

Name: _____

Student ID: _____

University of California

Los Angeles

Computer Science Department

CSM51A/EEM16 Midterm Exam #1

Winter Quarter 2019

February 6th 2019

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

Problem (Possible Points)	Points
1(20)	17.5
2(20)	16 + 3 = 19
3(20)	15
4(20)	20
5(20)	20
Total (100)	88.5 → 91.5

17.5

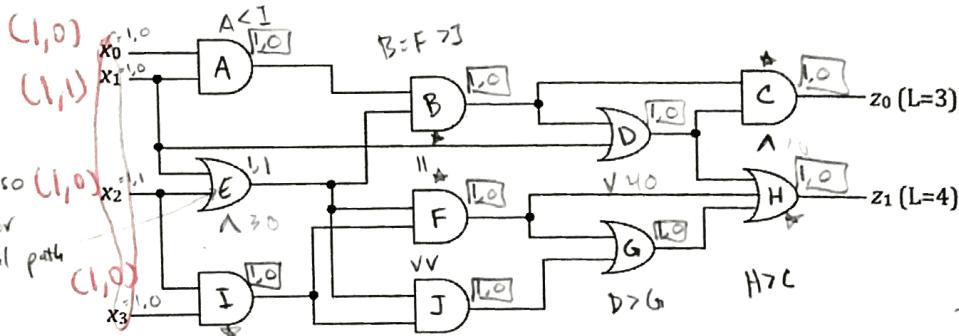
Problem 1 (20 points):

Given the network below, find the critical path and calculate critical path delay. Assume the values of (x_3, x_2, x_1, x_0) are initially $(1, 1, 1, 1)$ and they change to $(0, 0, 1, 0)$ in the next clock cycle. Now, choose a gate on the critical path which maximally decreases overall delay when the gate decreases its delay by 20%. Finally, find the critical path in the new network and its length.

Gate	Fan-in	t_{PLH}	t_{PHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$

~~$x_3 x_2 x_1 x_0 = 1111$
 $x_3 x_2 x_1 x_0 = 0010$~~

$$x_3 x_2 x_1 x_0 = 1111 \\ x_3 x_2 x_1 x_0 = 0010$$



$$z_0 = L1 \\ z_1 = HL$$

x_2 doesn't change so $(1,0)$
 \Rightarrow doesn't change or
contribute to critical path

$$\frac{1.87}{3.37} \\ \frac{3.37}{2.24} \\ + \frac{3.7}{2.61} \\ \hline 7.61$$

$$\frac{1.20}{0.71} \\ \frac{0.71}{0.94} \\ + \frac{3.7}{7.61} \\ \hline 10.24$$

$$\begin{aligned} A &: 0.15 + 0.037(1) = 0.187 \\ B &: 0.15 + 0.037(2) = 0.224 \\ C &: 0.15 + 0.037(3) = 0.261 \\ D &: 0.12 + 0.037(2) = 0.194 \\ E &: N/A \end{aligned}$$

$$\begin{aligned} F &: 0.15 + 0.037(2) = 0.224 \\ G &: 0.12 + 0.037(1) = 0.157 \\ H &: 0.12 + 0.038(4) = 0.272 \\ I &: 0.15 + 0.037(2) = 0.224 \\ J &: 0.15 + 0.037(1) = 0.187 \end{aligned}$$

$$\begin{array}{r} 3 \\ 3 \\ \hline 1.20 \\ .152 \\ \hline .272 \\ 2.24 \\ 2.24 \\ 1.57 \\ + 2.22 \\ \hline .877 \end{array}$$

Paths: $I - F - G - H = 0.224 + 0.224 + 0.157 + 0.272 = .877$
 $A - B - D - H = 0.187 + 0.224 + 0.194 + 0.272 = .877$

• Both are critical paths: $I - F - G - H \rightarrow \text{Critical Delay} = 0.877$

max decrease by decreasing gate delay by 20% = slowest gate = H

$$80\% H = .272 \times .8 = .218 = \text{new } H \text{ value}$$

$$C > H$$

$$\begin{array}{r} 0.187 \\ 0.224 \\ 0.194 \\ + 0.261 \\ \hline 0.866 \end{array}$$

• New critical Path: $A - B - D - C \rightarrow \text{Critical Delay} = 0.866, \text{Length} = 4$

$$A - B - D = I - F - G$$

& now $C > H$

so

$$A - B - D - C > I - F - G - H$$

Problem 2 (20 points):

You are given the following Boolean function.

$$F(x_6, x_5, x_4, x_3, x_2, x_1, x_0) = x_6x_5x_4x_3 + x_6x_5'x_2 + x_6x_5'x_3'x_2' + x_6'x_1x_0 + x_6'x_5x_0'$$

Given the universal operation E as specified in the table, implement F using only the gates specified by E. Have constants 0,1. Don't have complement.

$$E(x, y) = \begin{array}{c} x \\ y \end{array} \cdot \boxed{E} \cdot \begin{array}{c} x \\ y \end{array}$$

X	Y	E(X, Y)
0	0	1
0	1	1
1	0	0
1	1	1

$$E = x' + y$$

Create Modules using E & constants.

$$E(x, 0) = x' + 0 = x' = \text{NOT} \rightarrow$$

$$\begin{array}{c} x \\ 0 \end{array} \cdot \boxed{E} \cdot \begin{array}{c} x \\ 0 \end{array} := x \cdot \boxed{\text{NOT}}$$

* NOT Module

$$E(x', y) = x'' + y = x + y \rightarrow$$

CR →

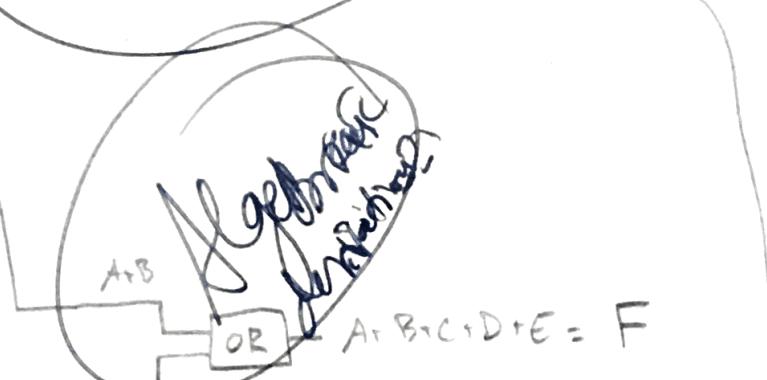
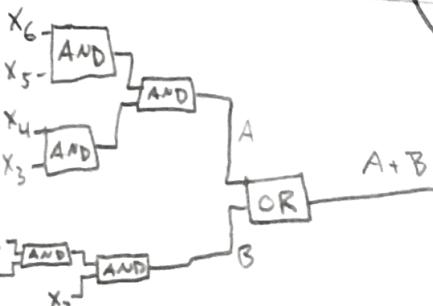
$$\begin{array}{c} x \\ x - \text{NOT} \end{array} \cdot \boxed{E} \cdot \begin{array}{c} y \\ y \end{array} := \begin{array}{c} x \\ y \end{array} \cdot \boxed{\text{OR}}$$

* OR Module

$$(x' + y')' = xy = \text{AND}$$

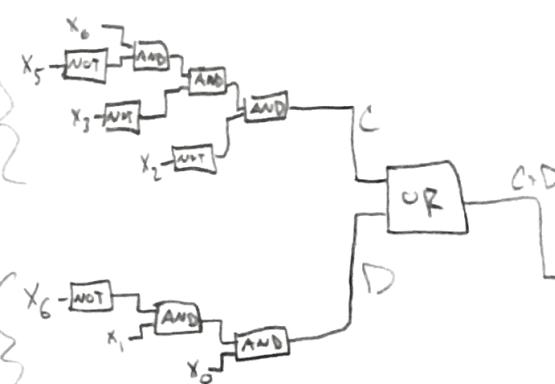
$$\begin{array}{c} x \\ x - \text{NOT} \end{array} \cdot \boxed{\text{OR}} \cdot \begin{array}{c} y \\ y - \text{NOT} \end{array} \cdot \boxed{\text{NOT}} \cdot \begin{array}{c} xy \\ xy \end{array} := \begin{array}{c} x \\ y \end{array} \cdot \boxed{\text{AND}}$$

* AND Module

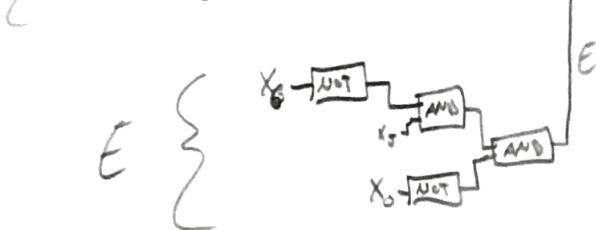


Algebraic Simplification

$$A + B + C + D + E = F$$

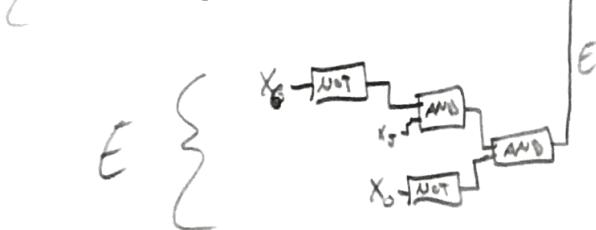


$$C + D + E$$



$$D$$

$$C + D + E$$

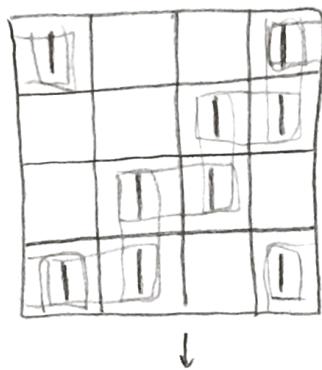
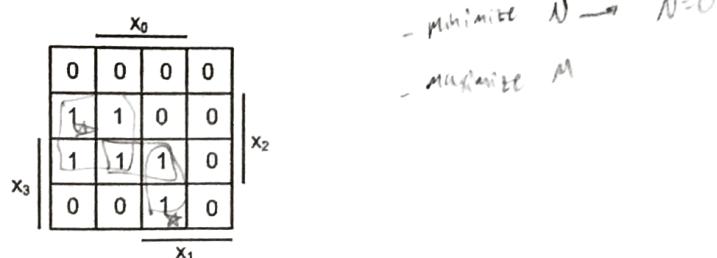


$$E$$

Problem 3 (20 points):

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of $P=M-N$ among all the 4×4 K-maps.

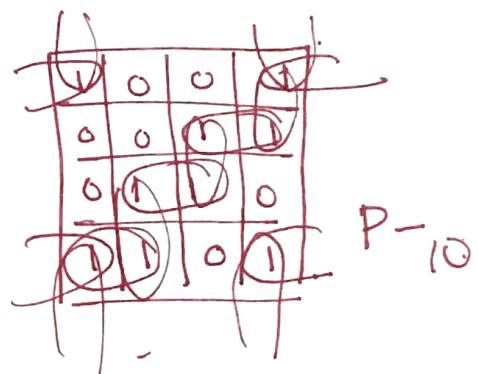
For example, in the following 4×4 K-map, $M=3$, $N=2$, $P=M-N=1$.



$$\begin{array}{c}
 M \\
 \cancel{\cancel{\cancel{1}}} \\
 \end{array} \qquad
 \begin{array}{c}
 N \\
 \cancel{\cancel{0}} \\
 \end{array}$$

$$P = M - N = 10 - 0 = 10$$

1	0	0	1
0	0	1	1
0	1	1	0
1	1	0	1



20

Problem 4 (20 points):

Given an input stream X, we want to recognize interchangably patterns A and B. We recognize A first, then B, followed by A again, then B again and so on.

For example,

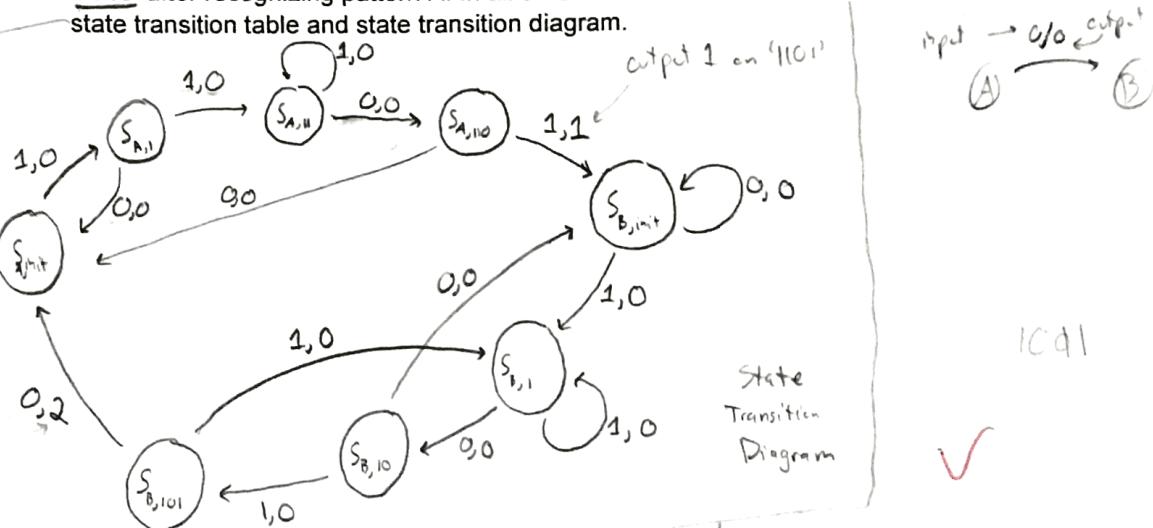
1. Assume $X = 010111010010101$, $A=101$ and $B=001$.

We will first recognize A, then look for B. Please note that we ignore the second '101' (A) in X and we only search for B once we have found A. After finding B, we again search for A.

2. Assume that we have $X = 1011$, $A = 101$ and $B = 011$

We recognize A, but we do not recognize B as we only start looking for B once we have detected A. In other words, A and B do not overlap.

Now, you are given any input stream X. Design a finite state machine such that the system outputs 1 when it recognizes pattern A = '1101' and outputs 2 when it recognizes pattern B = '1010' after recognizing pattern A. In all other cases the machine should output 0. Show the state transition table and state transition diagram.



	Input	
	0	1
$S_{A,\text{init}}$	$S_{A,\text{init}}, 0$	$S_{A,1}, 0$
$S_{A,1}$	$S_{A,\text{init}}, 0$	$S_{A,11}, 0$
$S_{A,11}$	$S_{A,110}, 0$	$S_{A,11}, 0$
$S_{A,110}$	$S_{A,\text{init}}, 0$	$S_{B,\text{init}}, 1$
$S_{B,\text{init}}$	$S_{B,\text{init}}, 0$	$S_{B,1}, 0$
$S_{B,1}$	$S_{B,10}, 0$	$S_{B,1}, 0$
$S_{B,10}$	$S_{B,\text{init}}, 0$	$S_{B,101}, 0$
$S_{B,101}$	$S_{A,\text{init}}, 2$	$S_{B,1}, 0$

State
Transition
Table

✓

10/11

✓

Problem 5 (20 points):

Perform the following conversions:

- a) $(B2451)_{16} \rightarrow (x)_8$
- b) $(354)_7 \rightarrow (y)_5$

a) $(B2451)_{16} = (1011\underbrace{0010}_{3}\underbrace{0100}_{2}\underbrace{0101}_{4}\underbrace{0001}_{5}\underbrace{1}_{1})_2$

$(10110010010001010001)_2 : (2622121)_8 \rightarrow \boxed{x = 2622121}$

$$\begin{array}{r} 147 \\ + 34 \\ \hline 186 \end{array}$$

$$\begin{array}{r} 49 \\ \times 3 \\ \hline 147 \end{array}$$

b) $(354)_7 = (3 \cdot 49) + (5 \cdot 7) + 4 = 147 + 35 + 4 = (186)_{10}$

$(186)_{10} = (1221)_5 \rightarrow \boxed{Y = 1221}$

$$\begin{array}{r} 186 \\ \hline 37 \\ 37 \\ \hline 34 \\ 34 \\ \hline 1 \end{array} \quad \begin{array}{r} 37 \\ \hline 37 \\ 37 \\ \hline 0 \end{array} \quad \begin{array}{r} 1 \\ \hline 7 \\ 7 \\ \hline 0 \end{array} \quad \begin{array}{r} 5 \\ \hline 1 \\ 1 \\ \hline 0 \end{array}$$