X_1	X_1	

P = M - N = 10 - 0 = 10

with prime implicants shown

		X_0	X_0		
	1	0	1	1	
	1	1	1	0	X_2
X_3	0	1	1	1	X_2
X_3	1	1	0	1	
			X_1	X_1	

M = 10
N = 0

10

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Problem 4 (20 points):

Given an input stream X, we want to recognize interchangably patterns A and B. We recognize A first, then B, followed by A again, then B again and so on.

For example,

1. Assume X= 01011010010101, A=101 and B=001.

   We will first recognize A, then look for B. Please note that we ignore the second '101' (A) in X and we only search for B once we have found A. After finding B, we again search for A.

2. Assume that we have X= 1011, A = 101 and B=011

   We recognize A, but we do not recognize B as we only start looking for B once we have detected A. In other words, A and B do not overlap.

Now, you are given any input stream X. Design a finite state machine such that the system outputs 1 when it recognizes pattern A= '1101' and outputs 2 when it recognizes pattern B= '1010' after recognizing pattern A. In all other cases the machine should output 0. Show the state transition table and state transition diagram.

(output/NS)

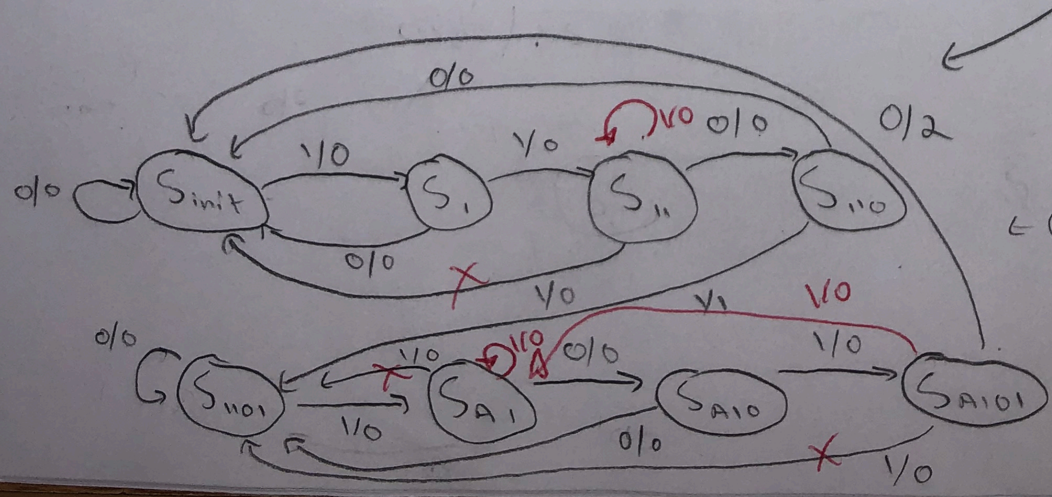
{0,1} inputs
{0,1,2} outputs

A = 1101
B = 1010

PS	0	1
S_{init}	$0/S_{init}$	$0/S_1$
S_1	$0/S_{init}$	$0/S_{11}$
✗ S_{11}	$0/S_{110}$	$0/S_{init}$
S_{110}	$0/S_{init}$	$1/S_{1101}$
S_{1101}	$0/S_{1101}$	$0/S_{A1}$
✗ S_{A1}	$0/S_{A10}$	$0/S_{1101}$
S_{A10}	$0/S_{1101}$	$0/S_{A101}$
✗ S_{A101}	$2/S_{init}$	$0/S_{1101}$

← STATE TRANSITION TABLE

STATE TRANSITION DIAGRAM



← (input/output)

A=10, B=11, C=12, D=13, E=14, F=15

Problem 5 (20 points):

Perform the following conversions:

a) $(B2451)_{16} \rightarrow (X)_8$
b) $(354)_7 \rightarrow (y)_5$

(a) $(B2451)_{16} \rightarrow (x)_8$

to binary

B 2 4 5 1

(0)1011, 0010, 0100, 0101, 0001

2 6 2 2 1 2 1

$\boxed{(x)_8 = (2622121)_8}$

(b) $(354)_7 \rightarrow (y)_5$

$(354)_7 \rightarrow 3(7^2) + 5(7^1) + 4(7^0)$
$= 3(49) + 5(7) + 4 = 147 + 35 + 4$
$= (186)_{10}$

$(186)_{10} \rightarrow (y)_5$:
186 − 1(125) = 61
61 − 2(25) = 11
11 − 2(5) = 1
1 − 1(1) = 0

$\boxed{(y)_5 = (1221)_5}$

(+1)
147
+ 35
182
+ 4
186

186
−125
61

61
−50
11
−10
1