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CSM51A/EEM16 Midterm Exam

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This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

**Time allowed 100 minutes.**

Problem (possible points)	Points
1 (20)	20
2 (20)	20
3 (20)	20
4 (20)	11
5 (20)	20
Total (100)	91

**Problem 1 (20 points)**

Use only the "E" gate defined below to implement Boolean function:

$$F = w'xy' + wxz + w'x'z + wx'y'z'$$

You may also use constants 0 and 1 as inputs.

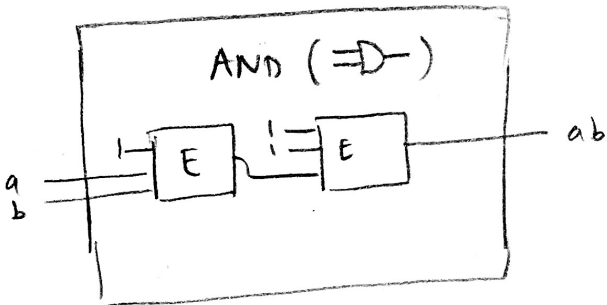
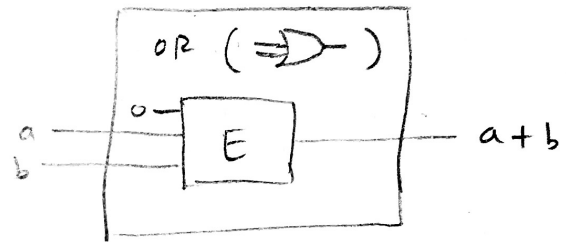
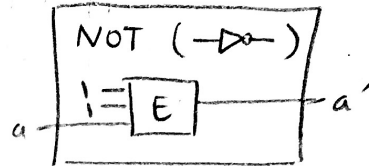
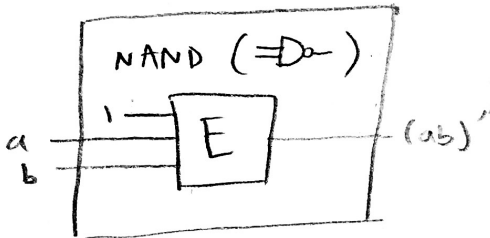
a	b	c	E(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$E(1,1,a) = a' \quad (\text{NOT})$$

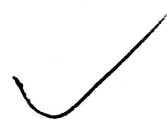
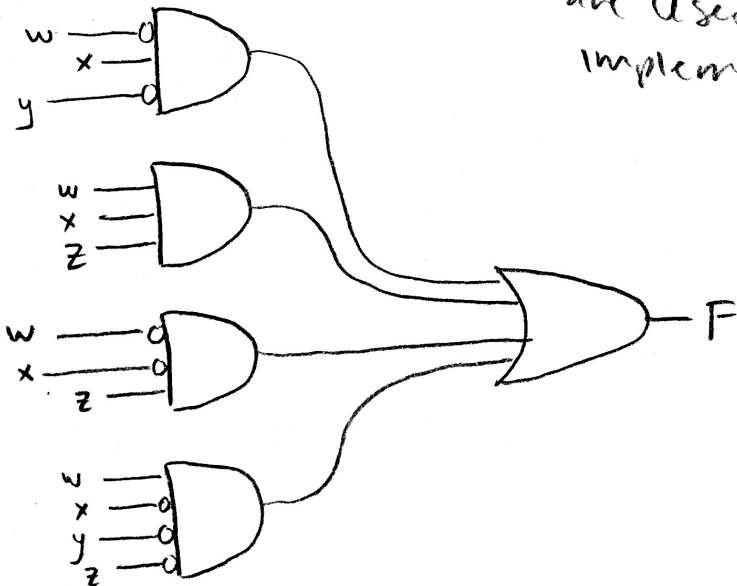
$$E(0,a,b) = a + b \quad (\text{OR})$$

$$E(1,a,b) = \text{NAND}(a,b) \quad (\text{NAND})$$

$$E(1,1,E(1,a,b)) = ((ab)')' = ab \quad (\text{AND})$$



Using these gate definitions (provided only with gate E), we can implement F. Regular gate symbols are used, but assume they have been implemented as Above.

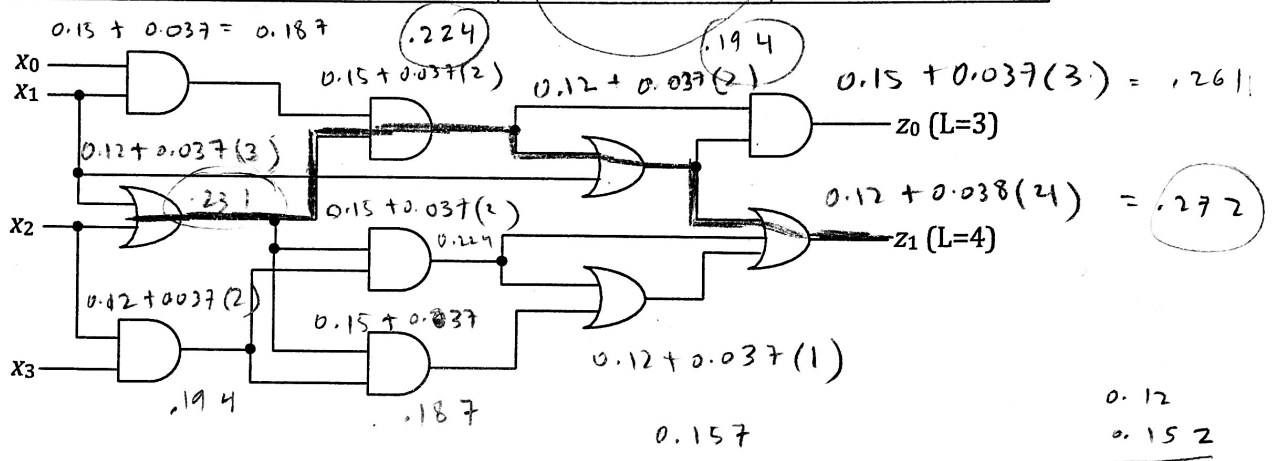


**Problem 2 (20 points)**

Given the network below, calculate the critical path delay. Consider  $L \rightarrow H$  delay when calculating the critical path.

Gate	Fan-in	$t_{pLH}$	$t_{pHL}$
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$

0.15  
0.481



0.12  
0.152  
0.272

0.231  
0.224  
0.194  
0.261  
0.910

0.272  
0.224  
0.157  
0.224  
0.231  
0.887

**CRITICAL PATH DELAY: 0.921**

0.272  
0.194  
0.224  
0.231  
0.921

20

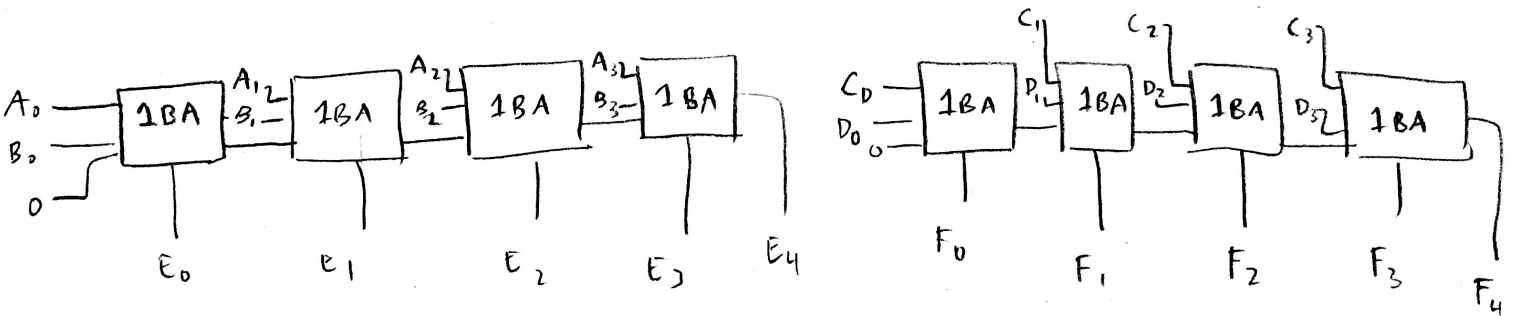
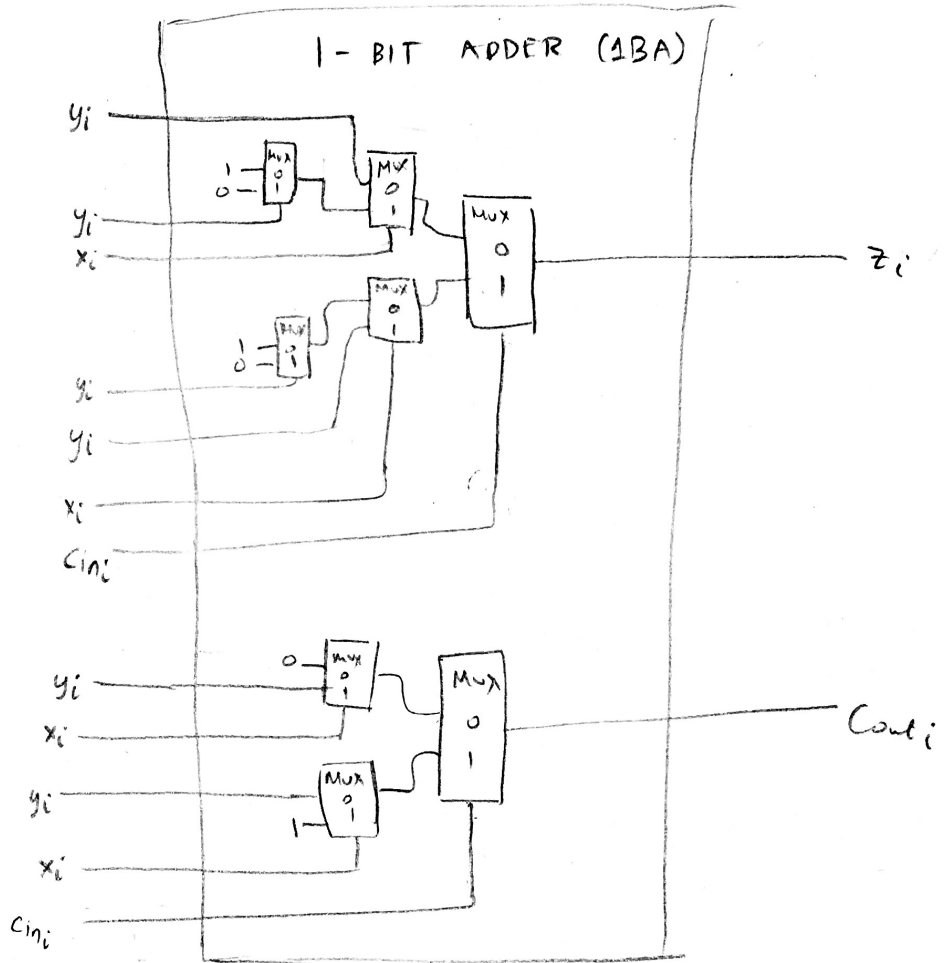
**Problem 3 (20 points)**

Four 4-bit numbers A,B,C, and D are given as inputs.  $E=A+B$ ,  $F=C+D$ . Design a system that outputs the larger number between E and F. If  $E=F$ , output either E or F. You can use any type of gates to implement your design.

$C_{in_i}$	$x_i$	$y_i$	$z_i$	$Count_i$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$z_i = C_{in_i}' (x_i' y_i + x_i y_i') + C_{in_i} (x_i' y_i' + x_i y_i)$$

$$Count_i = C_{in_i}' x_i y_i + C_{in_i} (x_i' y_i + x_i)$$

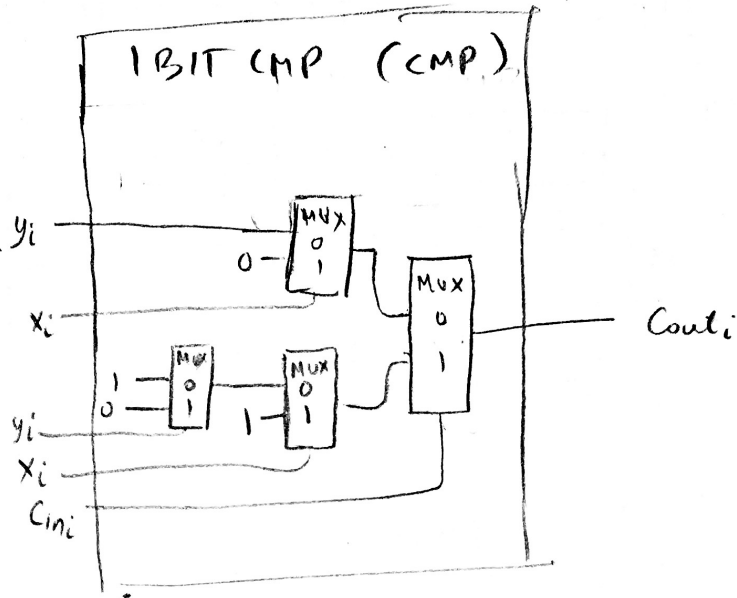


Problem 3) Extra Page

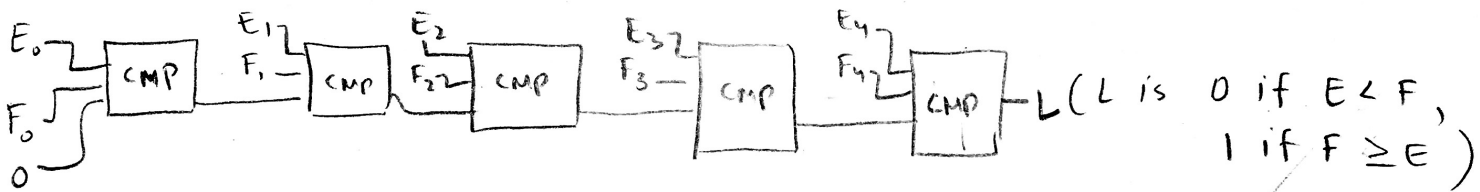
$$C_{in}/C_{out} = \begin{cases} 0 & \text{if } x > y \\ 1 & \text{if } y \leq x \end{cases}$$

$C_{in_i}$	$x_i$	$y_i$	$C_{out_i}$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

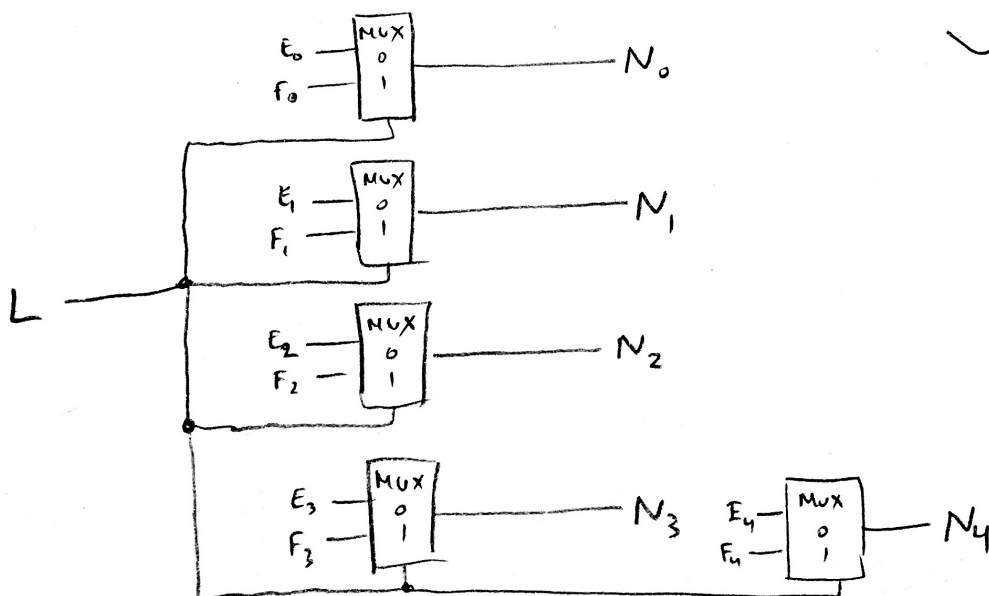
$$C_{out_i} = C_{in_i}'x_i'y_i + C_{in_i}(x_i'y_i' + x_i)$$



Using E and F as found on prev page:



OUTPUT: ( $N_4N_3N_2N_1N_0$  are digits of larger #)



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**Problem 4 (20 points)**

For a K-map,  $M$  denotes the number of prime implicants of the K-map, and  $N$  denotes the number of essential prime implicants of the K-map. Draw a  $4 \times 4$  K-map that has the largest value of  $P=M-N$  among all the  $4 \times 4$  K-maps.

For example, in the following  $4 \times 4$  K-map,  $M=3$ ,  $N=2$ ,  $P=M-N=1$ .

$$P = M - N$$

↑                    ↑  
P.I.                    E.P.I.

	$x_0$				
	0	0	0	0	
	1	1	0	0	$x_2$
$x_3$	1	1	1	0	
	0	0	1	0	
	$x_1$				

	$x_0$				
	0	1	1	1	
	1	1	0	1	$x_2$
$x_3$	1	0	1	0	
	1	1	1	1	
	$x_1$				

$N = 1$   
 $M = 8$

~~f 11~~

**Problem 5 (20 points)**

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Use only multiplexers to design a system with input  $x \in (0,1,2, \dots, 8)$ , outputs  $y$  and  $z$  that implements the following equation

$$(x)_{10} = (yz)_3$$

In the system,  $x$  is encoded as  $x_3x_2x_1x_0$  in binary.  $y$  is encoded as  $y_1y_0$  in binary, and  $z$  is encoded as  $z_1z_0$  in binary.

Note that the outputs  $y$  and  $z$  represent the two digits of a base-3 number.

For example, if  $x=7$  ( $x_3x_2x_1x_0=0111$ ), then the system will solve:  $(7)_{10} = (21)_3$ . Thus  $y = 2$  ( $y_1y_0=10$ ) and  $z = 1$  ( $z_1z_0=01$ ).

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	2	10	11	12	20	21	22	100	101	102	110	111	112	120

$x_3$	$x_2$	$x_1$	$x_0$	$y_1$	$y_0$	$z_1$	$z_0$
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	1	0	0	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	1	0

$$y_1 = x_3'x_2x_1 + x_3x_2'x_1'x_0'$$

$$y_0 = x_3'(x_2'x_1x_0 + x_2x_1')$$

$$z_1 = x_3(x_2'x_1'x_0' + x_2x_1'x_0) + x_3x_2'x_1'x_0'$$

$$z_0 = x_3'x_2'x_1'x_0 + x_3'x_2x_1'x_0' + x_3'x_2x_1x_0$$

$$= x_3'x_2'x_1'x_0 + x_3'x_2(x_1'x_0 + x_1x_0)$$

$$= x_3'[x_2'x_1'x_0 + x_2(x_1'x_0 + x_1x_0)]$$

