20

Problem 1 (20 points)

Use only the "E" gate defined below to implement Boolean function:
F=w'xy'+wxz+w'x'z+wx'y'z'
You may also use constants 0 and 1 as inputs.

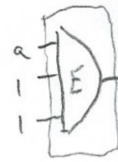
a	b	c	E(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$(a+b+c)(a'+b'+c')$

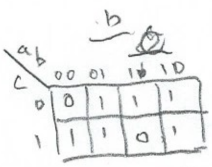
$ab'+ac'+a'b+bc'+a'c+b'c$

$E(a,1,1)=a'$ NOT

NOT



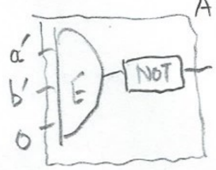
OR



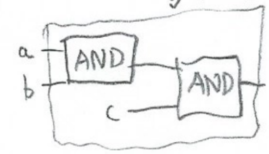
$ab'+bc'+a'c$

$E(a,b,0)=ab'+b=a+b$ OR

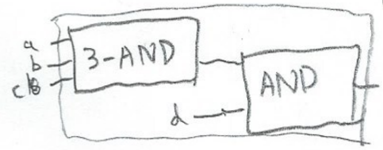
AND

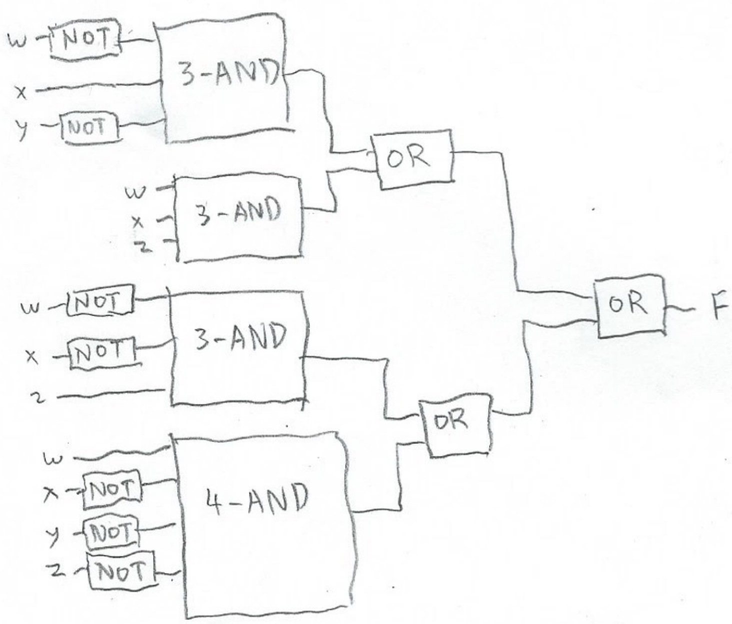


3-AND



4-AND



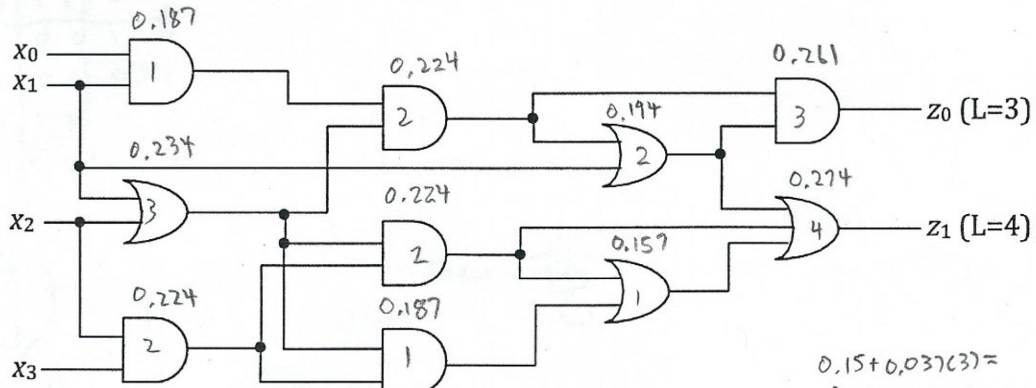


7

Problem 2 (20 points)

Given the network below, calculate the critical path delay. Consider L → H delay when calculating the critical path.

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$



0.15+0.037(1)=
0.187

0.12+0.037(2)=
0.12+0.074=
0.194

0.15+0.037(3)=
0.15+0.111=
0.261

0.12+0.038(3) =
0.12+0.114 =
0.234

0.12+0.037(1)=
0.157

0.12+0.038(4)=
0.12+0.152=
0.274

0.15+0.037(2) =
0.15+0.074 =
0.224

0.187
0.224
0.194
+ 0.274

~~0.234~~
~~0.224~~
~~0.194~~
~~+0.274~~
~~0.926~~

~~$\boxed{0.926}$~~

0.224
0.224
0.157
+ 0.274

0.231
0.224
0.194
0.274
0.923

$\boxed{0.923}$ X

+7

20

Problem 3 (20 points)

Four 4-bit numbers A,B,C, and D are given as inputs. E=A+B, F=C+D. Design a system that outputs the larger number between E and F. If E=F, output either E or F. You can use any type of gates to implement your design.

Full Adder:

c_{in}	x	y	z	c_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

z:

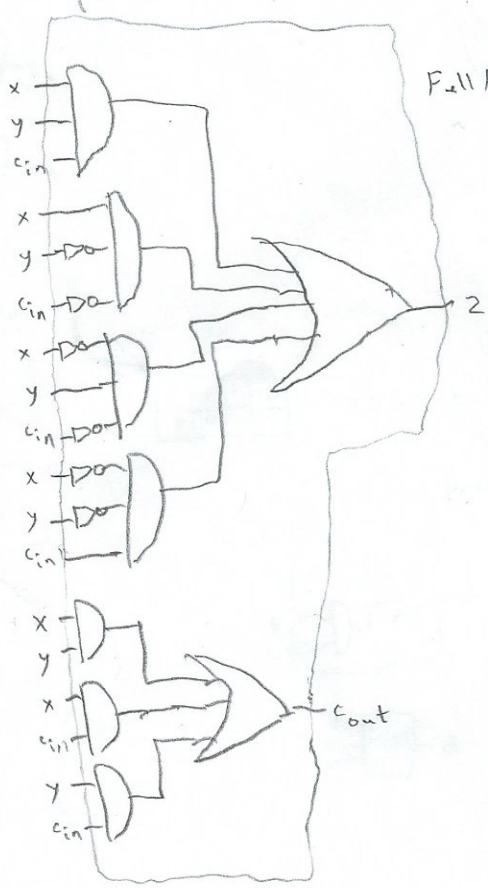
c_{in} \ xy				
0	0	1	0	1
1	1	0	1	0

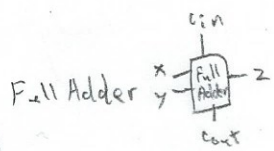
c_{out}:

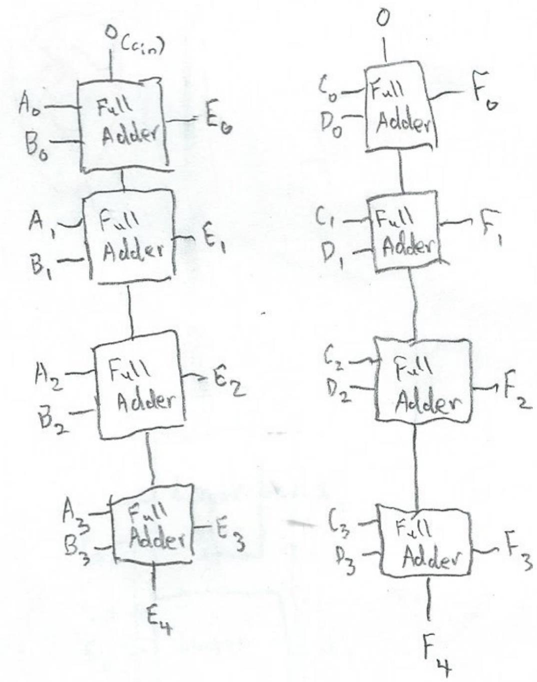
c_{in} \ xy				
0	0	0	1	0
1	0	1	1	1

$z = xyc_{in} + xy'c_{in}' + x'yc_{in}' + x'y'c_{in}$

$c_{out} = xy + xc_{in} + yc_{in}$







+7

Comparator:

c_{in}: 0 if x<y so far
1 if x>=y so far

z: x if c_{out}=1
y if c_{out}=0

c_{in}	x	y	z	c_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

z:

c_{in} \ xy				
0	0	1	1	1
1	0	1	1	1

c_{out}:

c_{in} \ xy				
0	0	0	0	1
1	1	0	1	1

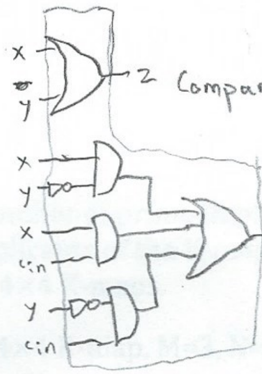
$z = x + y$

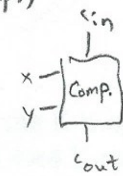
$c_{out} = xy' + xc_{in} + y'c_{in}$

Next Page

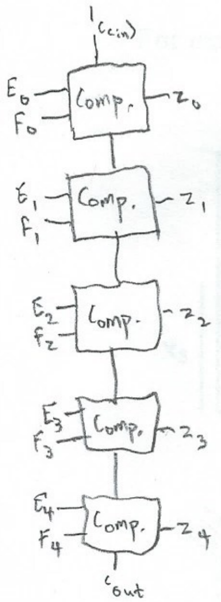
$z = x + y$

$c_{out} = xy' + xc_{in} + y'c_{in}$



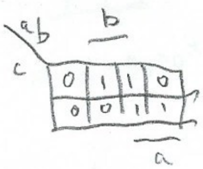


Problem 3) Extra Page



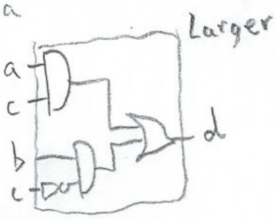
+7

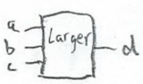
a	b	c	d
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

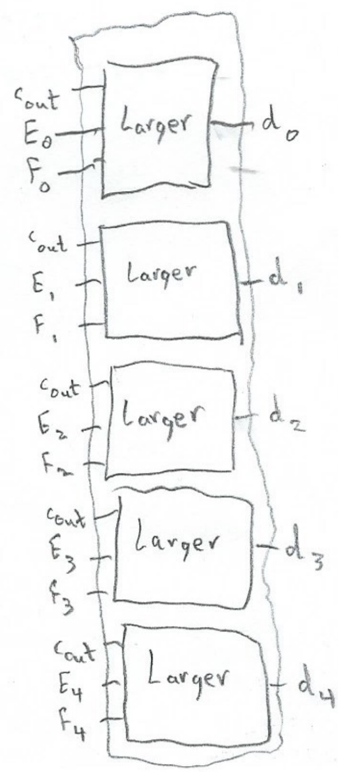


$d = ac + bc'$

+6







20

Problem 4 (20 points)

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of P=M-N among all the 4×4 K-maps.

For example, in the following 4×4 K-map, M=3, N=2, P=M-N=1.

X_0

0	0	0	0
1	1	0	0
1	1	1	0
0	0	1	0

X_2

X_3

X_1

x_2x_3 \ x_0x_1	00	01	11	10
00				
01				
11				
10				

$x_1'x_2 + x_1x_0x_3$

0	0	0	0
1	1	1	0
1	0	1	0
1	1	1	0

M=8
N=0
P=8-0=8

0	1	0	0
1	1	1	0
1	0	1	1
1	1	1	0

M=12
N=0
P=12-0=12

Problem 5 (20 points)

20

Use only multiplexers to design a system with input $x \in (0,1,2,\ldots 8)$, outputs y and z that implements the following equation

$$(x)_{10} = (yz)_3$$

In the system, x is encoded as $x_3x_2x_1x_0$ in binary. y is encoded as y_1y_0 in binary, and z is encoded as z_1z_0 in binary.

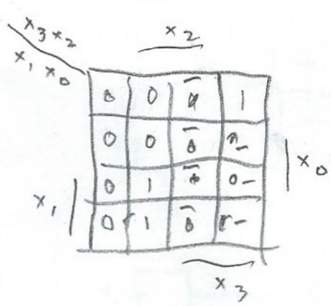
Note that the outputs y and z represent the two digits of a base-3 number.

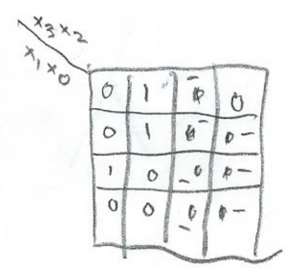
For example, if x=7 ($x_3x_2x_1x_0$=0111), then the system will solve: $(7)_{10} = (21)_3$. Thus $y = 2$ (y_1y_0=10) and $z = 1$ (z_1z_0=01).

x_3	x_2	x_1	x_0	y_1	y_0	z_1	z_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	1	0	0	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	1	0

~~$y_1 = x_3'x_2x_1x_0' + x_3x_2x_1x_0 + x_3x_2'x_1'x_0'$~~

~~$= x_3'x_2x_1 + x_3x_2'x_1'x_0'$~~

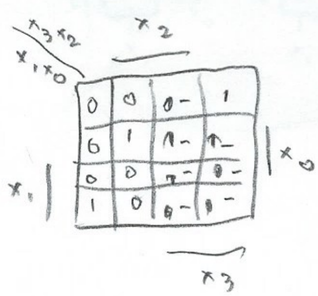


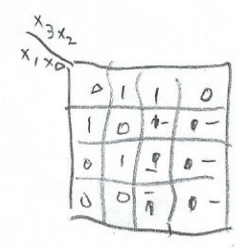


$y_1 = x_3 + x_2x_1$

$y_0 = x_1'x_2 + x_2'x_0x_1$

~~$y_0 = x_1'x_2 + x_1x_3 + x_0x_3 + x_2'x_0x_1$~~





$z_0 = x_2x_0'x_1' + x_0x_1x_2 + x_2'x_0x_1'$

~~$z_1 = x_3 + x_2'x_3'x_1' + x_2x_0x_1'$~~

$z_1 = x_3 + x_2'x_0'x_1 + x_2x_0x_1'$

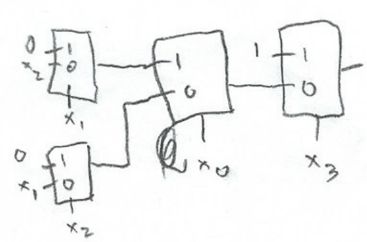
$f_{x_3} = 1$

$f_{x_3'} = x_2'x_0'x_1 + x_2x_0x_1'$

$f_{x_3'x_0} = x_2x_1'$: $f_{x_3'x_0x_1} = 0$, $f_{x_3'x_0x_1'} = x_2$

$f_{x_3'x_0'} = x_2'x_1$: $f_{x_3'x_0'x_2} = 0$, $f_{x_3'x_0'x_2'} = x_1$

Next Page



Replace z_1 portion of answer with this.

$y_1 = x_3 + x_2 x_1$

$y_0 = x_1' x_2 + x_2' x_0 x_1$

$z_1 = x_3 + x_2' x_3 x_1 + x_2 x_0 x_1'$

$z_0 = x_2 x_0' x_1' + x_0 x_1 x_2 + x_2' x_0 x_1'$

Problem 5) Extra Page

y_1

$f_{x_3} = 1$

$f_{x_3'} = x_2 x_1 < \begin{matrix} f_{x_3' x_2} = x_1 \\ f_{x_3' x_2'} = 0 \end{matrix}$

y_0

$f_{x_1} = x_0 x_2' < \begin{matrix} f_{x_1 x_2} = 0 \\ f_{x_1 x_2'} = x_0 \end{matrix}$

$f_{x_1'} = x_2$

z_1

$f_{x_3} = 1$

$f_{x_3'} = x_2 x_0 x_1' < \begin{matrix} f_{x_3' x_0} = x_2 x_1' < \begin{matrix} f_{x_3 x_0 x_1} = 0 \\ f_{x_3 x_0 x_1'} = x_2 \end{matrix} \\ f_{x_3' x_0'} = 0 \end{matrix}$

z_0

$f_{x_2} = x_0' x_1' + x_0 x_1 < \begin{matrix} f_{x_2 x_0} = x_1 \\ f_{x_2 x_0'} = x_1' < \begin{matrix} f_{x_2 x_0' x_1} = 0 \\ f_{x_2 x_0' x_1'} = 1 \end{matrix} \end{matrix}$

$f_{x_2'} = x_0 x_1' < \begin{matrix} f_{x_2' x_1} = 0 \\ f_{x_2' x_1'} = x_0 \end{matrix}$